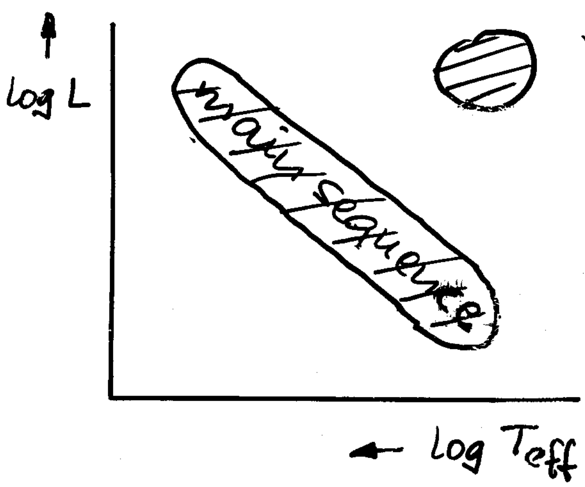
Red Giants – Why?

I, Post main sequence expansion of stars observation and theory

Hertzsprung – Russell – diagram:

correlation between absolute bolometric magnitude and spectral type of stars



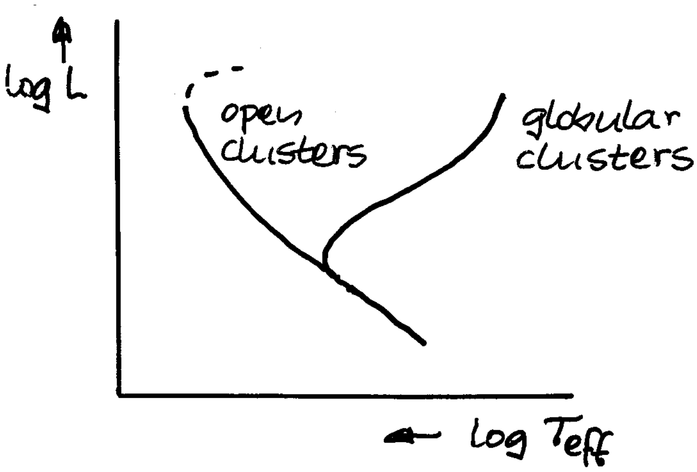
red giants: high luminosity but low surface brightness
→ large radius

L ← stellar distances

trig. paralaxes only for stars within ~ 25 pc

very inhomogeneos sample

investigation of stellar clusters:
stars with equal distance and equal age



open clusters – young:
main sequence completely occupied

globular clusters – old:
main sequence void of luminous stars

turn off → age

in the course of evolution:
main sequence stars → red giants

Why?

Theoretical interpretation:

investigation of a sequence of stellar models of fixed mass and varying chemical composition

→ important: location of nuclear energy production

central hydrogen burning
→ **main sequence stars**

after depletion of central hydrogen content

hydrogen burning in **shell** around helium core
→ evolution towards **red giants**

Why is a shell source model so vastly extended?

[Sun: post main sequence expansion swallows inner planets during next 10×10^9 years!]

Stellar structure equations:

hydrostatic equilibrium
energy equilibrium
energy transport

together with equation of state
opacity
nuclear rates

→ stellar models in good agreement with observations
but no simple answer because equations are
very general
very complicated

II, Investigation of a simplified model

- only hydrostatic equilibrium is strictly satisfied

  very short relaxation time $\sim [G\bar{\rho}]^{-1/2}$

  [relaxation time of energy equilibr. Helmholtz – Kelvin time $\sim GM^2/L$]

- composite configuration

  He – core and H – envelope

  interface : chemical discontinuity
  nuclear burning shell source

- temperature at interface $T_f = T_{nuc} = 2 \times 10^7$ K

  T_{nuc} fixed due to high temp. sensitivity of nuclear burning

  "thermostatic action of nuclear burning"

  → energy equilibrium satisfied

A) Schoenberg – Chandrasekhar model (1942)

core : $T = const$ (no energy production inside)

envelope : polytrope $n = 3$

evolutionary sequence :

M fixed , M_{core} growing , $T_f = T_{nuc}$

→ moderate expansion until $M_{core} = M_{SC}$

$$\frac{M_{SC}}{M} \sim 0.1$$

no hydrostatic equilibrium for $M_{core} > M_{SC}$

not surprising because there is no isothermal hydrostatic configuration of finite mass

Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$$

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

$$P = K\rho^{(n+1)/n}$$

2 equations for 3 dependant unknowns P, ρ, M_r

← closed by polytropic rel. (n polytropic index)

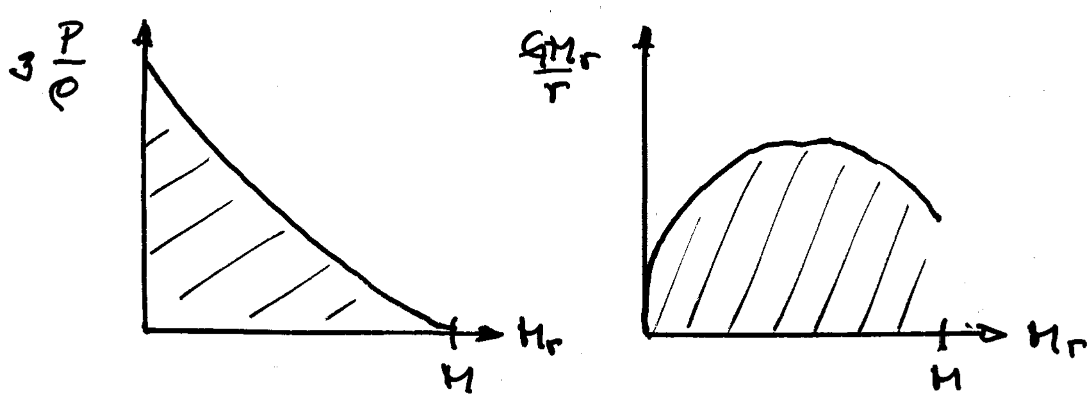
special cases:

$n = 0$	$\rho = \text{const}$ (analytical solution)
$n = 1.5$	equation of state for degenerate electron gas
$n = 3$	approx. satisfying radiative energy transport (Eddington)
$n \to \infty$	$P = K\rho$ $T = \text{const}$ isothermal conf. not possible for finite mass

integration of hydrostatic equation yields

$$\underbrace{2\int_0^M \frac{3}{2}\frac{P}{\rho}\, dM_r}_{\text{spec. internal energy}} = \underbrace{\int_0^M \frac{GM_r}{r}\, dM_r}_{-\text{spec. gravitational energy}}$$ virial theorem

$$2\, E_{int} = -E_{grav} \qquad E_{grav} \sim -\frac{GM^2}{R}$$



hydrostatic equilibrium requires equal areas

Escape from Schoenberg – Chandrasekhar trap by contraction (non-local energy sources)

important difficulty:
uniform contraction
→ uniform increase of temperature.

but:
temperature of shell source fixed by thermostatic action of nuclear burning

→ non-uniform contraction starting at $M_{core} = M_{SC}$

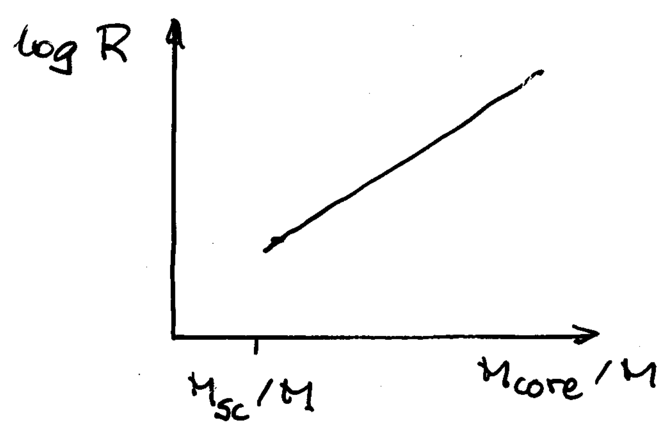
further evolution dependant on stellar mass

B) $M \lesssim 1.5\,M_\odot$: Schwarzschild – Hoyle model (1955)

high central density at main-sequ.-state
contraction leads to electron degeneracy
→ hydrostatic equilibrium possible!

