# Fundamentals on light scattering, absorption and thermal radiation, and its relation to the vector radiative transfer equation

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# Outline of talk:

Motivation: Provide some background on scattering and thermal radiation to researchers interpreting observations of asteroid and comet surfaces

1. Transversal waves → Polarization must be taken into account! Characterization of polarization.
2. Scattering of electromagnetic radiation by an arbitrary, finite particle in the far-field zone
3. The optical theorem
4. Thermal radiation
5. Diagrammatic representation of scattering processes – scattering by random, discrete scatterers
6. Vector radiative transfer equation

# Literature:

Mishchenko, M.I., L.D. Travis, A. A. Lacis: “Scattering, absorption and emission of light by small particles”, Cambridge Univ. Press, 2002

Bohren, C. F., D. R. Huffman: “Absorption and scattering of light by small particles”, Wiley, 1983

Tsang, L., J. A. Kong, R. T. Shin, “Theory of microwave remote sensing”, Wiley, 1985

Tsang, L., J. A. Kong, K.-H. Ding, “Scattering of electromagnetic waves, Theories and applications”, Wiley, 2000

Ishimaru, A., “Wave propagation and scattering in random media”, Academic Press, 1978

Morse, Ph. M., and H. Feshbach, “Methods of theoretical physics”, 2 Vols., McGraw-Hill, 1953-

*In my presentation I follow mainly the book of Mishchenko et al.*

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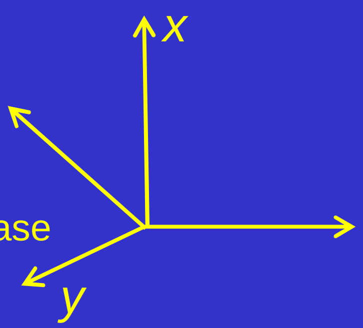
# Polarization

The electromagnetic waves are transversal → Polarization must be taken into account!

Polarization plane of the two superimposed waves of same phase




$z$ = propagation direction of the wave

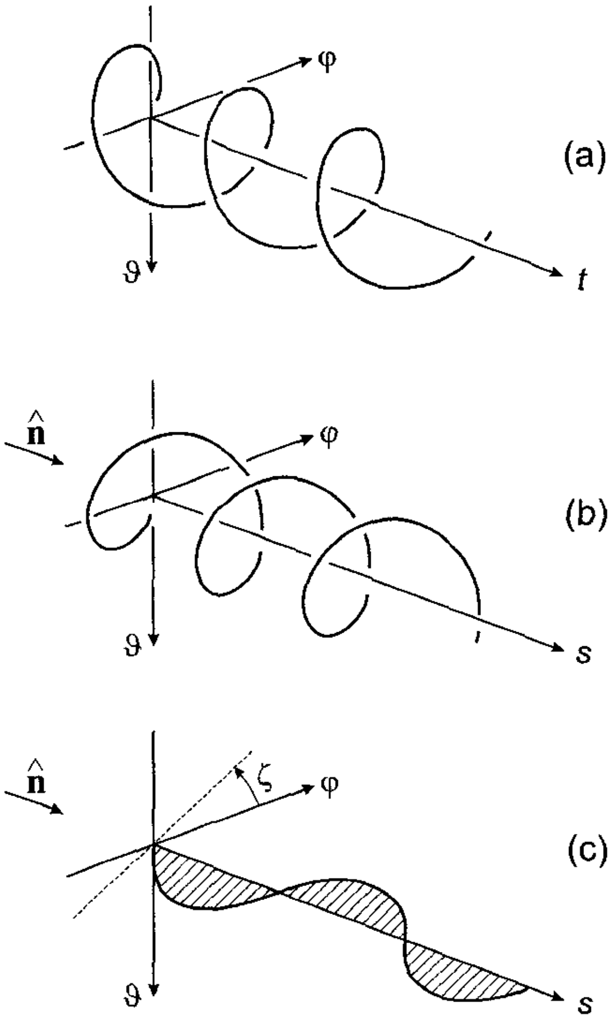
*Superimpose two plane waves, one polarized perpendicular to the other:*
*same phase*:
$E_x = a \exp(\mathrm{i}k\cdot z - \mathrm{i}\omega t)$ and $E_y = a \exp(\mathrm{i}k\cdot z - \mathrm{i}\omega t)$,
The resulting wave has amplitude $a\sqrt{2}$, polarization angle 45° in the $x,y$ plane and is in phase with the two constituting waves.

90° *phase shift:*
$E_x = a \exp(\mathrm{i}k\cdot z - \mathrm{i}\omega t)$ and $E_y = a \exp(\mathrm{i}k\cdot z - \mathrm{i}\omega t + \pi/2)$,
The resulting wave is circularly polarized.

*Superimpose two plane waves in the same plane, one the negative of the other, i.e. a phase shift of π:*

$E_x = a \exp(\mathrm{i}k\cdot z - \mathrm{i}\omega t)$ and $E_x = a \exp(\mathrm{i}k\cdot z - \mathrm{i}\omega t + \pi)$ :
These two waves will interfere and extinguish each other
*(like longitudinal waves do).*

Superposition of two waves with orthogonal linear polarization and different pase.



# Description of the polarization state 1:

Coherency (or density) matrix

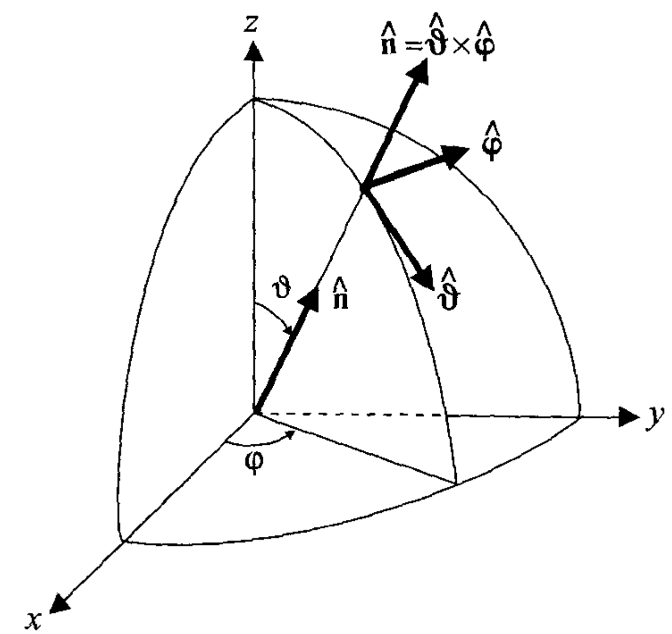



**Figure 1.2.** Coordinate system used to describe the direction of propagation and the polarization state of a plane electromagnetic wave.

$$\mathbf{E}_c(\mathbf{r}, t) = \mathbf{E}_0 \exp(\mathrm{i}k\hat{\mathbf{n}}\cdot\mathbf{r} - \mathrm{i}\omega t).$$

$\mathbf{E}_0 = [E_{0\theta}, E_{0\varphi}] = [a_\theta \exp(\mathrm{i}\Delta_\theta), a_\varphi \exp(\mathrm{i}\Delta_\varphi)]$ with $a$ and $\Delta$ real.

