



Magnetic field in electrically  
conductive medium (fluid, gas)

Electric field

$$\vec{E} = \underbrace{-\vec{v} \times \vec{B}}_{\text{convective}} + \underbrace{\frac{1}{\sigma} \vec{j}}_{\text{ohmic}} \quad \dots (1)$$

$$= -\vec{v} \times \vec{B} + \frac{1}{\mu_0 \sigma} \nabla \times \vec{B} \quad \dots (2)$$

(Ampère's law)

Induction eq.

$$\partial_t \vec{B} = -\nabla \times \vec{E} \quad \dots (3)$$

Combine (2) and (3) and  $\nabla \cdot \vec{B} = 0$

↘

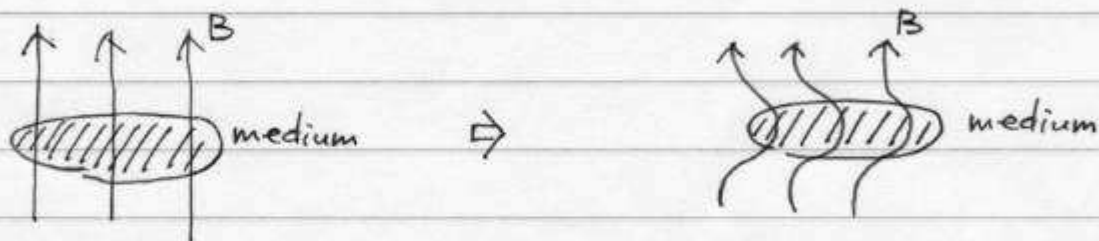
$$\partial_t \vec{B} = \nabla \times (\vec{v} \times \vec{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \vec{B} \quad \dots (4)$$

Evolution of  
mag. field

(A) frozen-in

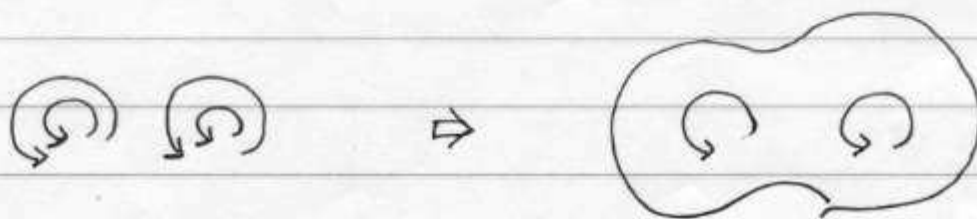
(B) diffusion

## (A) Frozen-in magnetic field



Magnetic field moves with flow/medium.

## (B) Diffusion



Magnetic field becomes weaker.

Space plasma ... collisionless, conductive medium.  
Conductivity  $\sigma$  very large.

↘ Frozen-in magnetic field

Consequences

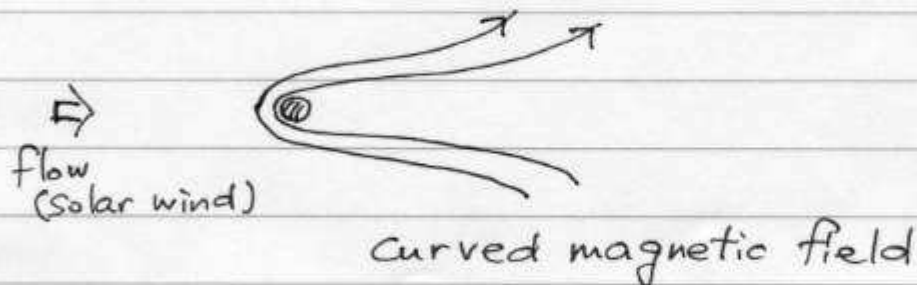
(1) Magnetic field moves with flow  
(e.g. solar wind)

(2) Different media cannot be mixed.

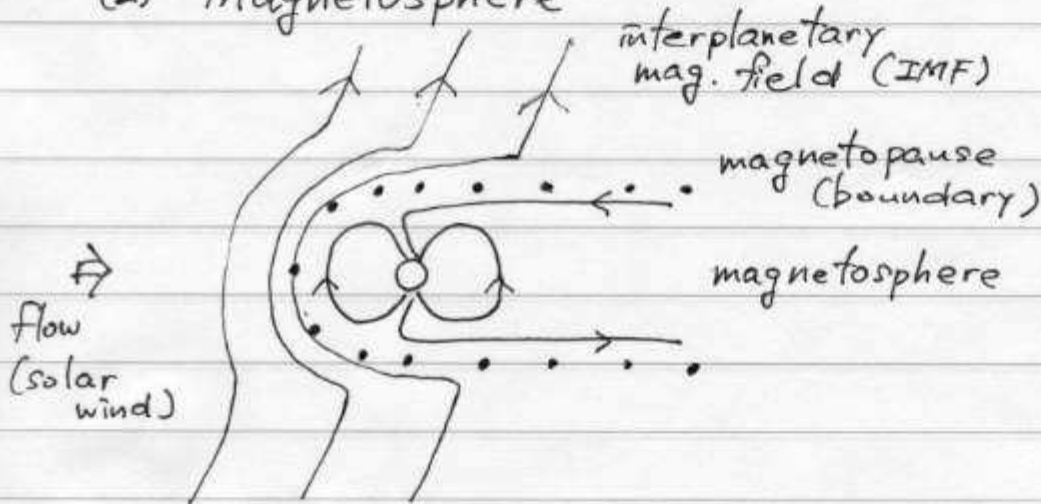
↘ Formation of boundaries  
(e.g. magnetopause)

## Applications

(1) Comet tail (Alfvén's idea, 1957)



(2) Magnetosphere



How large is magnetosphere?