core: $T = $ const
equation of state $P(\varrho)$ allowing for degeneracy
two asymptotic branches:
complete degeneracy $P \sim \varrho^{5/3}$
no degeneracy $P \sim \varrho$

envelope: polytrope $0 \leq n \leq 5$ $(n = 0)$



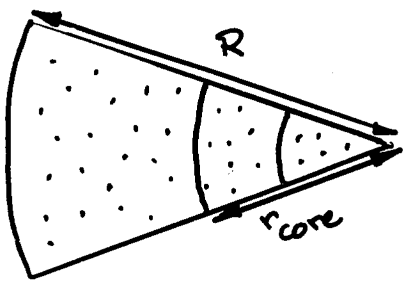
evolutionary sequence:
M fixed, M_{core} growing
$T_f \approx T_{nuc}$

time scale = nuclear burning time

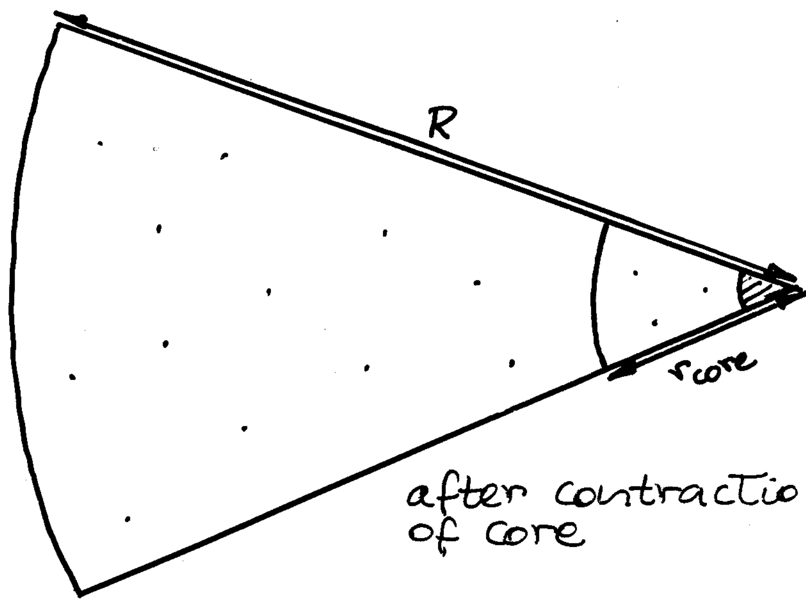
Simplified model is successful – but what does it teach us?

Close inspection of solutions reveals two-fold evolutionary behaviour of core due to two branches of equation of state:

- only inner part of core is highly contracted
  → high density (highly degenerate)
- (almost) no contraction of interface – T_f fixed!
- outer part of core is rarefied accordingly (non degenerate)



before contraction of core



after contraction of core

- envelope must expand to restore equality of density at interface

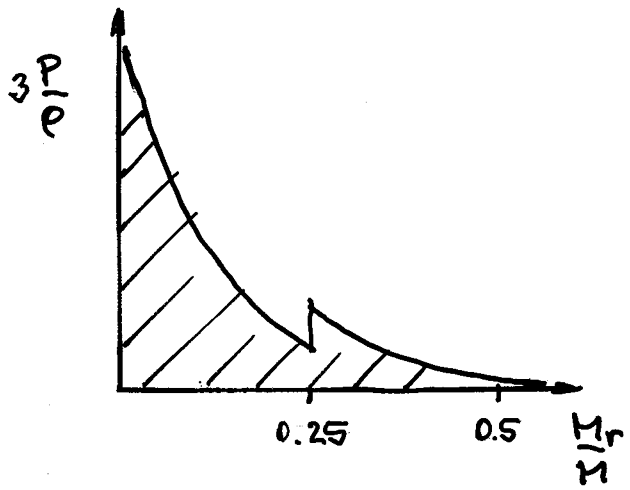
envelope only responding to actions of core

