From Measurement to Scientific Data Analysis

IMPRS Lecture Series “Space Instrumentation”

Alex Feller

2010-10-25

Outline

What is a measurement?

"Classical" definition

The process of estimating or determining the magnitude of a physical quantity relative to a unit of measurement.

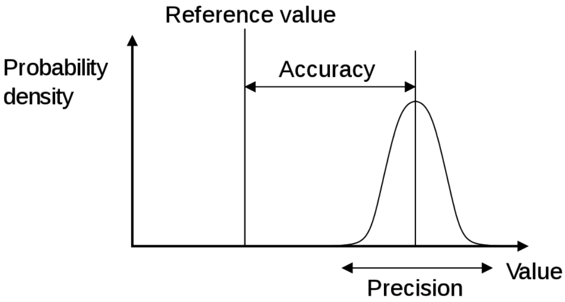
Representational theory

The correlation of numbers with entities that are not numbers.

Information theory

A set of observations that reduce uncertainty where the result is expressed as a quantity.

Accuracy vs. precision



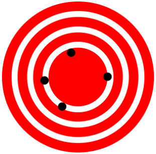
Accuracy

- Systematic errors
- Can be improved by calibration

Precision

- Random errors (noise)
- Can be improved by repeated measurements

Accuracy vs. precision



Accurate but not precise

Precise but not accurate

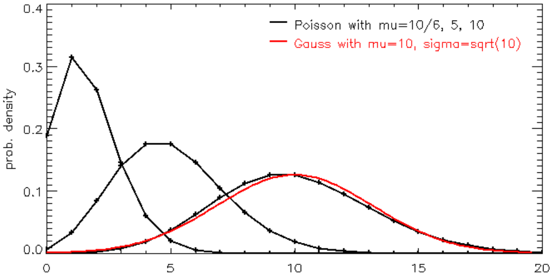
Accuracy

- Systematic errors
- Can be improved by calibration

Precision

- Random errors (noise)
- Can be improved by repeated measurements

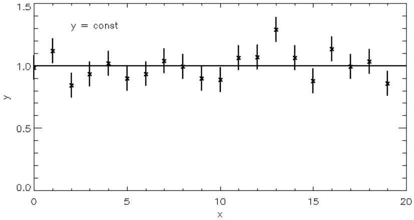
Gaussian and Poisson error distributions



Poisson distribution

- Photon noise!
- From "dice game" (binomial dist.) to radiative transition (Poisson dist.)
- $P_p(x;\mu) \rightarrow P_G(x;\mu,\sigma=\sqrt{\mu})$ for x large

Estimates and uncertainties



Mean value as maximum likelyhood estimate (MLE)

- Model for measurements $y_i, i = 1 \dots N$:

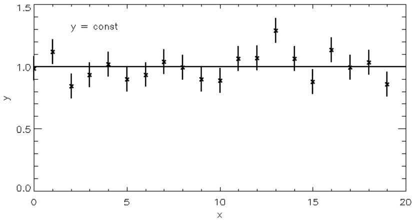
$$y_i = \mu + n_i$$

  with i.i.d. Gaussian noise n_i and free parameter μ.

- Estimate:

$$\hat{\mu} = \bar{y} = (1/N) \sum_{i=1}^{N} y_i$$

Estimates and uncertainties



Mean value as maximum likelyhood estimate (MLE)

- Probability for realizing the observed set of measurements $y_1, \ldots, y_N$:

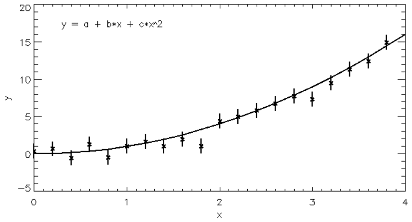
$$P(y_1, \ldots, y_N) = \prod_i P_G(y_i - \mu; 0, \sigma) \propto \exp\left\{ -\frac{1}{2\sigma^2} \sum_i (y_i - \mu)^2 \right\}$$