Pressure balance

$$\frac{1}{2} \rho v^2 = \frac{B^2}{2\mu_0} \quad \dots (5)$$

dynamic pressure	=	magnetic pressure
(kinetic energy density)		(mag. energy density)

solar wind

planet

## Dipole magnetic field (for planet)

$$B \propto \frac{1}{r^3}$$

$$\Rightarrow \frac{B}{B_{\text{surf}}} = \left( \frac{R_{\text{surf}}}{R_{\text{mp}}} \right)^3 \quad \dots (6)$$

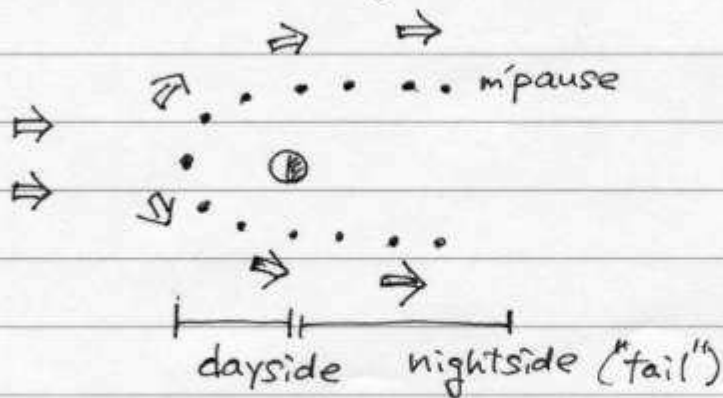
Combine (5) and (6)

$$\Rightarrow \frac{R_{\text{mp}}}{R_{\text{surf}}} = \left( \frac{B_{\text{surf}}}{\mu_0 \rho v^2} \right)^{1/6} \quad \dots (7)$$

## Classification of planets

	$R_{\text{mp}} > R_{\text{surf}}$	$R_{\text{mp}} < R_{\text{surf}}$
Solid surface	Mercury	Earth Moon
gas surface (atmosphere)	Earth Jupiter, Saturn Uranus, Neptune	Venus, Mars

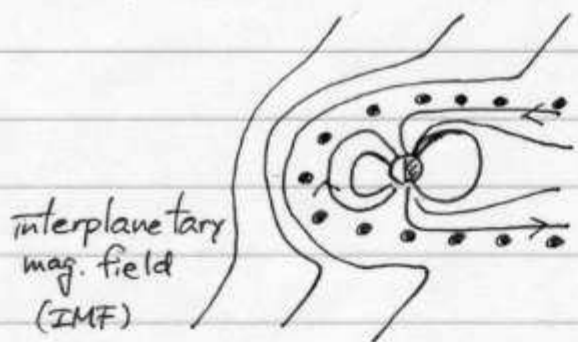
• Flow around magnetosphere



Surface mag. field 30 000 nT

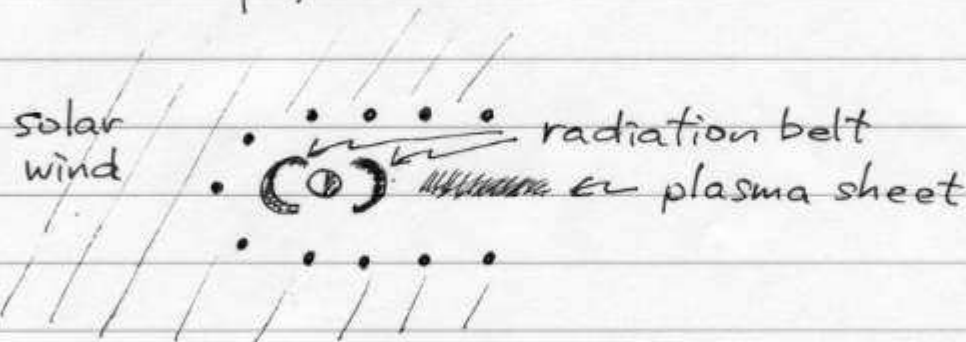
Magnetopause distance ... 10 ~ 11  $R_E$  (dayside)

• Magnetic field



solar wind ... curved mag. field  
 dayside magnetosphere  
 ... dipolar field  
 nightside ... elongated / stretched field