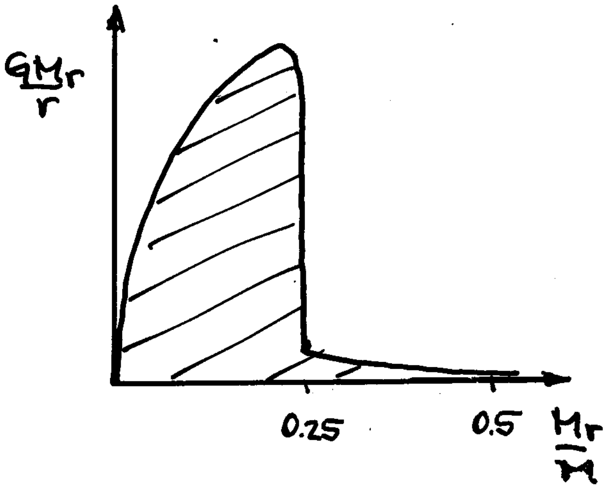
Working of thermostat:

- contraction after reaching Schoenberg-Chandrasekhar mass-limit
  - → increasing temperature, especially T_f
  - → increasing energy production
- local temperature gradient limited by hydrostatics cannot transport increased energy
  - → surplus energy drives expansion of matter against gravity
    adiabatic expansion until $T_f = T_{nuc}$ is restituted

Application of virial theorem:

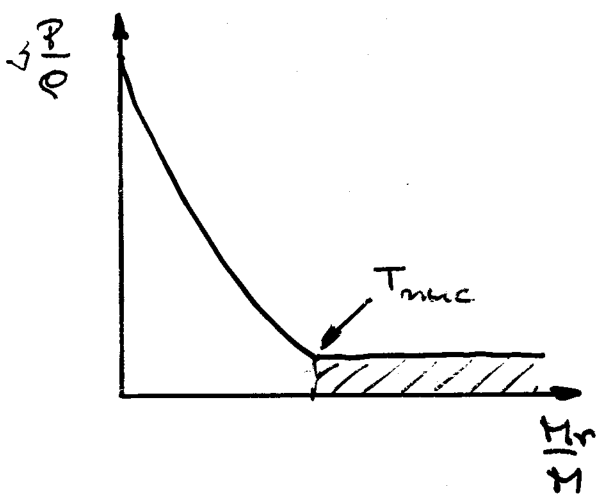
$$3\int_0^M \frac{P}{\varrho}\, dM_r = \int_0^M \frac{GM_r}{r}\, dM_r$$

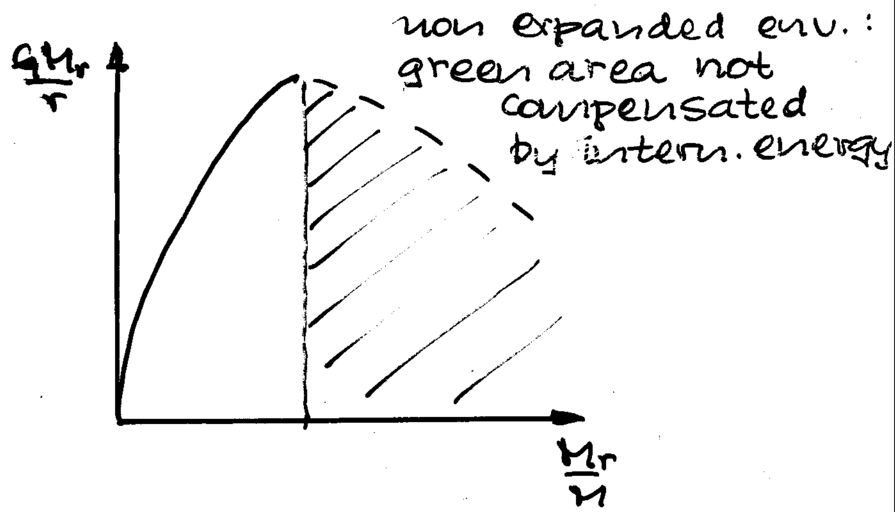




- only core is energetically important
- virial theorem satisfied due to strong decrease of GM_r/r at interface – resulting from rarefied outer part of core

  it is forced by $T_f = T_{nuc}$





almost identical situation obtained from actual stellar evolution simulations by M. Stix

age 14.1×10^9 ys $M = 1\, M_\odot$ $M_{core} = 0.85 M_\odot$ $R = 8.98\, R_\odot$