To describe polarization, let us form products of quantities that do not vary as fast as the wave itself:

$$E_{c\vartheta}E_{c\vartheta}^* = E_{0\vartheta}E_{0\vartheta}^*, \quad E_{c\vartheta}E_{c\varphi}^* = E_{0\vartheta}E_{0\varphi}^*, \quad E_{c\varphi}E_{c\vartheta}^* = E_{0\varphi}E_{0\vartheta}^*, \quad E_{c\varphi}E_{c\varphi}^* = E_{0\varphi}E_{0\varphi}^*.$$

$$\boldsymbol{\rho} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}} \begin{bmatrix} E_{0\vartheta}E_{0\vartheta}^* & E_{0\vartheta}E_{0\varphi}^* \\ E_{0\varphi}E_{0\vartheta}^* & E_{0\varphi}E_{0\varphi}^* \end{bmatrix}.$$

# Description of the polarization state 2:

Coherency vector *J* (from the coherency matrix):

$$\mathbf{J} = \begin{bmatrix} \rho_{11} \\ \rho_{12} \\ \rho_{21} \\ \rho_{22} \end{bmatrix} \overset{\text{complex}}{=} \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}} \begin{bmatrix} E_{0\vartheta}E^*_{0\vartheta} \\ E_{0\vartheta}E^*_{0\varphi} \\ E_{0\varphi}E^*_{0\vartheta} \\ E_{0\varphi}E^*_{0\varphi} \end{bmatrix}.$$

ε permittivity (√ε refractive index)
*μ* permeability

The factor $\sqrt{\varepsilon/\mu}$ is frequently omitted.

The Stokes parameters *I*, *Q*, *U*, and *V* are then defined as the elements of a 4 × 1 column vector **I**, otherwise known as the Stokes vector, as follows:

$$\mathbf{I} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}} \begin{bmatrix} E_{0\vartheta}E^*_{0\vartheta} + E_{0\varphi}E^*_{0\varphi} \\ E_{0\vartheta}E^*_{0\vartheta} - E_{0\varphi}E^*_{0\varphi} \\ -E_{0\vartheta}E^*_{0\varphi} - E_{0\varphi}E^*_{0\vartheta} \\ \mathrm{i}(E_{0\varphi}E^*_{0\vartheta} - E_{0\vartheta}E^*_{0\varphi}) \end{bmatrix} \overset{\text{real}}{=} \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}} \begin{bmatrix} E_{0\vartheta}E^*_{0\vartheta} + E_{0\varphi}E^*_{0\varphi} \\ E_{0\vartheta}E^*_{0\vartheta} - E_{0\varphi}E^*_{0\varphi} \\ -2\,\mathrm{Re}\,E_{0\vartheta}E^*_{0\varphi} \\ 2\,\mathrm{Im}\,E_{0\vartheta}E^*_{0\varphi} \end{bmatrix},$$

Other combinations of $E_{0\varphi}$ and $E_{0\theta}$ are possible and are in use. It is, however, important, that always pairs with conjugate complex partners are combined.

# Quasi-monochromatic light 1:

The previous definition of the coherency and Stokes vectors implies that the waves are strictly monochromatic, i. e. $\mathbf{E}_0 = [E_{0\theta}, E_{0\varphi}] = [a_\theta \exp(i\Delta_\theta), a_\varphi \exp(i\Delta_\varphi)]$ and the real amplitudes $a_\varphi$ and $a_\theta$, and phases $\Delta_\varphi$ and $\Delta\theta$ are exactly constant.

But in reality there is no strictly monochromatic wave. Amplitudes and phases vary, but much less than the oscillating electric field itself.

We must replace the amplitudes and phases themselves by their time averages.

$$I = \langle E_{0\vartheta} E^*_{0\vartheta} \rangle + \langle E_{0\varphi} E^*_{0\varphi} \rangle = \langle a^2_\vartheta \rangle + \langle a^2_\varphi \rangle,$$

$$Q = \langle E_{0\vartheta} E^*_{0\vartheta} \rangle - \langle E_{0\varphi} E^*_{0\varphi} \rangle = \langle a^2_\vartheta \rangle - \langle a^2_\varphi \rangle,$$

$$U = -\langle E_{0\vartheta} E^*_{0\varphi} \rangle - \langle E_{0\varphi} E^*_{0\vartheta} \rangle = -2\langle a_\vartheta a_\varphi \cos \Delta \rangle,$$

$$V = \mathrm{i}\langle E_{0\varphi} E^*_{0\vartheta} \rangle - \mathrm{i}\langle E_{0\vartheta} E^*_{0\varphi} \rangle = 2\langle a_\vartheta a_\varphi \sin \Delta \rangle,$$

where we have omitted the common factor $\frac{1}{2}\sqrt{\varepsilon/\mu}$ and

$$\langle f \rangle = \frac{1}{T} \int_t^{t+T} \mathrm{d}t' f(t')$$

# Quasi-monochromatic light 2:

For strictly monochromatic light we have $I^2 = Q^2 + U^2 + V^2$.

For quasi-monochromatic light we have

$$
\begin{aligned}
I^2 &- Q^2 - U^2 - V^2 \\
&= 4[\langle a_\vartheta^2 \rangle \langle a_\varphi^2 \rangle - \langle a_\vartheta a_\varphi \cos \Delta \rangle^2 - \langle a_\vartheta a_\varphi \sin \Delta \rangle^2] \\
&= \frac{4}{T^2} \int_t^{t+T} \mathrm{d}t' \int_t^{t+T} \mathrm{d}t'' \{ [a_\vartheta(t')]^2 [a_\varphi(t'')]^2 \\
&\qquad - a_\vartheta(t') a_\varphi(t') \cos[\Delta(t')]\, a_\vartheta(t'') a_\varphi(t'') \cos[\Delta(t'')] \\
&\qquad - a_\vartheta(t') a_\varphi(t') \sin[\Delta(t')]\, a_\vartheta(t'') a_\varphi(t'') \sin[\Delta(t'')] \} \geq 0
\end{aligned}
$$

# Quasi-monochromatic light 3:

## Corollary:

Highly monochromatic light like laser light easily produces speckles, caused by interference. We expect in such cases the polarization to be high:

$I^2 = Q^2 + U^2 + V^2 \approx 1.$

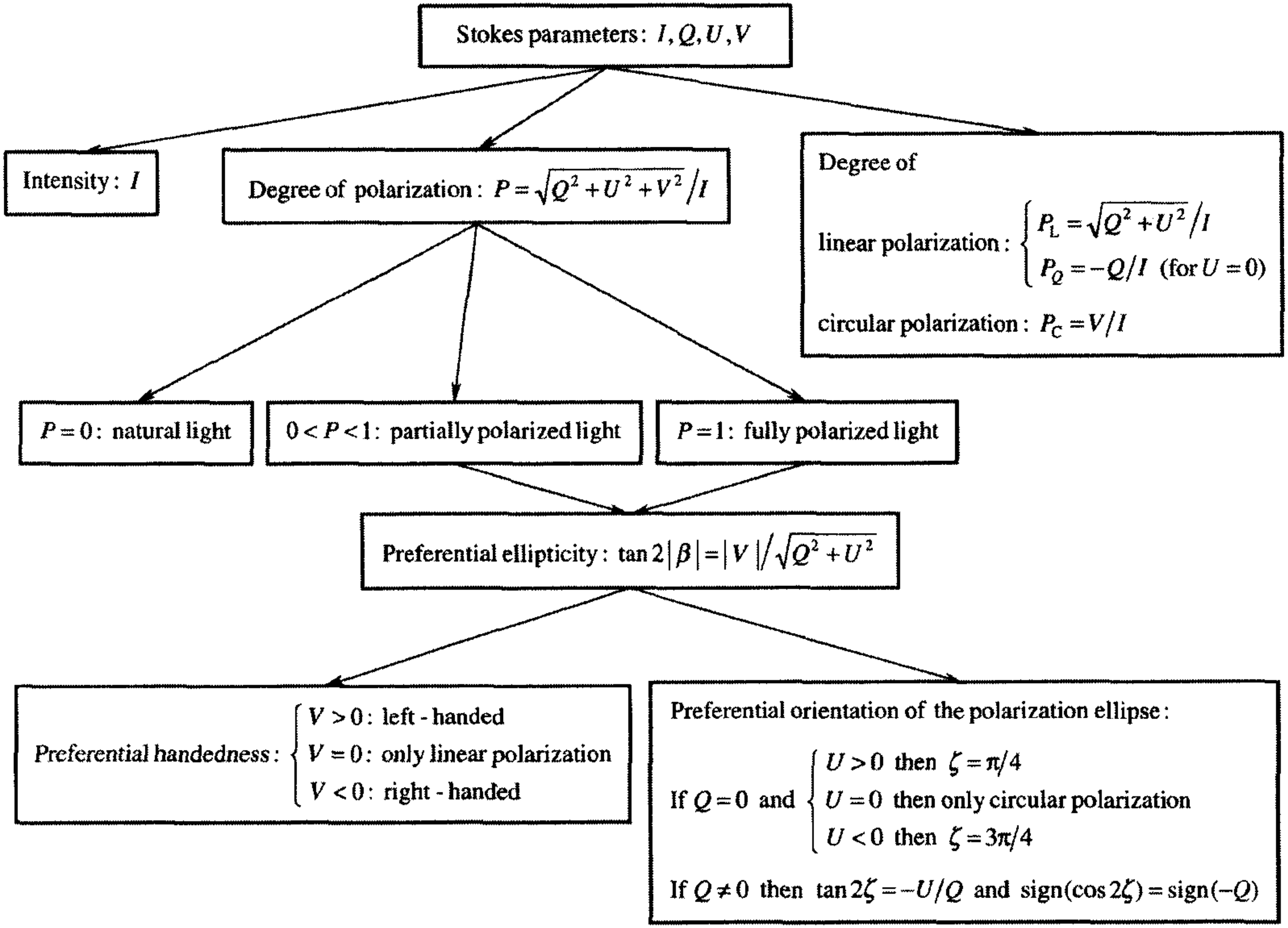



**Figure 1.6.** Analysis of a quasi-monochromatic beam with Stokes parameters $I$, $Q$, $U$, and $V$.

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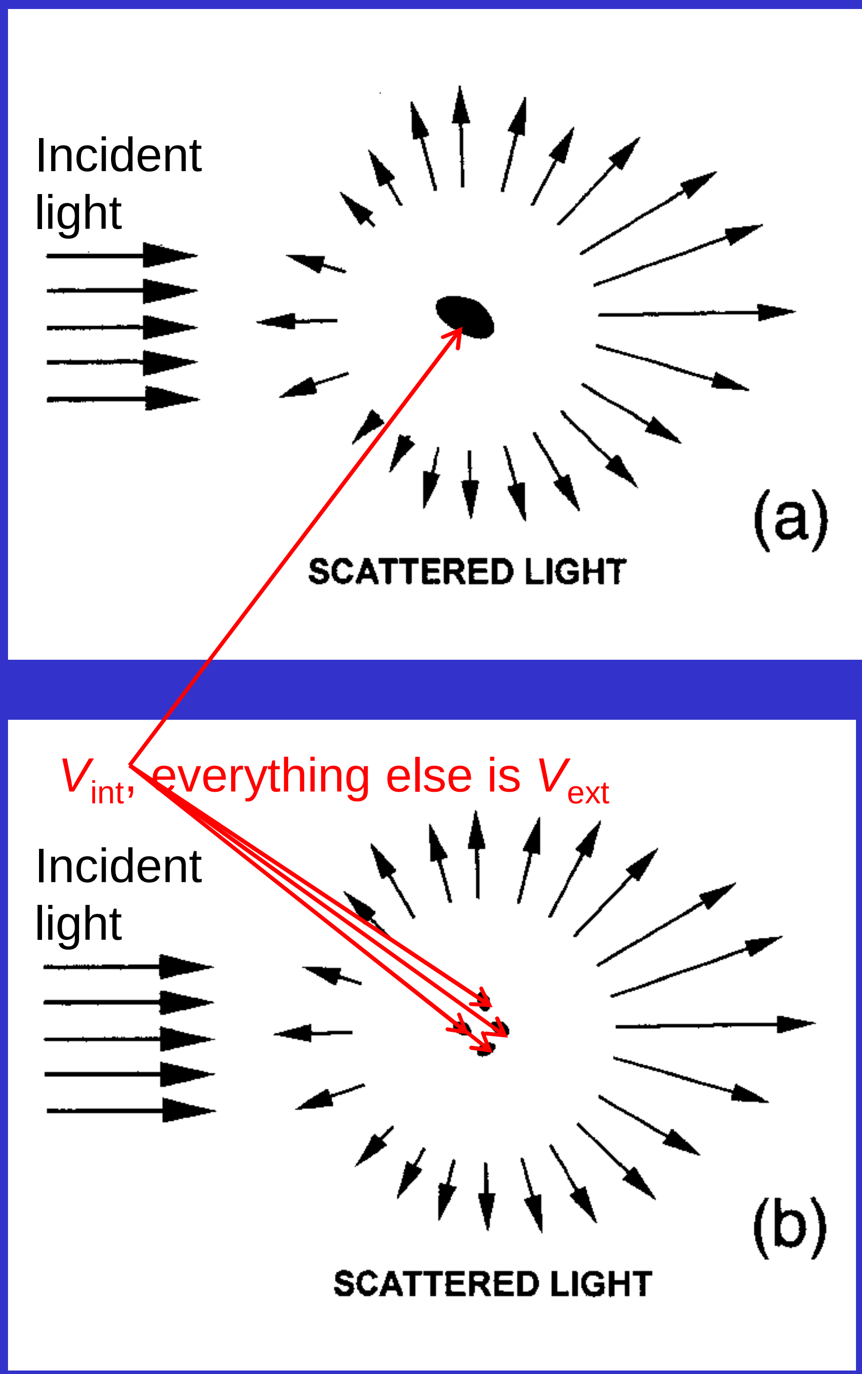
# Scattering by an arbitrary, finite particle:

The “particle“ can be a single entity or a small group of separate entities.