• Plasma populations

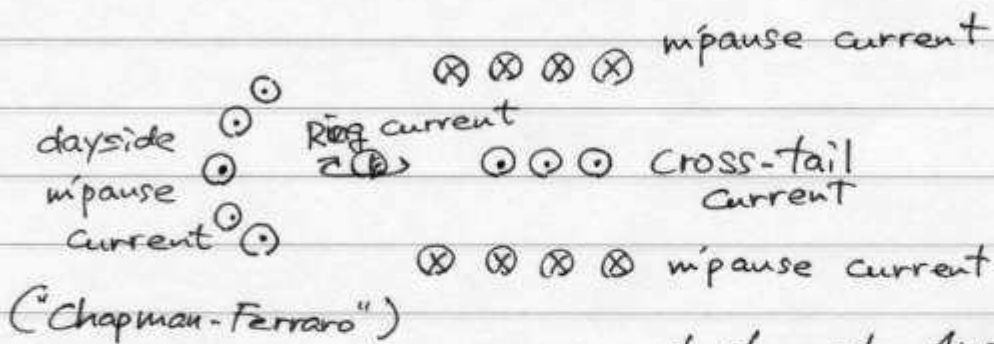


solar wind

radiation belt

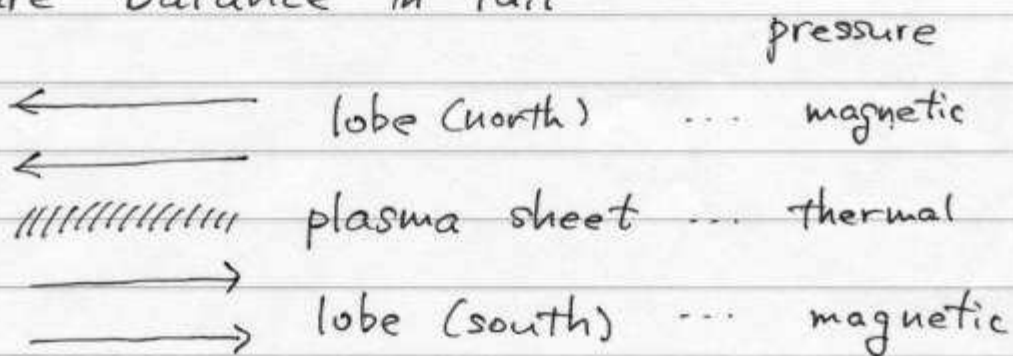
plasma sheet

• Electric current



consistent with Ampère's law  
~~discontinuous current~~  $\vec{j} = \frac{1}{\mu_0} \nabla \times \vec{B}$

• Pressure balance in tail

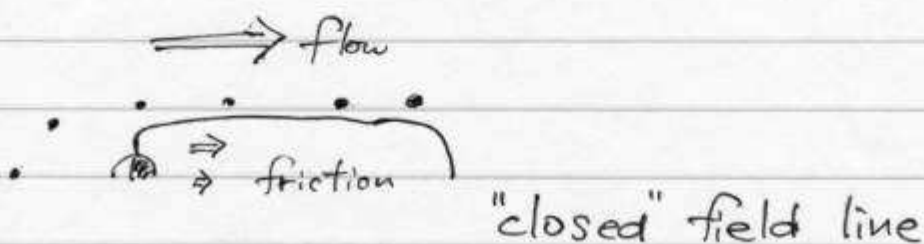


$$P_{\text{th plasma sheet}} = P_{\text{mag lobe}}$$

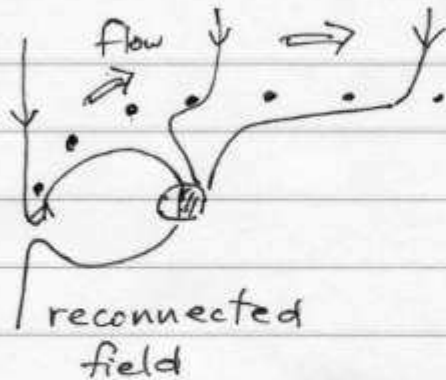
• Why is tail-field stretched?

possibility 1 ... viscosity/friction

(Axford and Hines, 1961)

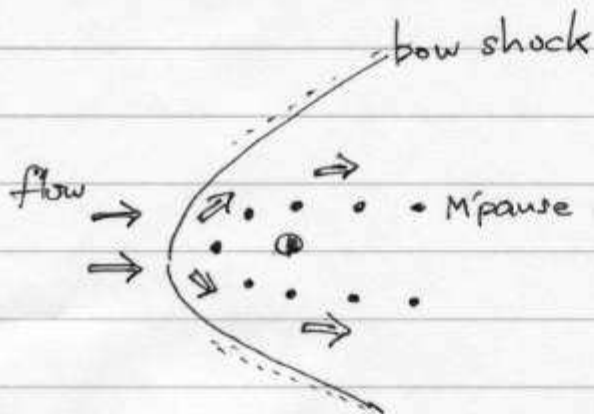


possibility 2 ... reconnection (Dungey 1961)