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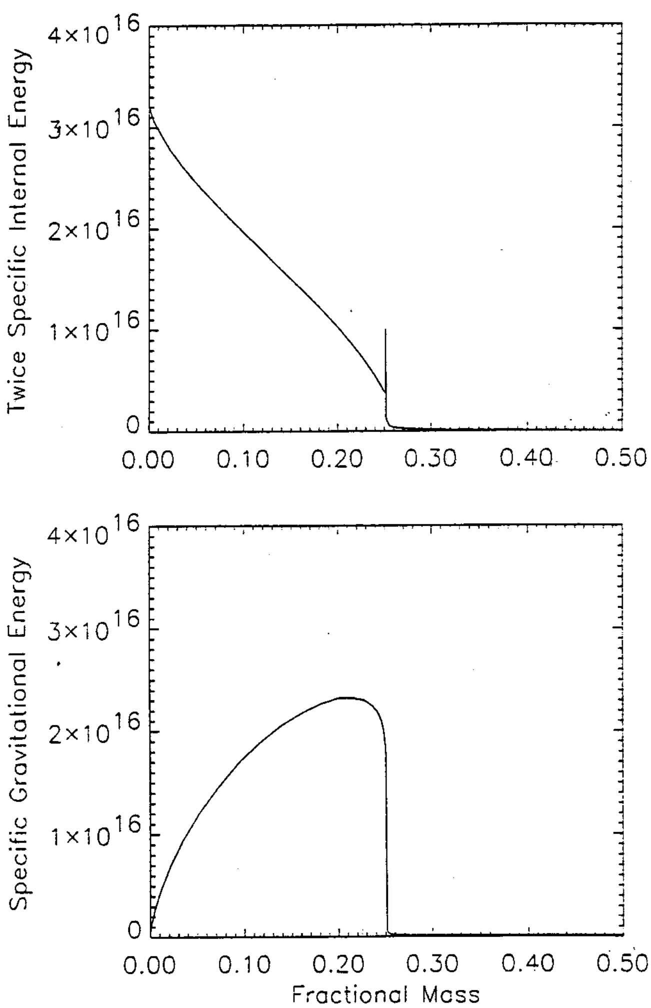


Figure 2. Specific internal and gravitational energy u and w as functions of fractional mass M_r/M for the $1M_{\odot}$-model with $M_{core}/M = 0.25$.

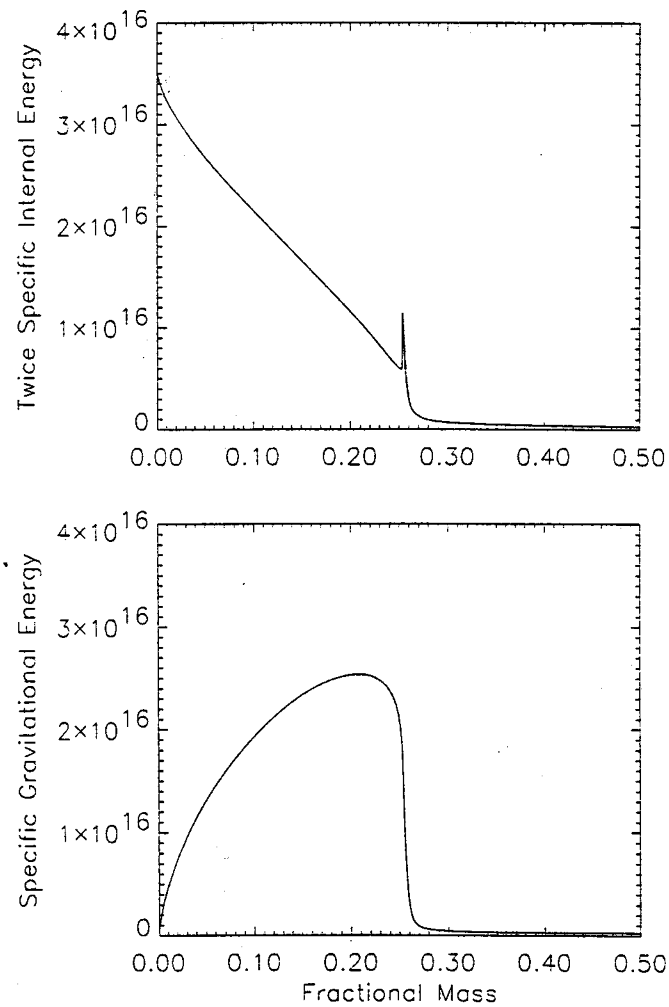


Figure 3. Specific internal and gravitational energy u and w as functions of fractional mass M_r/M for the sun with $M_{core}/M = 0.25$ as obtained by numerical simulation (Stix, 1997).

