A study on diffusion plateaus of protons in the fast solar wind

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Abstract

Prominent features of velocity distribution functions of ions in the fast solar wind such as temperature anisotropies and relative streaming are commonly explained by the interaction of the solar wind particles with plasma waves. These plasma waves are thought to be of solar origin and to propagate away from the sun parallel to the interplanetary magnetic field. The interaction of waves and particles is described by the mean field approach to the Vlasov-Maxwell equations. Assuming weak turbulence, the linear closure leads to the equations of quasi-linear diffusion of particles in velocity space. The turbulent and mean field scales must be separate. But it is unclear whether weak turbulence or scale separation hold in the solar wind. The validity of quasi-linear theory in the solar wind and of the assumptions underlying it is investigated by comparison of measured velocity distribution function of ions with self-consistently determined resonant diffusion plateaus. In this respect, we have studied several thousands of proton velocity distribution functions from fast solar wind streams between 0.3 AU and 1 AU. These velocity distribution functions have been measured on Helios 2 during the solar minimum 1976.

Resonant velocity space diffusion

- The equations of resonant quasi-linear diffusion of particles interacting with waves propagating parallel to the magnetic field may be summarized as follows

$$\partial_t f(v_\parallel, v_\perp) = \frac{1}{v_\perp}\,\partial_a\, v_\perp\, D_{ab}\,\partial_b\, f(v_\parallel, v_\perp), \quad a,b = \parallel, \perp$$

$$D_{ab} = \frac{\pi}{2}\left(\frac{e}{m_p}\right)^2 \int \frac{d\omega\, dk_\parallel}{\sqrt{2\pi}}\;\delta(\omega - k_\parallel v_\parallel - \Omega_p)\, P_E(\omega, k_\parallel)\;\vartheta_a\,\vartheta_b$$

$$\vartheta_\parallel = v_\perp / v_{\mathrm{ph}}, \quad \vartheta_\perp = 1 - v_\parallel / v_{\mathrm{ph}}\,.$$

- The magnetic field is a symmetry axis of the velocity distribution function. Electrostatic waves are neglected and only cyclotron waves are retained. The wave spectrum P_E is supposed to fulfill a linear dispersion relation.

Diffusion time scale and diffusion plateaus

- It can be argued that the time scale of resonant diffusion $\tau_d \approx 100$ is much smaller than all other time scales on which the velocity distribution function might evolve.
- The velocity distributions are nearly constant along the direction $\vartheta(v_\parallel, v_\perp)$ of diffusion if the resonant wave mode correponding to $v_\parallel$ is sufficiently excited.
- These diffusion plateaus coincide with the level lines of the function

$$P(v_\parallel, v_\perp) = v_\parallel^2 + v_\perp^2 - 2\int^{v_\parallel} dv'_\parallel\, v_{\mathrm{ph}}(v'_\parallel)$$

which solely depends on the dispersion relation.

Scope of this study

- The dispersion relation of cyclotron waves is calculated from the measured velocity distribution functions. The generic warm plasma dispersion relation is applied

$$\omega^2 - c^2 k_\parallel^2 + \sum_s \omega_{ps}^2 / n_s \int dv_\parallel\, \frac{k_\parallel/2}{\omega - v_\parallel k_\parallel - \Omega_p}\,\Big(\partial_\parallel f_1(v_\parallel) - 2(v_{\mathrm{ph}} - v_\parallel) f_0(v_\parallel)\Big) = 0$$

$$f_n(v_\parallel) = \int_0^\infty dv_\perp\, v_\perp^{2n+1}\, f(v_\parallel, v_\perp)\,.$$

- The electron particle density and mean velocity are choosen such that quasi-neutrality and a vanishing electric current density are guaranteed. The electron temperature is taken from measurement. Finally, the electron velocity distribution function is assumed to be Maxwellian.