"open" field line

• bow shock



- standing shock wave
- misphere as obstacle
- flow speed changes from super-magnetosonic to sub-magnetosonic

cf. magnetosonic speed

$$V \sim \sqrt{V_A^2 + C_s^2}$$

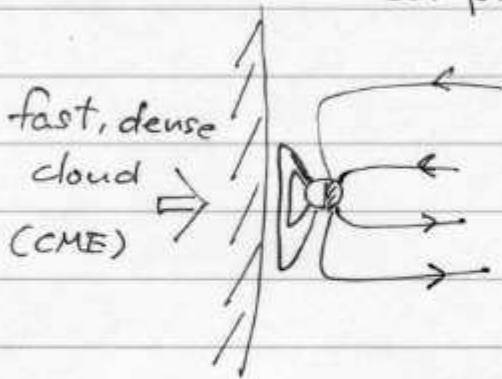
- plasma is <sup>(1)</sup> compressed, <sup>(2)</sup> heated

- magnetic field also compressed



Geomagnetic storm

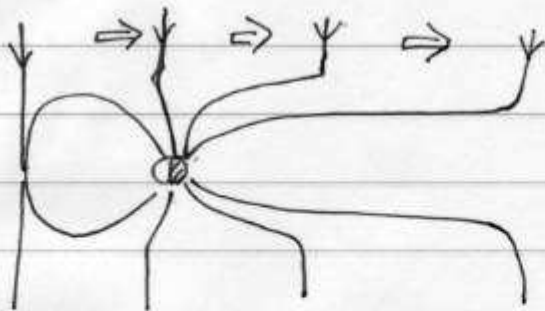
... compression of magnetosphere  
(Chapman, 1930)



1. dayside compression ~ minutes
2. nightside compression ~ hours
3. recovery phase (expansion) ~ days

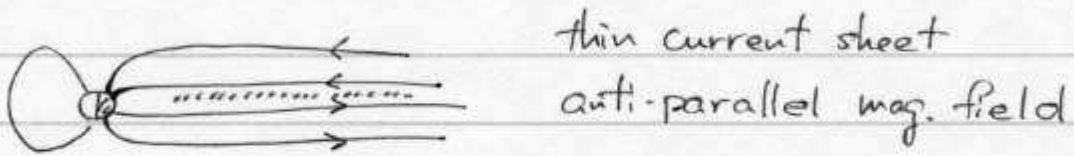
Substorm (reconnection model)

1. growth phase ( $T \sim 40$  min.)

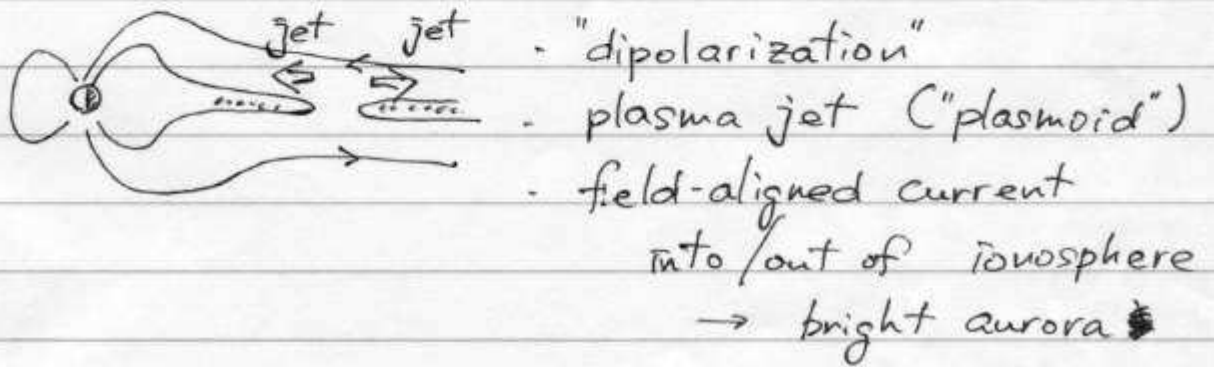


- magnetic field transport from dayside to tail (energy charge in tail)
- reconnection on dayside (southward IMF)

2. expansion phase ( $T \sim 10$  min.)

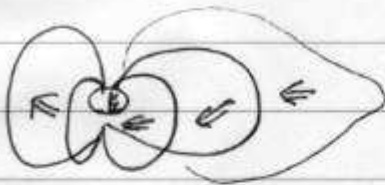


⇓ tail reconnection (energy discharge)