c) $M \gtrsim 1.5\,M_\odot$: Schwarzschild – Sandage model 1952

continued contraction of core after reaching Schoenberg – Chandrasekhar mass

continued release of gravitational energy which must be transported to the outside

escape from Schoenberg – Chandrasekhar trap due to energy transport by radiation

$$\frac{d\left(\frac{a}{3}T^4\right)}{dP} = \frac{\varkappa}{4\pi c G}\;\frac{L_r}{M_r}$$

L_r from non-local energy sources
→ problem becomes time – dependant

considerable complication – avoided by global approximation by Schwarzschild – Sandage

$$\frac{L_r}{M_r} = \frac{L_{core}}{M_{core}} \qquad L_{core} = \frac{1}{2}\,\frac{1}{\Delta t}\,\Delta\int_0^{M_{core}} \frac{GM_r}{r}\,dM_r$$

now energy transport equation may be integr.

$$\frac{a}{3}T^4 = \frac{\varkappa}{4\pi c G}\;\frac{L_{core}}{M_{core}}\;P + \underbrace{const}_{\approx \frac{a}{3}T_{nuc}^4}$$

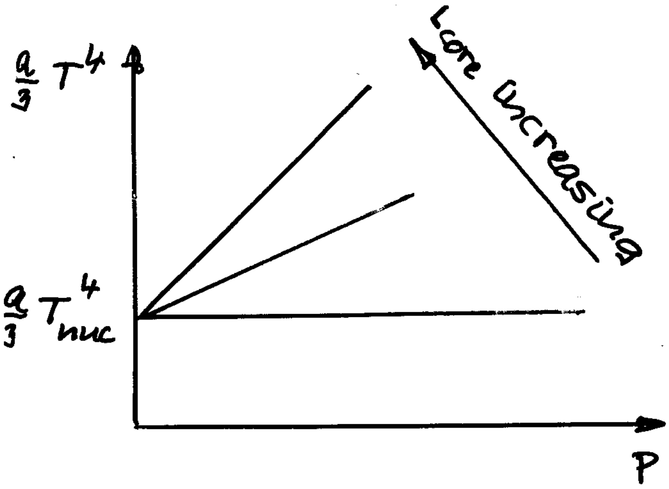
two asymptotic branches:

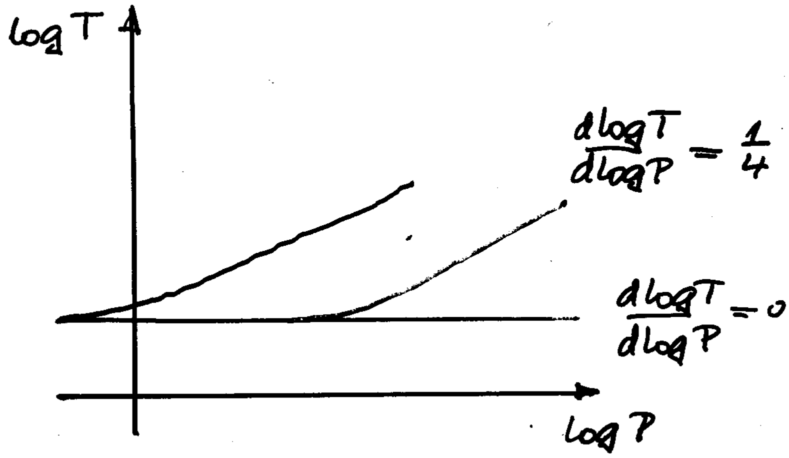
$$\frac{\varkappa}{4\pi c G}\;\frac{L_{core}}{M_{core}}\;P \gg \frac{a}{3}T_{nuc}^4 \qquad \frac{d\log T}{d\log P} = \frac{1}{4} \qquad (n=3)$$

$$\frac{\varkappa}{4\pi c G}\;\frac{L_{core}}{M_{core}}\;P \ll \frac{a}{3}T_{nuc}^4 \qquad \frac{d\log T}{d\log P} = 0 \qquad (n \to \infty)$$