- Maximum probability i.e. maximum likelihood for model to yield the measurements y_i:

$$\chi^2 = \sum_i (y_i - \hat{\mu})^2 min. \Rightarrow \hat{\mu} = \bar{y}$$

"Least squares"

Estimates and uncertainties



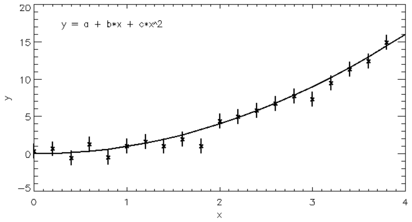
General maximum likelihood estimates

- Model for measurements $y_i, i = 1 \dots N$:

$$y_i = y(x_i; \mu_j) + n_i(\sigma_i)$$

- with Gaussian noise $n_i(\sigma_i)$ of standard deviations σ_i
- and with free parameters μ_j.

Estimates and uncertainties



General maximum likelihood estimates

- Probability for realizing the observed set of measurements $y_1, \ldots, y_N$:

$$
\begin{aligned}
P(y_1, \ldots, y_N) &= \prod_i P_G(y_i - y(x_i); 0, \sigma_i) \\
&\propto \exp\left\{ -\frac{1}{2} \sum_i \left(\frac{y_i - y(x_i)}{\sigma_i} \right)^2 \right\}
\end{aligned}
$$

- Maximum likelihood estimates for parameters μ_j:

$$\chi^2 = \sum_i \left(\frac{y_i - y(x_i; \mu_j)}{\sigma_i} \right)^2 \; min.$$

Estimates and uncertainties

Uncertainty of the mean value?

- Estimate:

$$\hat{\mu} = \bar{y} = (1/N) \sum_{i=1}^{N} y_i$$

- Error propagation: $\sigma_\mu = \sigma/\sqrt{N}$
- 68.3% confidence interval: $\bar{y} \pm \sigma_\mu$

Uncertainty of a general model parameter?

- $\chi^2 = \sum_i \left(\frac{y_i - y(x_i; \mu_j)}{\sigma_i} \right)^2$ $min. \rightarrow \hat{\mu}_j$
- Evaluate variations of χ^2 as a function of the parameter
- Monte Carlo simulation
- ...

Calibration

Goal: reduce systematic errors!

But calibrations are measurements of their own which come with their proper measurements errors $\rightarrow$ error propagation!

2 types of calibration measurements

1. Determine the correlation between the actual measurand and the representative measurand.
   Examples:
   - wavelength calibration of spectra: wavelength $\longleftrightarrow$ pixels
   - polarimetric calibration: polarization $\longleftrightarrow$ intensity
2. Determine the influence quantities which
   - alter the physical quantity directly (e.g. movement of the spacecraft $\rightarrow$ Doppler shifts)
   - or which bias the output signal of the instrument (e.g. dark current, flatfield).

Summary - Questions to be asked about a measurement

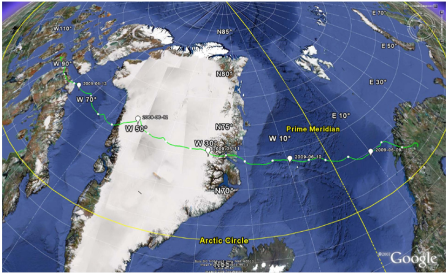
- Exact measurement task and quantity to be measured?
- Measurement principle?
- Absolute or relative measurement?
- Direct or indirect measurement?
- Required accuracy and precision? Sensitivty?
- What is the measuring environment? What are the influencing quantities? Which calibrations are needed?

Data reduction example: Sunrise

Key Parameters

- Telescope diam.: 1m
- Spatial res.: 0.1" (coin in 45 km!)