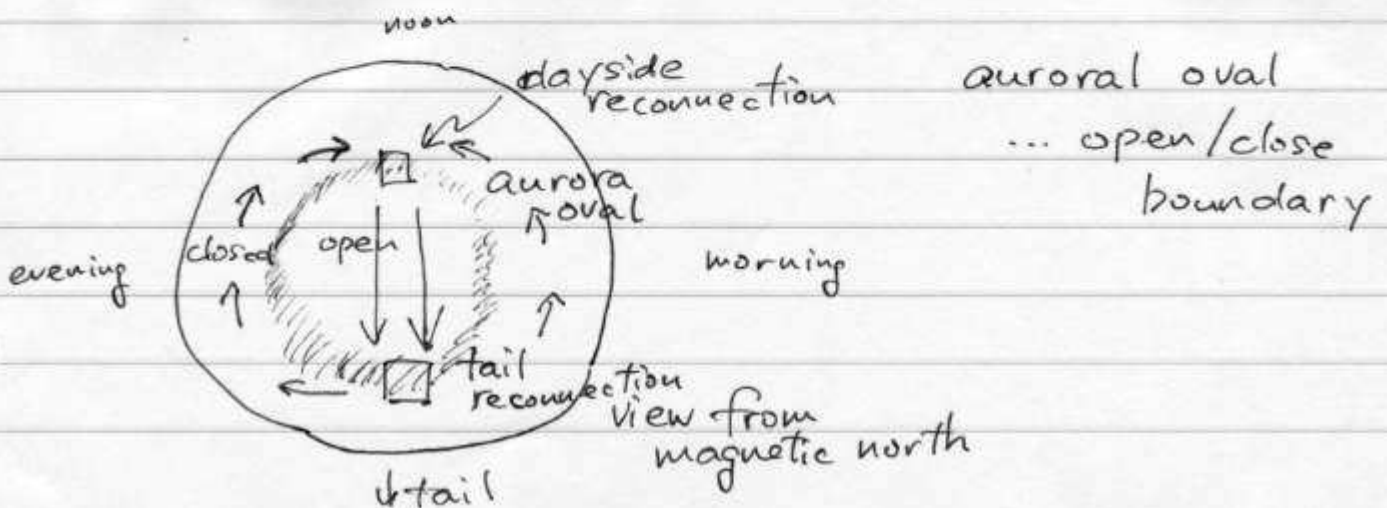


3. recovery phase ( $T \sim$  hours)

- magnetic field transport back to dayside



ionospheric convection pattern (southward IMF)



Magnetized, non-atmospheric body.

Surface magnetic field  $\sim 200 \text{ nT}$

(Mariner 10, Messenger)

cf.  $30000 \text{ nT}$  at Earth surface

Magnetopause distance

$$\frac{1}{2} \rho v^2 = \frac{B^2}{2\mu_0} \quad (\text{pressure balance})$$

$$B \propto \frac{1}{r^3} \quad (\text{dipole field})$$

$$B_0 = 200 \text{ (nT)} \quad (\text{surface})$$

$$\rho = m_p \times 20 \text{ (cm}^{-3}\text{)} \quad (\text{solar wind mass density})$$

$$v = 300 \text{ (km/s)} \quad (\text{solar wind speed})$$

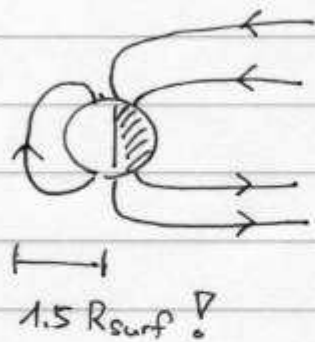
$$\rightarrow R_{mp} \approx 1.5 R_{surf} \quad !$$

(from planet center)

surface - magnetopause distance

$$0.5 R_{surf} = 1200 \text{ km}$$

## Magnetosphere



weak magnetic field



small magnetosphere

- No radiation belt (or ring current)
  - ... planet <sup>occupies</sup> ~~is occupying~~ a lot of volume
- Sodium (Na) sputtering
  - sodium-rich plasma
- "Hybrid" scale ... magnetosphere size of the order of ion gyro-radius
  - currents are carried by electrons? (mpause, plasma sheet)
- ~~Fast~~ Fast reaction, rapid reconfiguration

length dayside-tail  $l \sim 10 R_{surf}$  (24000 km)

solar wind speed  $v \sim 300$  km/s

↘

reaction time  $\tau = l/v \sim 80$  s

(cf. 40 min. at Earth)

- reconnection? substorm?

## Dynamo problem

Theoretical estimate of surface magnetic field

$$\Lambda = \frac{\text{Lorentz force}}{\text{Coriolis force}}$$

$$= \frac{|\sigma(\vec{v} \times \vec{B}) \times \vec{B}|}{|2\rho\vec{\Omega} \times \vec{v}|}$$

$$\sim \frac{\sigma B^2}{2\rho\Omega}$$

Assume  $\Lambda \sim 1 \Rightarrow B \propto \sqrt{\Omega}$ .

Scale Earth magnetic field into Mercury,

$$B(\text{earth}) = 30000 \text{ nT}$$

$$\Omega(\text{earth}) = \frac{2\pi}{1} \text{ (rad/day)}$$

$$\Omega(\text{merc}) = \frac{2\pi}{58} \text{ (rad/day)}$$

$$\Rightarrow B(\text{merc}) = 4000 \text{ nT.}$$

Mercury field is weaker than  
Earth-scaled field.

## Lost dynamo

Venus and Mars have atmosphere  
but no global magnetic field.  
(dipole)

Estimate of dipole moment (upper limit!)

Venus  $10^5 \times$  (Earth dipole)  
 $\rightarrow$  0.1 nT at surface pvo

Mars  $10^{15} \times$  (Earth dipole)  
 $\rightarrow$  ~~100~~ 0.5 nT at surface MGS

Mars have magnetized crust  
e.g. 200 nT at 400 km altitude

$\rightarrow$  Dynamo operating in early time?