→ two-fold structure of core due to fixed T_{nuc}

core : "equation of state"





envelope: polytrope $n = 3$

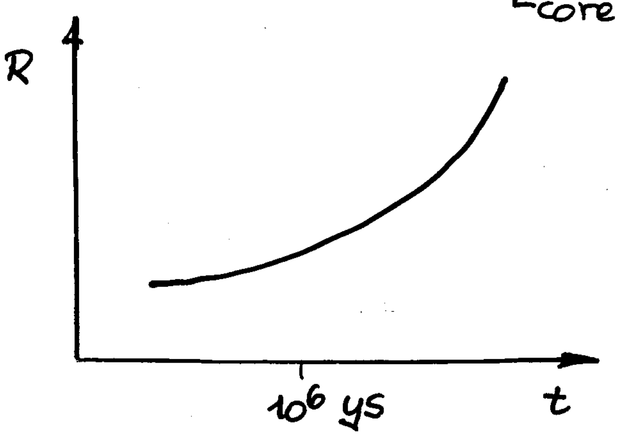
evolutionary sequence :

M fixed , $M_{core} = M_{sc}$, $T_f = T_{nuc}$

L_{core} growing (evolutionary parameter)

time step between succeeding models from

$$L_{core} = \frac{1}{2}\,\frac{1}{\Delta t}\,\Delta \int_0^{M_{core}} \frac{GM_r}{r}\, dM_r$$



$M = 5\,M_\odot$
$M_{sc}/M = 0.07$
$T_{nuc} = 2 \times 10^7$ K

simplified model is successful

thermostatic action of nuclear shell-source : strongly contracting inner part of core and fixed location of interface

→ rarefied zone below interface

→ adjustment of envelope by expansion

evident by comparing specific internal and specific gravitational energy

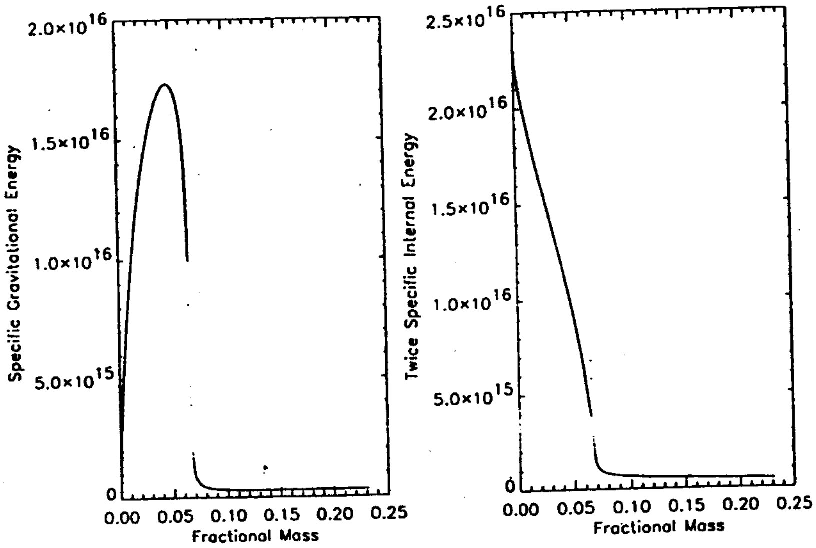


Fig. 4. Specific internal and gravitational energy as functions of fractional mass $\frac{M_r}{M}$ for a simplified stellar model of 5 $M_\odot$ at $t = 1.6 \times 10^6$ years.