Data reduction example: Sunrise



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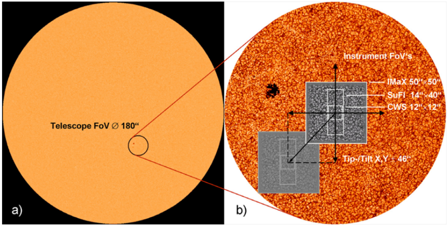

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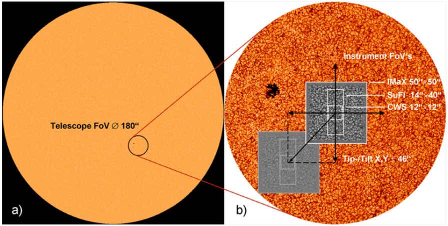

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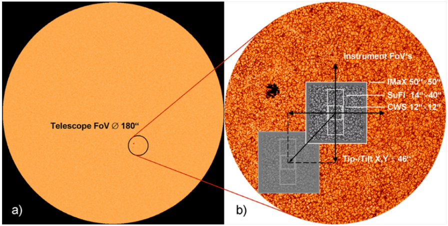
- Sunrise Filter Imager (SuFI):
  - 214 nm, 300 nm, 312 nm (OH), 388 nm (CN), 397 nm (Ca II)
  - FOV: 14" x 40"

Data reduction example: Sunrise



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  - FOV: 14" x 40"
- IMaging Magnetograph Experiment (IMaX):
  - Full Stokes spectropolarimetry in Fe I 525 nm
  - FOV: 50" x 50"

Sunrise/SuFI data - from raw to final

Before

After

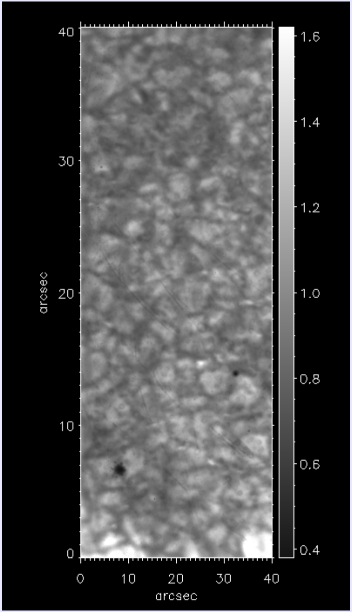
Sunrise/SuFI data - from raw to final

Before

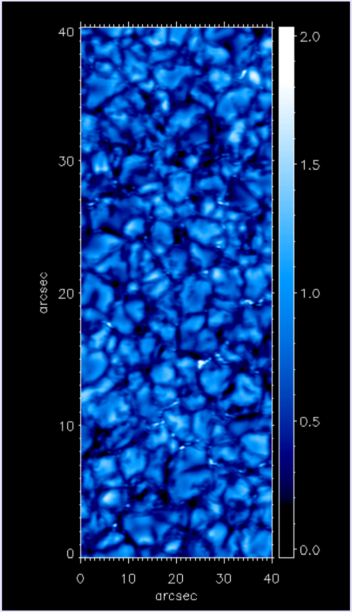
After

Sunrise/SuFI data - from raw to final

Before

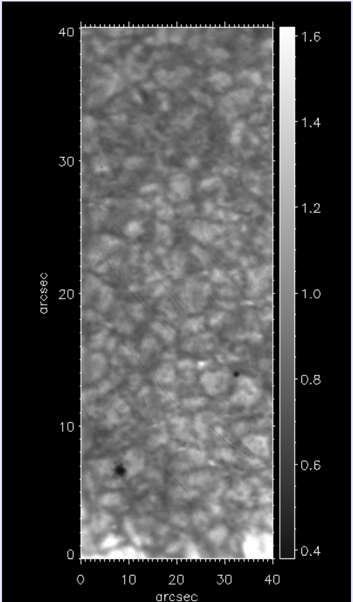


After



Sunrise/SuFI data - from raw to final

Cont. 300 nm

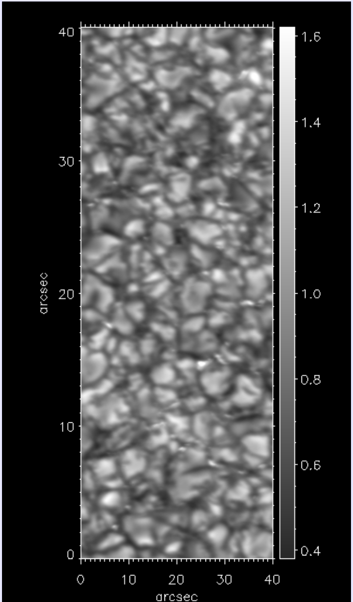


Level 0

Raw data!