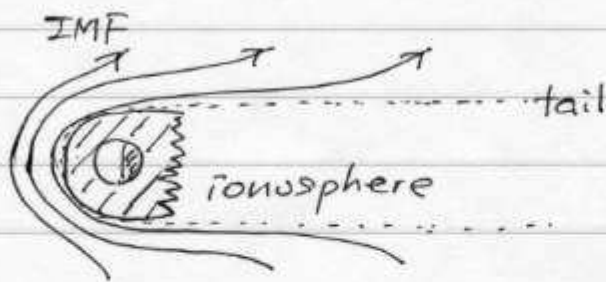
Why is dynamo missing?

Venus ... slow rotation (243 days)?

Mars ... core in solid state?

(Liquid core became solid,  
Small planet size)

# Magnetosphere



## Pressure balance

(1) magnetic boundary

$$\frac{1}{2} \rho v^2 = \frac{B^2}{2\mu_0}$$

solar wind  
dynamic  
pressure

solar wind  
magnetic pressure

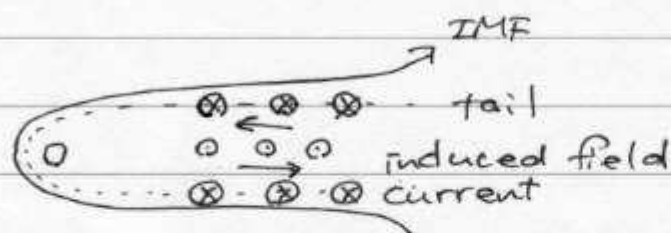
↕  
field compression  
on dayside

(2) ionopause

$$\frac{1}{2} \rho v^2 = nkT$$

ionosphere  
gas pressure

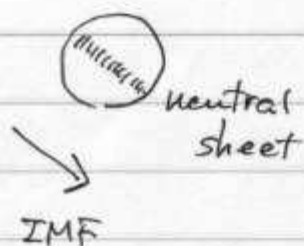
# Induced magnetic field



↳ "induced magnetosphere"

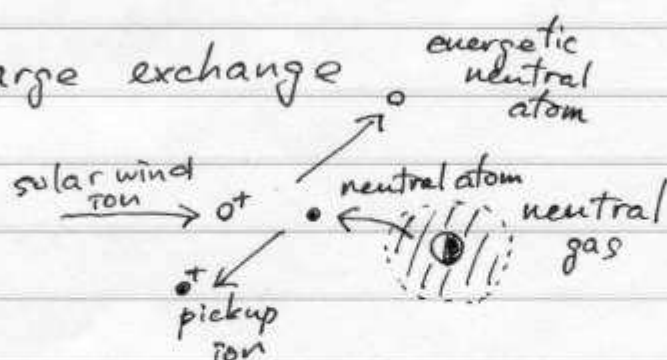
sensitive to IMF direction

tail cross section

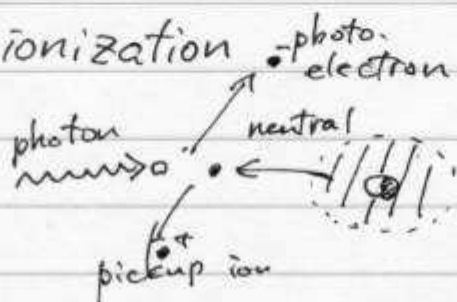


## Pickup process

(1) charge exchange



(2) photoionization



plasma gains additional mass



Giant magnetosphere

surface magnetic field

400 000 nT (Jupiter)

20 000 nT (Saturn)

metallic hydrogen core?

magnetopause distance

50 - 100  $R_{surf}$  (Jupiter) ca. 5  $R_{sun}$ !

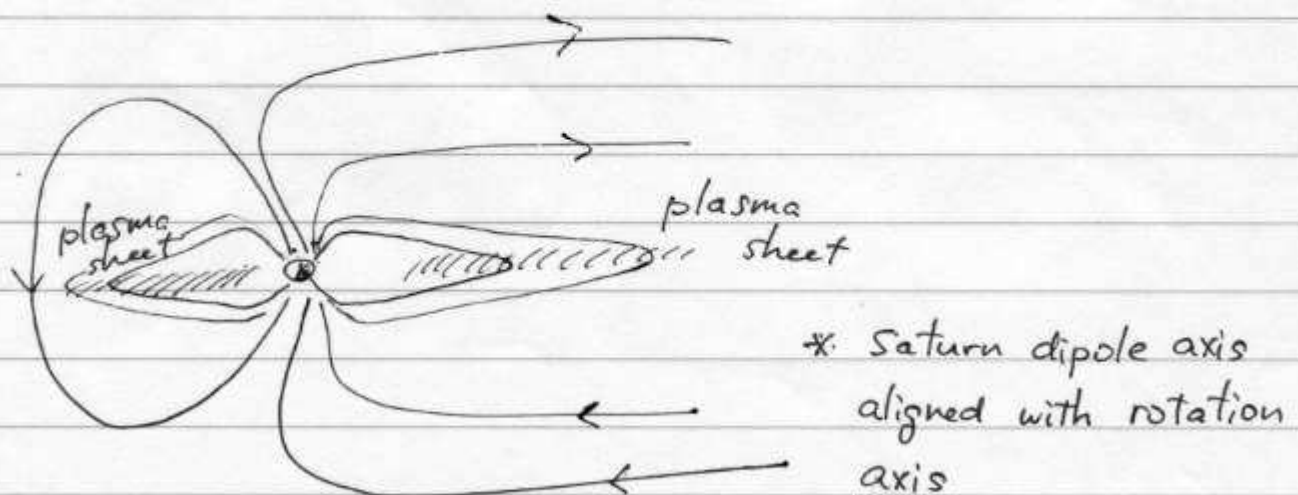
20  $R_{surf}$  (Saturn) ca. 2  $R_{sun}$