Ansatz for the proton velocity distribution functions

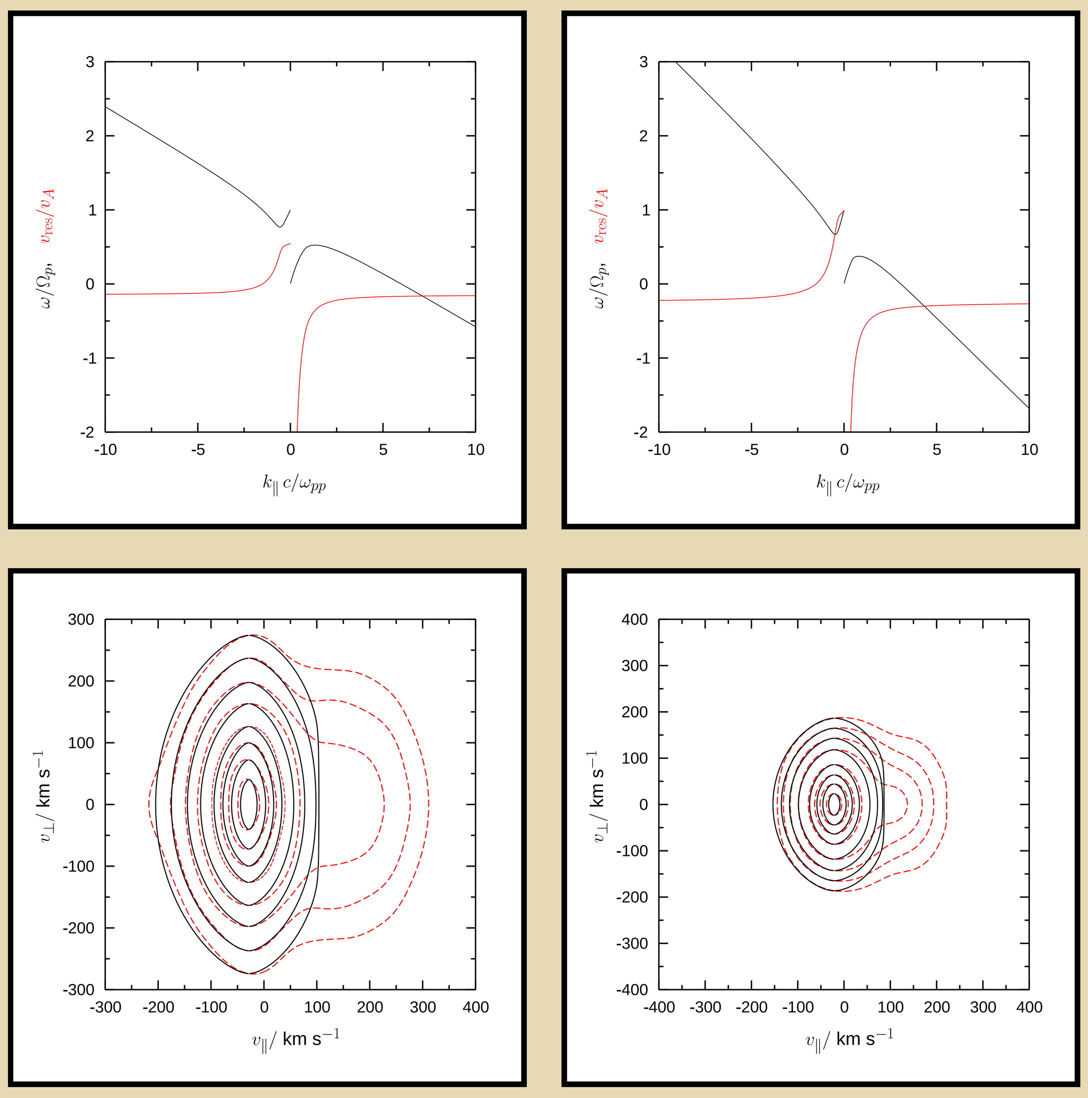
- The proton velocity distribution functions are modeled as superpositions of gaussian functions

$$f(\mathbf{v}) = \sum_{i=1}^{N} \frac{F_i}{h^3}\, e^{-(\mathbf{v}-\mathbf{v}_i)^2/2h^2}\,.$$

- The velocities $\mathbf{v}_i$ are given by the channels of the instrument which was used to measure the velocity distributions. The smoothing parameter h and the expansion coefficients F_i are fitted to the measured values of the velocity distribution functions.