Sunrise/SuFI data - from raw to final

Cont. 300 nm

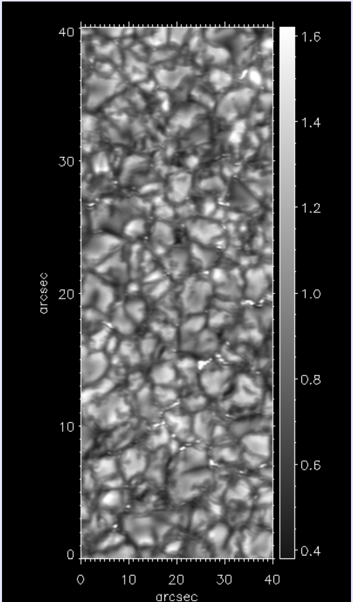


Level 1

- Dark image correction
- Flatfield correction
- Correction for residual defects
  - Median filtering (cosmic rays rem, . . .)
  - Low-pass filtering (scratches, . . .)
  - Fourier filtering (fringes, electronic interferences, . . .)

Sunrise/SuFI data - from raw to final

Cont. 300 nm



Level 2,3

Phase Diversity restoration

Sunrise/SuFI data - from raw to final

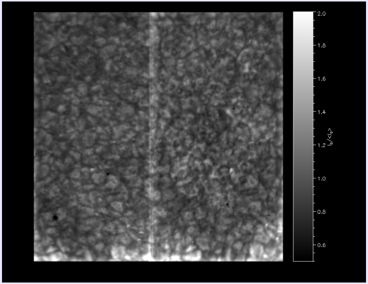
Cont. 300 nm

Time series

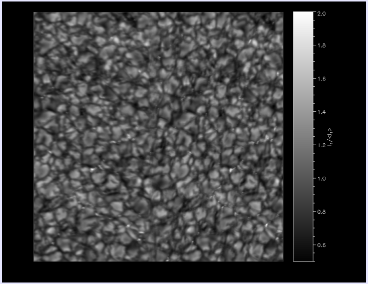
- Frame selection
- Cross-correlation

Dark and flatfield correction

Level-0 image (I_0)



Dark and flatfield corr. image (I)

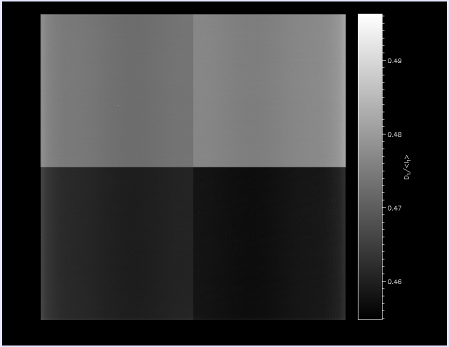


$$I_0(x, y) = F(x, y) \cdot I(x, y) + D(x, y; T, \Delta t)$$

I_0	obs. level-0 image
I	input image
F	flatfield
D	dark image
Δt	integration time
T	detector temperature

Dark and flatfield correction

Dark image ($I = 0$)



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I_0	obs. level-0 image
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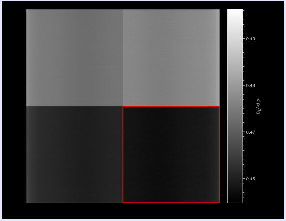
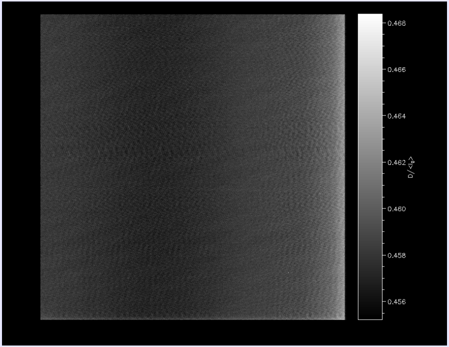
Dark and flatfield correction