The particle is embedded in a loss-free medium (e.g. vacuum).

The incident light enters the particle and induces electrical dipoles in it.
The dipoles radiate secondary waves and in this way generate the scattered radiation. In turn, the generated radiation influences the strength and direction of the dipoles.

The Discrete Dipole Approximation calculates the scattering by directly determining the dipoles, but I will use the approach of macroscopic electromagnetics, i.e. describe the particle by its index of refraction.



Assumptions:

1. Time-harmonic, quasi-monochromatic light ~ $e^{-i\omega t}$.
2. No frequency redistribution, scattered and incident light have the same frequency. If radiation energy is transferred to a different frequency this is considered as absorption.
3. Only finite particles are considered.
4. Only scattering in far-field zone, i.e. distance of observer » wavelength and extent of particle.
5. Bound host medium surrounding the scatterer is homogeneous, linear, isotropic, and nonabsorbing.
6. In the following we assume host medium and scattering object as nonmagnetic: $\mu_2(\mathbf{r}) \equiv \mu_1 = \mu_0$, where $\mu_0$ is the permeability of vacuum. ($V_{int} \leftarrow$ index 1, $V_{ext} \leftarrow$ index 2.)

At the surface of particles the well-known boundary conditions must be observed: $E_{tangential}$ and (in the absence of surface charges) $D_{normal} = E_{normal}/\varepsilon$ must be continuous.

Equation to be solved:

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r}) - k_1^2\mathbf{E}(\mathbf{r}) = \mathbf{j}(\mathbf{r}), \qquad \mathbf{r} \in V_{\mathrm{EXT}} \cup V_{\mathrm{INT}}, \tag{2.5}$$

where *k*: wave number, *m*: refractive index, **r** position vector

$$\mathbf{j}(\mathbf{r}) = k_1^2[\tilde{m}^2(\mathbf{r}) - 1]\mathbf{E}(\mathbf{r}), \tag{2.6a}$$

$$\tilde{m}(\mathbf{r}) = \begin{cases} 1, & \mathbf{r} \in V_{\mathrm{EXT}}, \\ m(\mathbf{r}) = k_2(\mathbf{r})/k_1 = m_2(\mathbf{r})/m_1, & \mathbf{r} \in V_{\mathrm{INT}}, \end{cases} \tag{2.6b}$$

and $m(\mathbf{r})$ is the refractive index of the interior relative to that of the exterior; the forcing function $\mathbf{j}(\mathbf{r})$ obviously vanishes everywhere outside the interior region.

Result equation:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{\mathrm{inc}}(\mathbf{r}) + k_1^2 \int_{V_{\mathrm{INT}}} \mathrm{d}^3\mathbf{r}' \overleftrightarrow{G}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{E}(\mathbf{r}')[m^2(\mathbf{r}') - 1]$$

$$= \mathbf{E}^{\mathrm{inc}}(\mathbf{r}) + k_1^2 \underbrace{\left(\overleftrightarrow{I} + \frac{1}{k_1^2}\nabla \otimes \nabla\right)}\cdot \int_{V_{\mathrm{INT}}} \mathrm{d}^3\mathbf{r}'[m^2(\mathbf{r}') - 1]\mathbf{E}(\mathbf{r}') \frac{\mathrm{e}^{ik_1|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r} - \mathbf{r}'|},$$

$$\mathbf{r} \in V_{\mathrm{INT}} \cup V_{\mathrm{EXT}}.$$

In brackets: tensor, i.e. linear vector function. $\mathbf{E}^{\mathrm{inc}}(\mathbf{r})$: incident plane wave.

The result equation can be solved, e.g. by iteration, entering $\mathbf{E} = \mathbf{E}^{\mathrm{int}}$ as a start on the right side. It must be solved for all possible polarization states of $\mathbf{E}^{\mathrm{inc}}(\mathbf{r})$.

*note:*
In electrostatics we have for the electric potential $\Phi$:

$$\Phi = \int d^3 \mathbf{r}\ \rho(\mathbf{r}') \,/\, |\mathbf{r} - \mathbf{r}'| \quad (\rho \text{ charge density}),$$

a similar Green function as in the previous projection.

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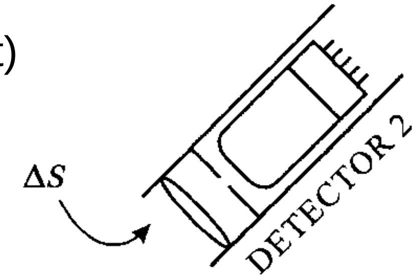
# Optical theorem 1:

Detector: ΔS is the detector area.
Size of particle « ΔS « r (distance to scattering object)
Lens and stop define the acceptance angle. Plane optics in front of the detector (not shown) allows to measure all Stokes parameters. The lens images the stop into infinity. Intensity is area x acceptance cone.
Detector 1 looks into direction of the incoming light and observes the extinction.
Detector 2 observes the scattered radiation.