Two exemplary cases

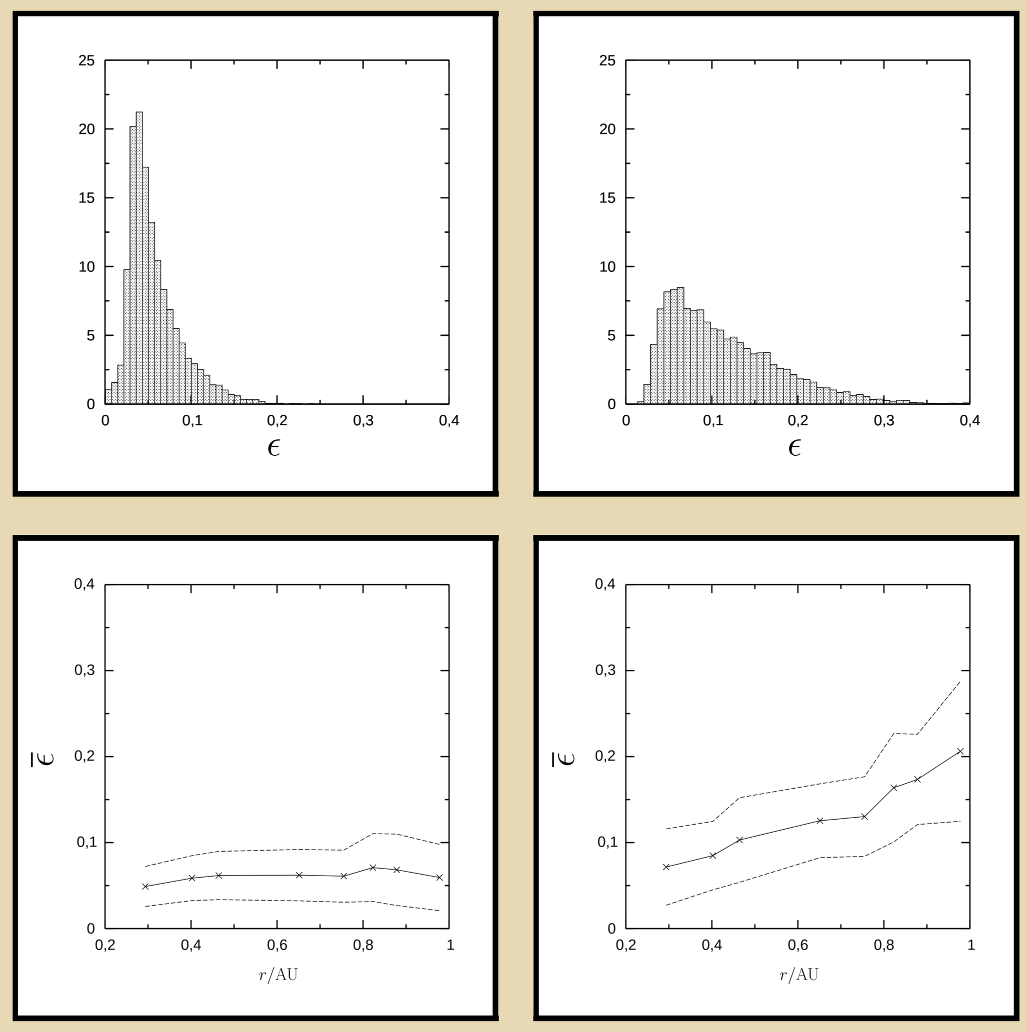
$r = 0.3$ AU, $v_A = 191.1$ km s^{-1} $v_{\mathrm{sw}} = 664$ km s^{-1} $\qquad$ $r = 0.74$ AU, $v_A = 88.1$ km s^{-1} $v_{\mathrm{sw}} = 627$ km s^{-1}



- There are two unique branches of cyclotron waves which resonate with the protons in the core of the velocity distribution function. These branches are shown in the upper figures together with the wave number dependency of the resonance velocity $v_{\mathrm{vres}} = (\omega - \Omega_p)/k_\parallel$ for two measured proton velocity distribution functions
- The lower figures show diffusion plateaus deduced from these dispersion branches. The level lines corresponding to fractions 0.9, 0.7, ..., 0.1, 0.03, 0,01 and 0.003 of the maximum of each symmetrized velocity distribution function are superimposed as dashed red curves.
- Both distribution functions have a proton beam. The wave modes of the considered dispersion branches cannot resonate with it.

Statistical results

anti-beamward plateaus $\qquad$ beamward plateaus



- In order to estimate the deviation of a velocity distribution function from the self-consistent diffusion plateaus we have averaged each distribution function along the plateaus within a sufficiently large range of $P(v_\parallel, v_\perp)$. The relative distance ϵ (in the sense of square integrable functions) of a distribution function to its plateau-averaged version is used as a quantitative measure.
- The upper figures show normalized histograms of ϵ for nearly 11000 proton velocity distribution functions from eight distinct fast solar wind streams. In the lower figures, the avarage value of ϵ in each stream is plotted against the distance to the sun where the measurements were made.
- The exemplary proton distribution functions deviate from their diffusion plateaus by $\epsilon = 0.05$ (anti-beamward), $\epsilon = 0.07$ (beamward) at 0.3 AU and $\epsilon = 0.04$ (anti-beamward), $\epsilon = 0.12$ (beamward) at 0.74 AU.

Conclusions

The level lines of the exemplary proton velocity distribution functions show a qualitatively good agreement with the corresponding resonant diffusion plateaus on both the beamward and anti-beamward side of the core. In this respect, they conform to the mean of all investigated measurements. Whereas the deviation of the proton velocity distributions from the diffusion plateaus on the anti-beamward side of the core does not change with distance to the sun, it clearly increases on the beamward side. Large deviations from the diffusion plateaus occur more often on this side of the core. Our findings confirm that the quasi-linear theory of wave-particle interaction does apply to the fast solar wind. The fast solar wind is weakly turbulent and scale separation holds, among the other assumptions underlying the existence of diffusion plateaus. In particular, the resonant wave modes of both considered branches of cyclotron waves are excited. The spectral energy density of the anti-beamward (inward) propagating wave modes does not suffice to to keep the deviation from the corresponding diffusion plateaus constant, though.