Dark image ($I = 0$), lower-right quadrant





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I_0	obs. level-0 image
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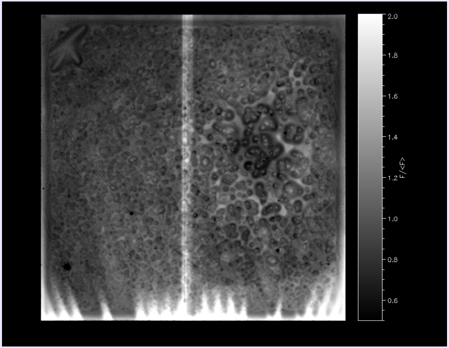
Dark and flatfield correction

Flatfield (I=const)



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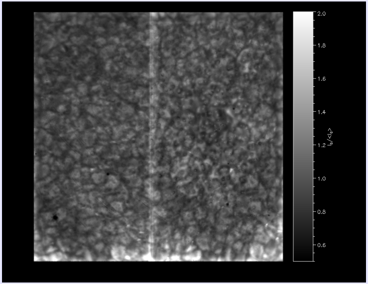
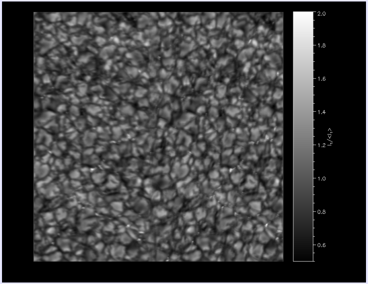
Dark and flatfield correction

Level-0 image (I_0)



Dark and flatfield corr. image (I)



$$I \propto \frac{I_0 - D}{F - D}$$

I_0	obs. level-0 image
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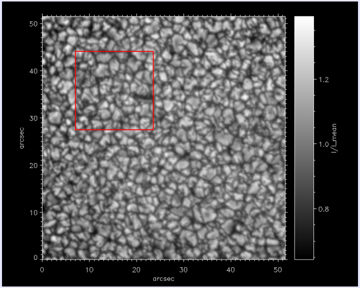
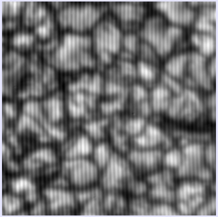
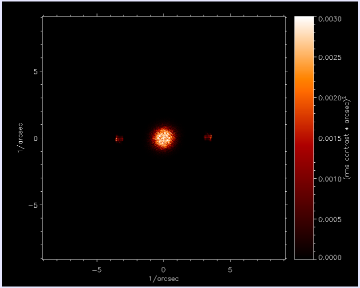
Fringe filtering