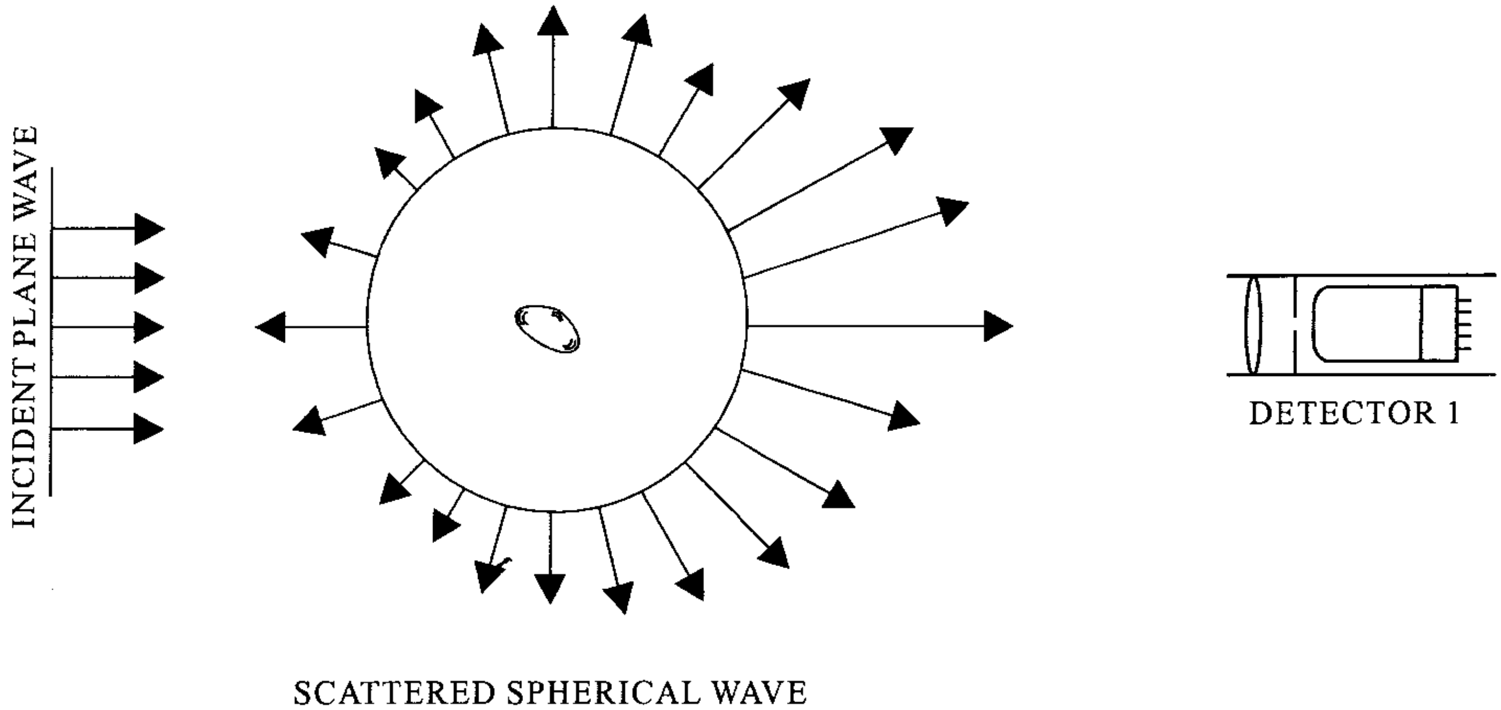




**Figure 2.3.** The response of the collimated detector depends on the line of sight.

# Optical theorem 2:

$$W_{\Delta S}(\hat{\mathbf{r}}) = \int_{\Delta S} \mathrm{d}S\, \hat{\mathbf{r}} \cdot \langle \mathbf{S}(\mathbf{r}') \rangle \approx \frac{1}{2}\sqrt{\frac{\varepsilon_1}{\mu_0}}\frac{\Delta S}{r^2}|\mathbf{E}_1^{\mathrm{sca}}(\hat{\mathbf{r}})|^2$$

$W_{\Delta S}$: Total electromagnetic power received by detector 2
**S**(**r**) is energy flow (Poynting vector) in any direction away from direction of incident wave.
dS element on detector surface. Δ*S* detector area.
$\mathbf{E}^{sca}_1$ electric field in embedding medium in the far field (depends on direction).

$$W_{\Delta S}(\hat{\mathbf{n}}^{\mathrm{inc}}) = \int_{\Delta S} \mathrm{d}S\, \hat{\mathbf{n}}^{\mathrm{inc}} \cdot \langle \mathbf{S}(\mathbf{r}') \rangle = \frac{1}{2}\Delta S \sqrt{\frac{\varepsilon_1}{\mu_0}}|\mathbf{E}_0^{\mathrm{inc}}|^2 - \frac{2\pi}{k_1}\sqrt{\frac{\varepsilon_1}{\mu_0}}\mathrm{Im}[\mathbf{E}_1^{\mathrm{sca}}(\hat{\mathbf{n}}^{\mathrm{inc}}) \cdot \mathbf{E}_0^{\mathrm{inc}*}] + O(r^{-2}),$$

$W_{\Delta S}$ ($n^{inc}$): Total electromagnetic power received by detector 1.
First term like scattered radiation,
second term is an attenuation term independent of Δ*S*.

# Optical theorem 3:

## Energy conservation in the scattering process:

Energy is removed from the incoming beam, partly absorbed in the particle (i.e. transferred to another frequency), partly scattered.

Extinction: The light in the incoming beam, after it has passed the particle (forward scattering):

Scattering (except forward scattering)

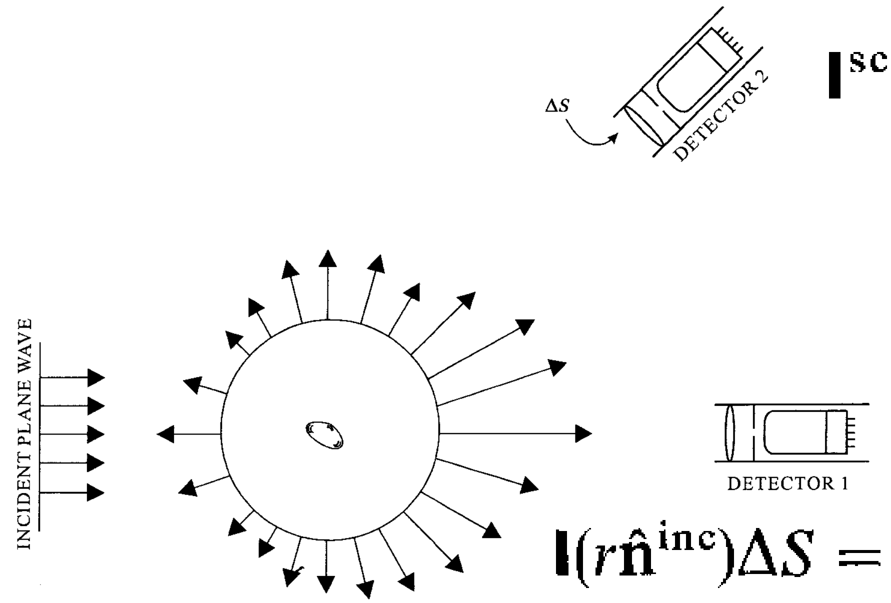
*Optical theorem:* The energy in the scattered light (except forward scattering) equals the energy removed by interference between incoming and forward scattered light.

$W_{ext} = W_{scat} + W_{abs}$

Divide by $\frac{1}{2}\sqrt{\varepsilon_1/\mu_0}\,|\mathbf{E}^{inc}|^2$ to get the cross-sections $C_{ext}$, $C_{scat}$, and $C_{abs}$.

The scattering cross-sections are always larger than the geometrical ones.