rotation - dominant

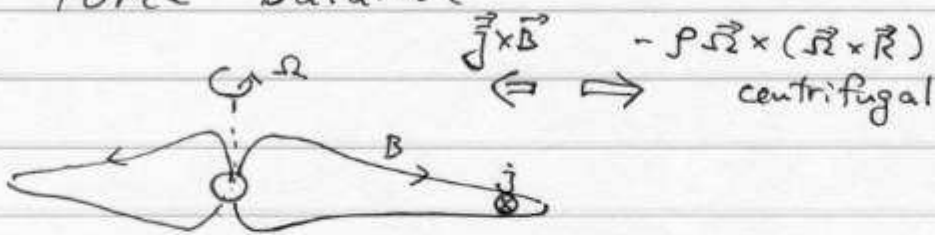
10 hours (Jup.), 10.5 hours (Sat.)

→ large centrifugal force

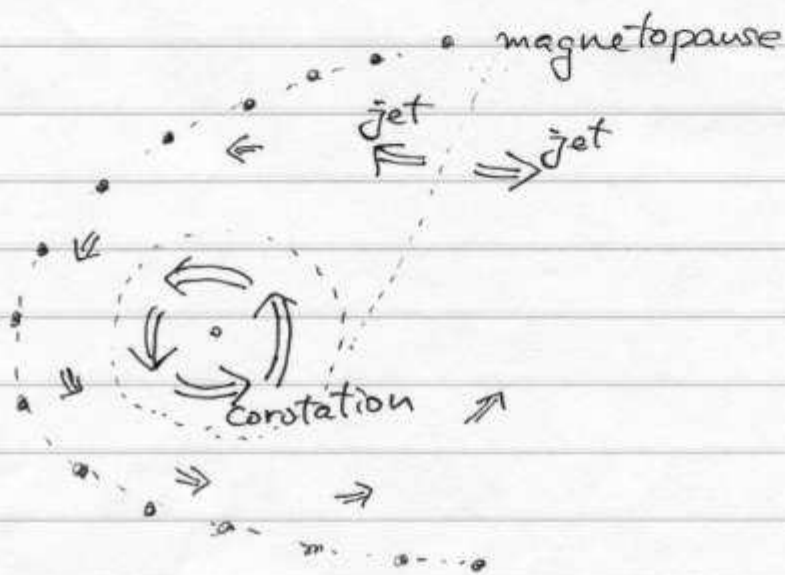
aurora ... magnetosphere-ionosphere coupling.  
radio emission source



## Force balance



## View from rotation axis (Jupiter)



periodic jet → substorm? reconnection?  
 on morningside ⚡  
 internally driven substorm?

## Plasma source from satellites

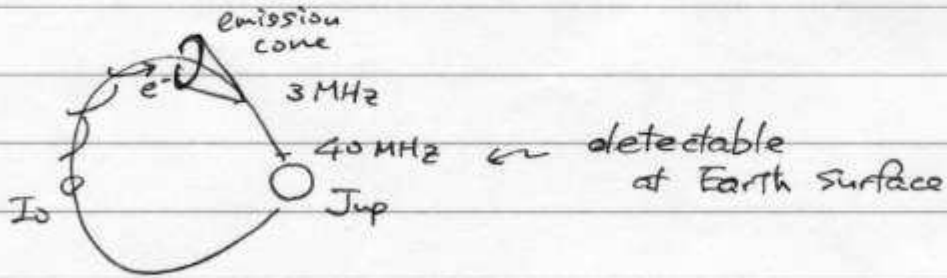
Io (Jup.) ... volcanism, sulfur-rich plasma  
 torus

Enceladus (Sat.) ... water ice, water-ion products

## Radio emission

- Auroral emission (cyclotron maser)
- radiation belt (synchrotron)

### Jupiter aurora emission



Uranus and Neptune

Magnetized, gas planets

Surface mag. field 23 000 nT (U), 14 000 nT (N)