IMaX Image



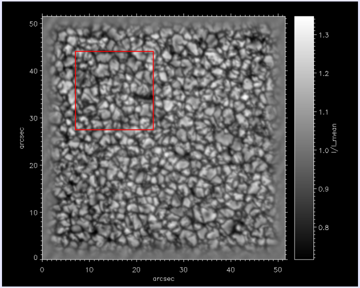


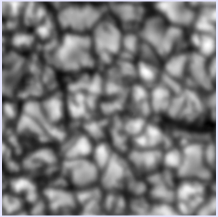
Power spectrum



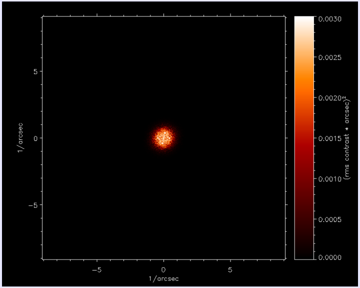
Fringe filtering

Filtered IMaX Image





Filtered power spectrum



Heritage from ground-based solar observations

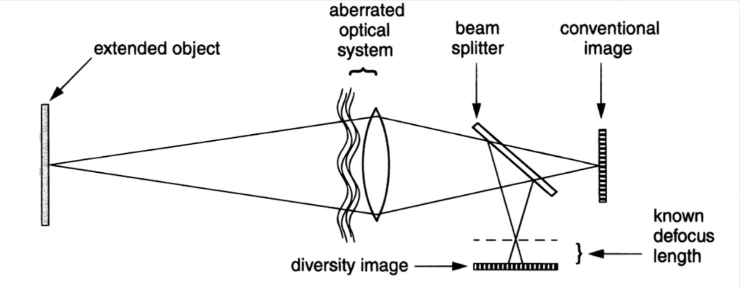
Ca II H line center, recorded at Swedish Solar Telescope; time span 10 min., cadence 4s (J. Hirzberger)

Image restoration

Heritage from ground-based solar observations

Ca II H line center, recorded at Swedish Solar Telescope; time span 10 min., cadence 4s (J. Hirzberger)

Phase Diversity - Principle

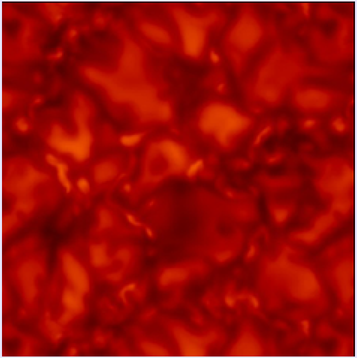


Paxman et al. 1992

Phase Diversity - Principle

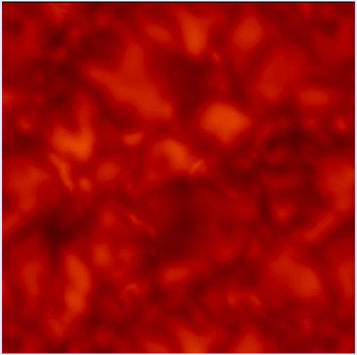
What we have

a conventional image



- of the unknown true Sun
- with unkown aberrations

a diversity image

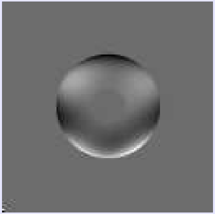


- of the same unknown true Sun
- with the same unkown aberrations
- plus a known defocus

Phase Diversity - Principle

This allows us to make maximum-likelihood estimates of . . .

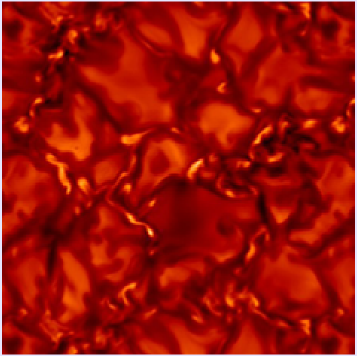
the aberrations



in terms of

- pupil function
- or Zernike coefficients

the true solar image



Phase Diversity - a glimpse of the beauty of the algorithm

Remember the earlier discussion of maximum-likelihood estimates?

- Model: $d_k = f * t_k + n_k, \quad k = 1, 2$
- Maximum-likelihood error metric (objective function):

$$L = \sum_{v} |D_1 - FT_1|^2 + |D_2 - FT_2|^2$$

$d_{1,2}$	conventional and diversity image
f	true solar image
$t_{1,2}$	point spread functions of conv. and div. image
n_k	Gaussian noise
D_k, F, T_k	FFTs of d_k, f and t_k

Phase Diversity - a glimpse of the beauty of the algorithm

But wait a minute . . . how many free parameters do we have? 10^5?

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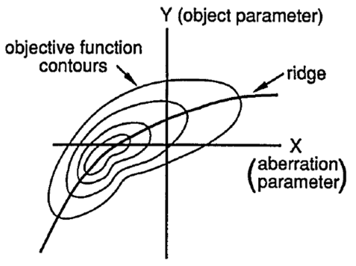
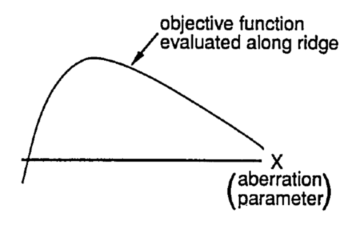
"Trick I": Solve explicitely for the aberrations only!