# Phase matrix and extinction matrix




$$\mathbf{I}^{\mathrm{sca}}(r\hat{\mathbf{n}}^{\mathrm{sca}}) = \frac{1}{r^2}\mathbf{Z}(\hat{\mathbf{n}}^{\mathrm{sca}}, \hat{\mathbf{n}}^{\mathrm{inc}})\mathbf{I}^{\mathrm{inc}},$$

$\mathbf{I}^{\mathrm{sca}}(r\hat{\mathbf{n}}^{\mathrm{sca}})$ Stokes vector of scattered radiation

**Z** Phase matrix

$$\mathbf{I}(r\hat{\mathbf{n}}^{\mathrm{inc}})\Delta S = \mathbf{I}^{\mathrm{inc}}\Delta S - \mathbf{K}(\hat{\mathbf{n}}^{\mathrm{inc}})\mathbf{I}^{\mathrm{inc}} + O(r^{-2}),$$

$\mathbf{I}(r\hat{\mathbf{n}}^{\mathrm{inc}})$ Stokes vector received by Detector 1

“Extinguished” radiation

**K** extinction matrix

Digression:

Extinction is possible without absorption:
Examples: Interstellar dust and Schott® shortcut filter glasses

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The thermal radiation emitted by the particle is characterized by a four-component column Stokes vector $\mathbf{K}_e(\hat{\mathbf{r}}, T, \omega)$ pointing radially away from the particle in the far field.
*In this thought experiment the particle is embedded in a cavity.* In the cavity the radiation is a collection of quasi-monochromatic, unpolarized, incoherent beams propagating in all directions with the Planck blackbody energy distribution

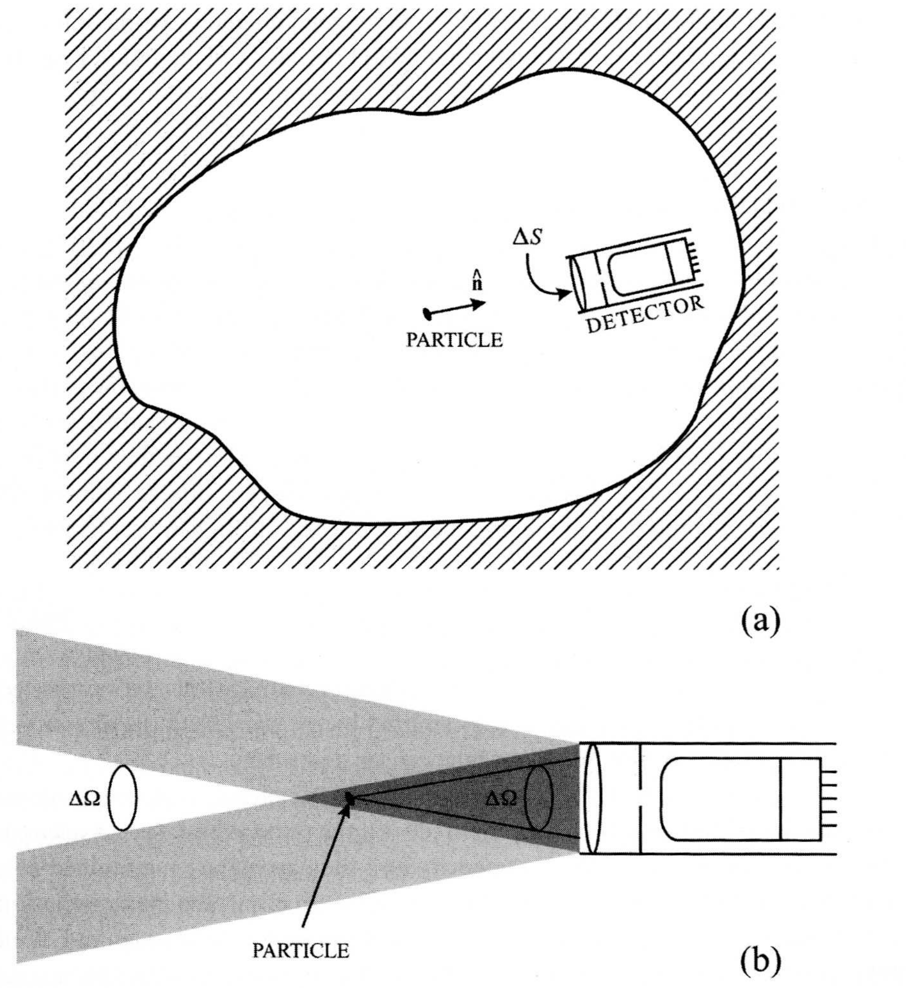


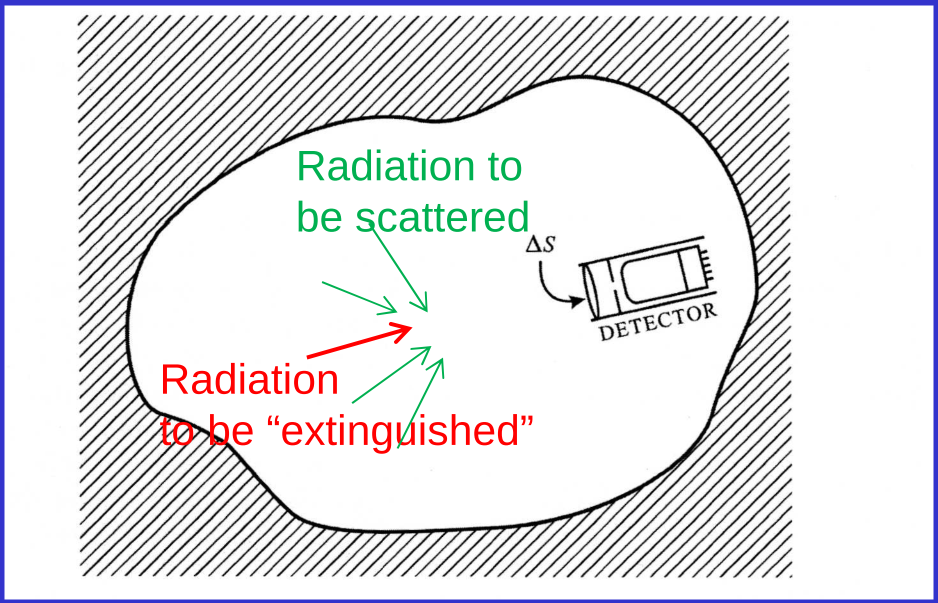

**Figure 2.4.** (a) Cavity, particle, and electromagnetic radiation field in thermal equilibrium. (b) Illumination geometry.

$$I_b(T,\omega)\Delta S\,\Delta\Omega = \frac{\hbar\omega^3}{4\pi^3 c^2\left[\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1\right]}\Delta S\,\Delta\Omega,$$

The detector is in the far field of the particle.
The cavity size is much larger than the distance of the detector from the particle.

The acceptance angle of the detector is selected to cover the particle in its full field.