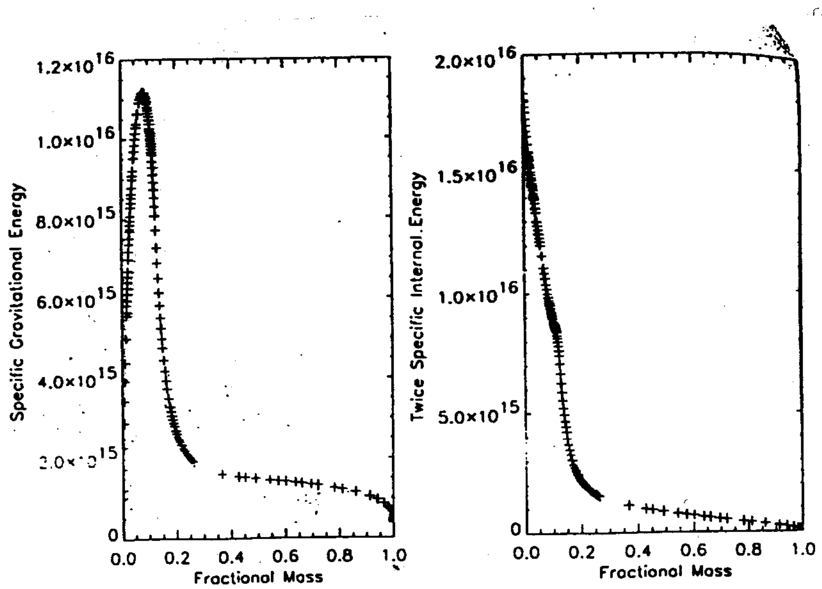


Fig. 5. Specific internal and gravitational energy as functions of fractional mass $\frac{M_r}{M}$ for an evolved star of 5 $M_\odot$ obtained by numerical simulation (Langer 1999).

III) Summary: red giants – why?

- He-core / H-envelope with nuclear shell source at interface
- hydrostatic equilibrium only possible if mass of isothermal core is smaller than Schoenberg-Chandrasekhar limit
- after that core must contract
  <u>but:</u>
  temperature of interface and consequently its position fixed by thermostatic action of nuclear shell source
- hydrostatic equilibrium requires two-fold structure of core:
  highly contracted inner part
  rarefied outer part
- hydrostatic equilibrium requires accordingly rarefied and thus expanded envelope
- post main sequence expansion driven by surplus nuclear energy released by thermostatic action of shell source

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On the post main sequence expansion of low mass stars

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Abstract. The post main sequence expansion of stars is investigated by means of a simple composite configuration: an isothermal He-core (allowing for non-relativistic electron degeneracy) is surrounded by a H-envelope of constant density (polytrope $n = 0$). Solving the equations of hydrostatic equilibrium for fixed values of total mass and temperature at the interface a one dimensional sequence of models is obtained with the mass of the core as parameter. As soon as the main part of the core becomes fully degenerate, the model stars expand rapidly. This behaviour is in good agreement with that of models obtained by numerical simulations.

The expansion is caused by an intermediate non-degenerate layer of large extension (but of very small mass content) just below the interface. It shifts the envelope to larger distances from the center and thus reduces the gravitational pull on it due to the highly contracted part of the core. Without this layer the thermal forces of the envelope – determined by the hydrogen burning temperatures at the interface – would be much too small to balance gravity. Such a loosely bound envelope extends to the large radii in question. Hence, the model suggests the fixed temperature required by hydrogen burning to be the ultimate reason for the post main sequence expansion.

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On the post main sequence expansion of stars with contracting helium cores

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Abstract. The post main sequence expansion of a 5 $M_{\odot}$-star is investigated by means of a simple composite configuration: a contracting He-core of Schoenberg-Chandrasekhar mass surrounded by an H-envelope of polytropic index $n = 3$. While the structure of the envelope is immediately obtained by solving the equations of hydrostatic equilibrium, the core requires some further simplification: if the actual non-local gravitational energy release due to contraction is replaced by its constant core-average, the equation of radiative energy transport may be easily integrated. Thus an explicit relation between pressure and temperature is obtained and the equations of hydrostatic equilibrium may be solved. Specifying $M, M_{\rm core}$ and T_0 (the temperature of the H-burning shell-source at the interface), a sequence of models follows with $L_{\rm core}$, the gravitational energy released from the core per second, and hence with t, the contraction time, as the parameter. The resulting simple models show very rapid expansion, a consequence of the thermostatic action of the shell-source. Its fixed temperature prevents the shell-source from participating in the contraction of the core – thus causing the outer parts of the core and hence the adjoining envelope to decrease in density. Accordingly, the envelope must expand. This consequence of a fixed temperature T_0 is clearly demonstrated by the distributions of the specific internal and gravitational energies. This characteristic behaviour is also found in stellar models obtained by elaborate numerical simulations.

Key words. stars: Hertzsprung-Russell diagram – stars: interiors – stars: evolution