Paxman et al. 1992



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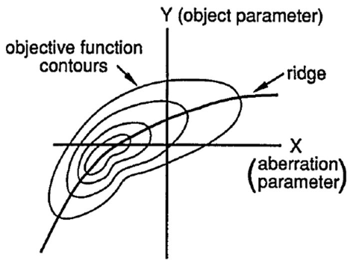

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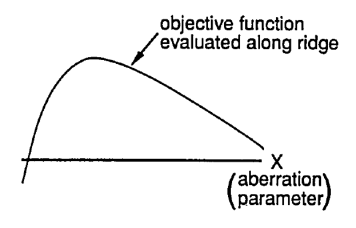

Paxman et al. 1992

"Trick II": Use a Zernike expansion for the aberrations!

Expand the pupil phase ϕ into a series of Zernike functions ϕ_i:

$$\phi = \sum_i c_i \phi_i \quad (1)$$

$$\rightarrow T_k = T_k(\phi) = T_k(c_i) \quad (2)$$

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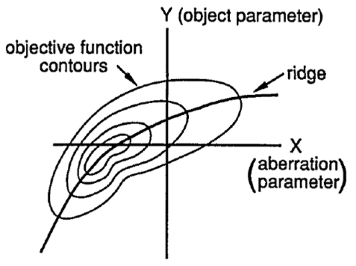

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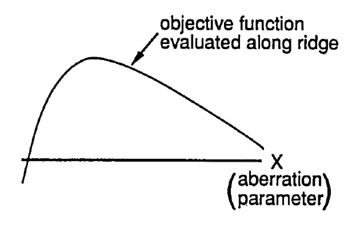

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This leaves us with typically 21 to 45 free parameters instead of 10^5!

Phase Diversity - What is it really good for?

- Structures that are not already “visible” in the raw images cannot be recovered!
- Aberrations re-distribute the intensity in the image
- By doing image restoration we want to recover the true intensity distribution (contrast)

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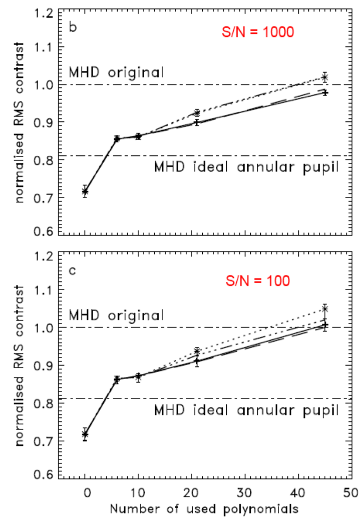
How reliable is the restoration?

Phase Diversity - What is it really good for?

Influence of noise



Hirzberger et al. 2010

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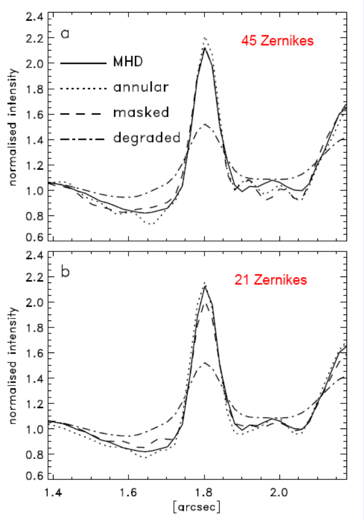
Restoration artifacts



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Where to go from here?

Recommended reading

- "Data Reduction and Error Analysis for the Physical Sciences", P.R. Bevington and D. K. Robinson, McGraw-Hill
- "Numerical Recipes", W.H. Press et al., Cambridge University Press

Looking for code?

- SolarSoft Library (http://www.lmsal.com/solarsoft/)
- Community has developed many IDL code snippets that can be easily re-used for different purposes - ask around at MPS!