In the absence of the particle the polarized signal per unit frequency interval measured by the detector is $\mathbf{I}_b(T,\omega)\Delta S\,\Delta\Omega,$




$$\mathbf{I}_b(T,\omega) = \begin{bmatrix} I_b(T,\omega) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

In the presence of the particle the polarized signal measured by the detector is

$$\mathbf{I}_b(T,\omega)\Delta S\,\Delta\Omega - \mathbf{K}(\hat{\mathbf{r}},\omega)\mathbf{I}_b(T,\omega)\Delta\Omega + \mathbf{K}_e(\hat{\mathbf{r}},T,\omega)\Delta\Omega + \Delta\Omega\int_{4\pi} d\hat{\mathbf{r}}'\,\mathbf{Z}(\hat{\mathbf{r}},\hat{\mathbf{r}}',\omega)\mathbf{I}_b(T,\omega)$$

| Planck cavity radiation | Radiation "extinguished" by particle | Radiation emitted by particle | Radiation scattered by particle |
|---|---|---|---|

But the cavity is in equilibrium with the particle, i.e. the particle should not add or take away radiation from the cavity

$$0 = -\mathbf{K}(\hat{\mathbf{r}},\omega)\mathbf{I}_b(T,\omega)\Delta\Omega + \mathbf{K}_e(\hat{\mathbf{r}},T,\omega)\Delta\Omega + \Delta\Omega\int_{4\pi} d\hat{\mathbf{r}}'\mathbf{Z}(\hat{\mathbf{r}},\hat{\mathbf{r}}',\omega)\mathbf{I}_b(T,\omega)$$

or

$$K_{ei}(\hat{\mathbf{r}},T,\omega) = I_b(T,\omega)K_{i1}(\hat{\mathbf{r}},\omega) - I_b(T,\omega)\int_{4\pi} d\hat{\mathbf{r}}' Z_{i1}(\hat{\mathbf{r}},\hat{\mathbf{r}}',\omega), \qquad i = 1,\ldots,4.$$

*This is the result.*

This equation must be considered for all directions $\hat{\mathbf{r}}$ and for all frequencies $\omega$.

Tell me your scattering properties and I will tell you the thermal radiation you emit.

The thermal emission can be anisotropic and polarized.

Because of its thermal radiation a force (analogous to the radiation force acting in case of scattering) is exerted on the particle.

$$\mathbf{F}_e(T) = -\frac{1}{c}\int_0^\infty d\omega \int_{4\pi} d\hat{\mathbf{r}}\,\hat{\mathbf{r}}K_{e1}(\hat{\mathbf{r}},T,\omega).$$

More on thermal emission can be found in the book:
Tsang, L., J. A. Kong, K.-H. Ding, “Scattering of electromagnetic waves, Theories and applications”, Wiley, 2000, Chapter 3.5, “Fundamentals of random scattering”, “Passive remote sensing”, in particular about the fluctuation dissipation theorem.

Also in the older book:
Tsang, L., J. A. Kong, R. T. Shin, “Theory of microwave remote sensing”, Wiley, 1985, there is a lot about thermal emission, as microwaves are mostly emitted via thermal emission.

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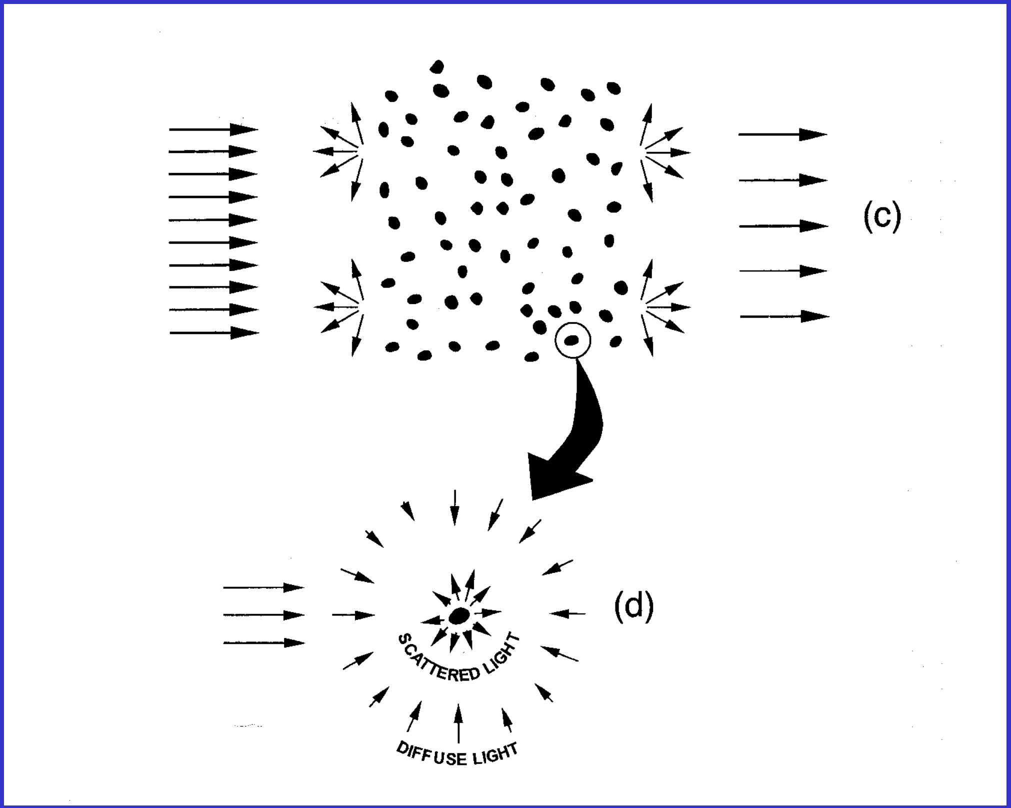
Up to now we have treated only the scattering in the far field.
The methods to be shown very briefly now, can in principle be applied to dense media as well.

In the past I have collaborated with Viktor Tishkovets, Charkiv, Ukraina.
There has been some progress concerning compact random media.

Scattering by many, not necessarily equal particles.
Particles may be densely packed.
Scattered light illuminates other particles and is scattered by them.