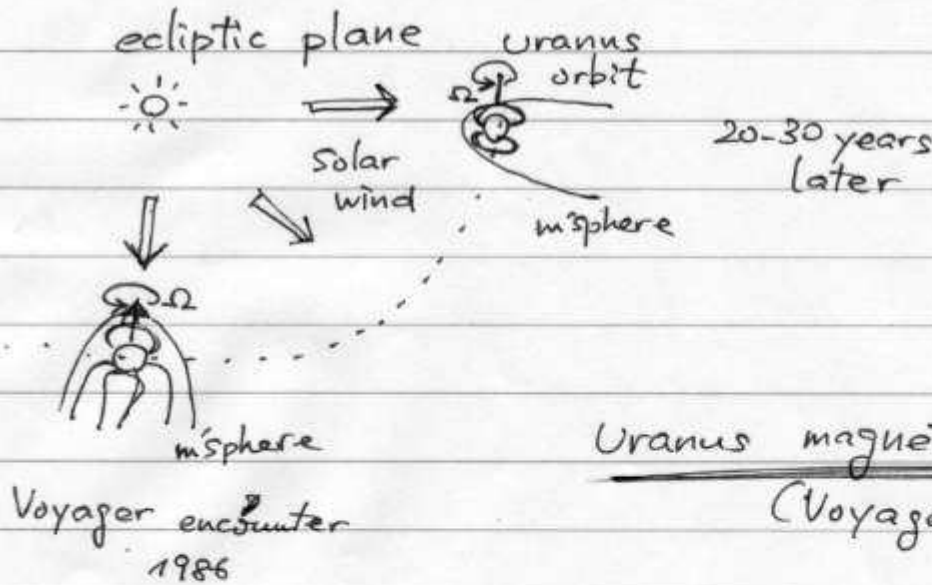
magnetopause distance 25  $R_{surf}$  (U) 26  $R_{surf}$  (N)

Large tilt angle (rotation axis - dipole axis)

59° (U), 47° (N)

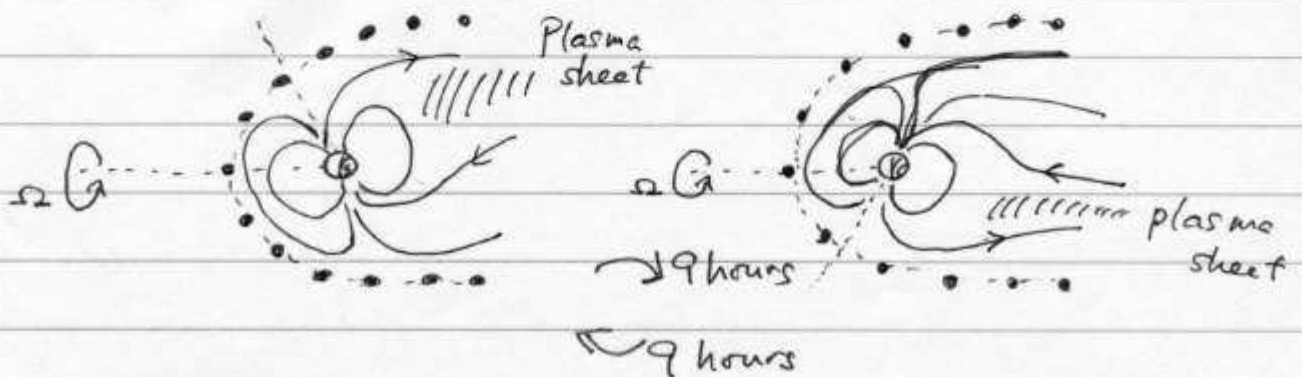
Uranus

Rotation axis almost in the orbital plane



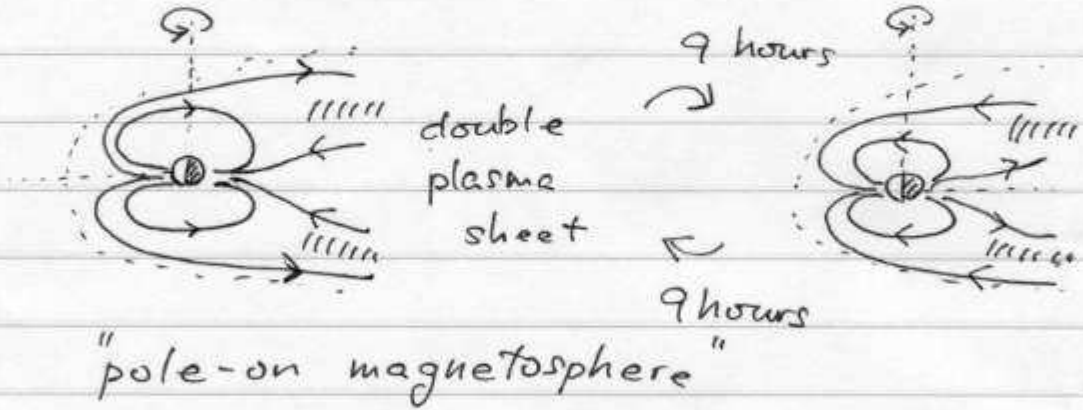
Uranus magnetosphere (1)

(Voyager encounter 1986)



Uranus magnetosphere (2)

20-30 years later



Neptune