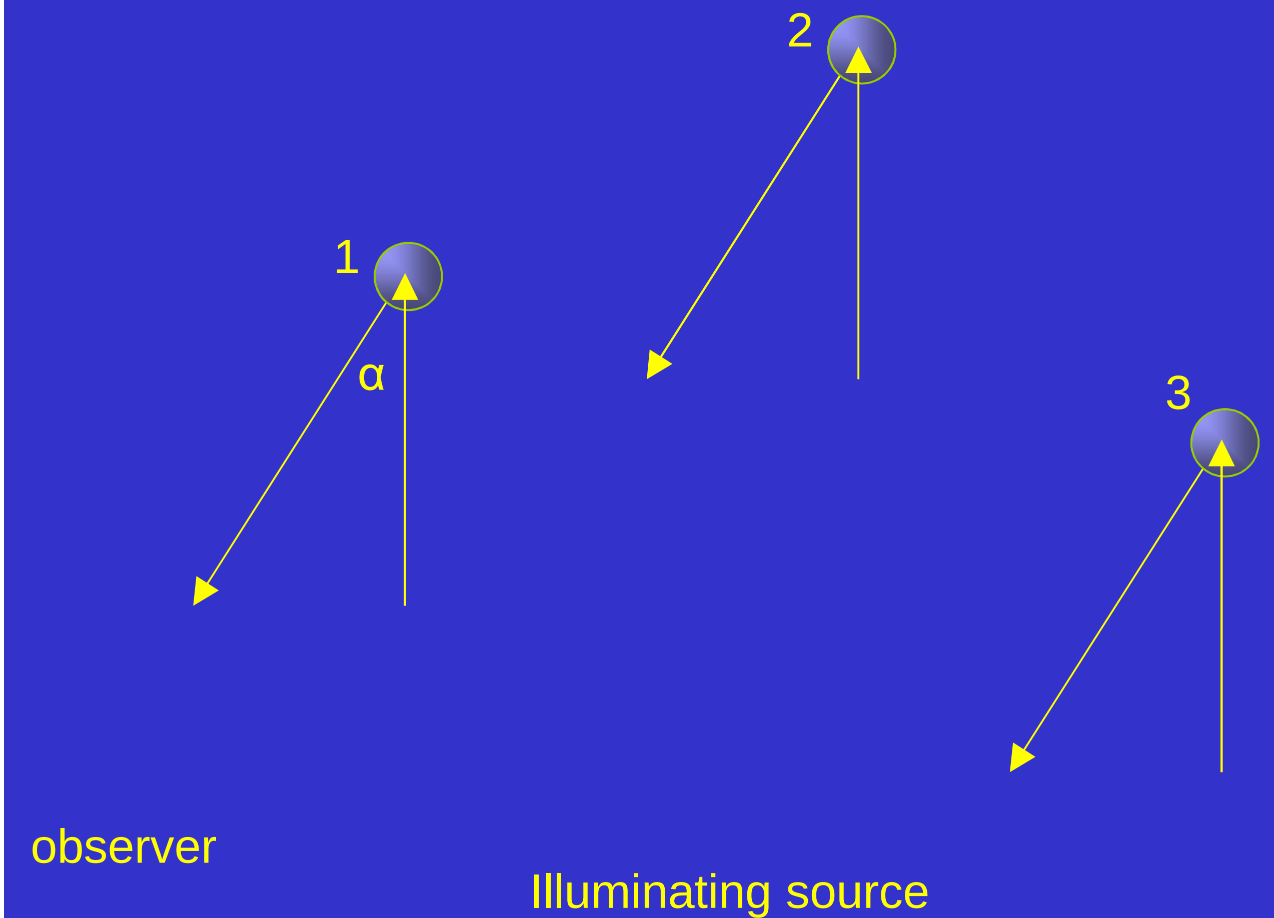



We consider the scattering in such a cloud as a sequence of scattering events on individual particles. Consequently we consider scattering orders, single scattering, double scattering, … , multiple scattering.

$\alpha$ is the phase angle, i.e. the angle light source – particle – observer.

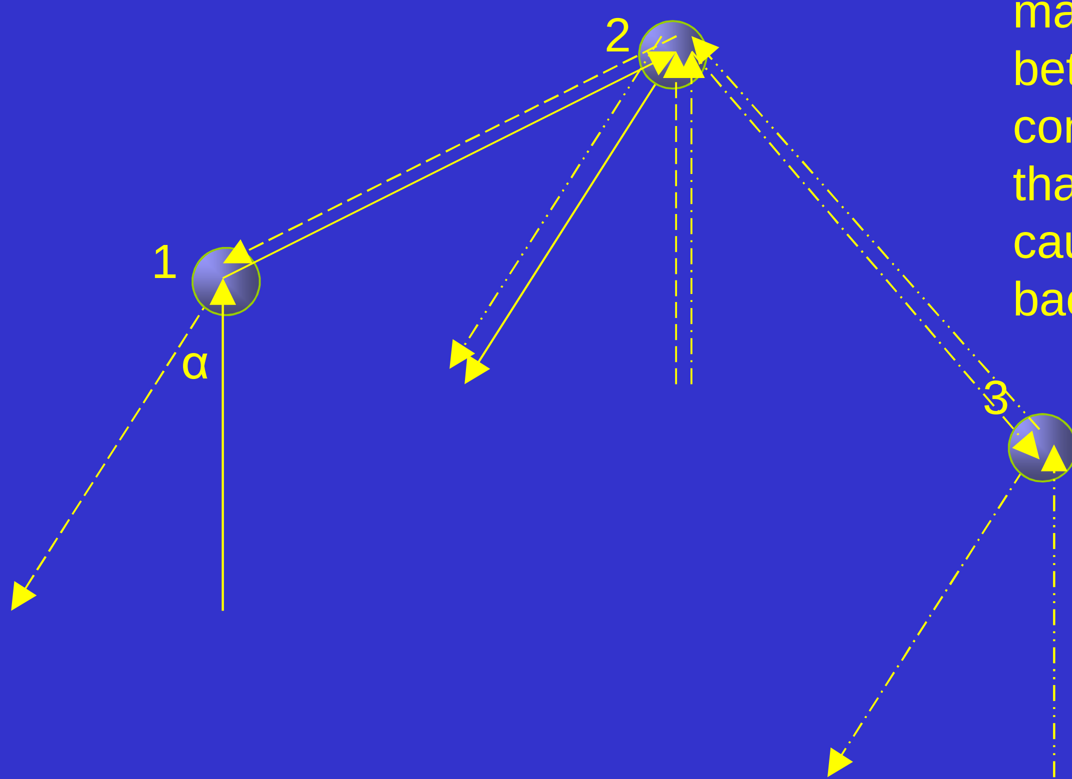
# Single scattering

Electric field = $E_1 + E_2 + E_3$



# Double scattering

Electric field = $E_{12} + E_{21} \quad + E_{23} + E_{32} + \ldots$




observer

Illuminating source

Note: For small phase angles α waves $E_{12}$ and $E_{21}$ will always be coherent, no matter if the distance between particles 1 and 2 is comparable with or larger than the wavelength. This causes the coherent backscattering effect.

If and only if the distance between particles 1 and 2 is comparable to λ waves $E_1$ and $E_2$ are coherent with waves $E_{12}$ and $E_{21}$.

## Triple scattering ...

For the electric field of our 3-particle cluster we have:

$$E = E_1 + E_2 + E_3 + E_{12} + E_{21} + E_{13} + E_{31} + E_{23} + E_{32} + \qquad \text{(eq. 1)}$$
$$+ E_{123} + E_{231} + E_{312} + E_{132} + E_{321} + E_{213} +$$
$$+ E_{121} + E_{131} + E_{212} + E_{232} + E_{313} + E_{323}$$

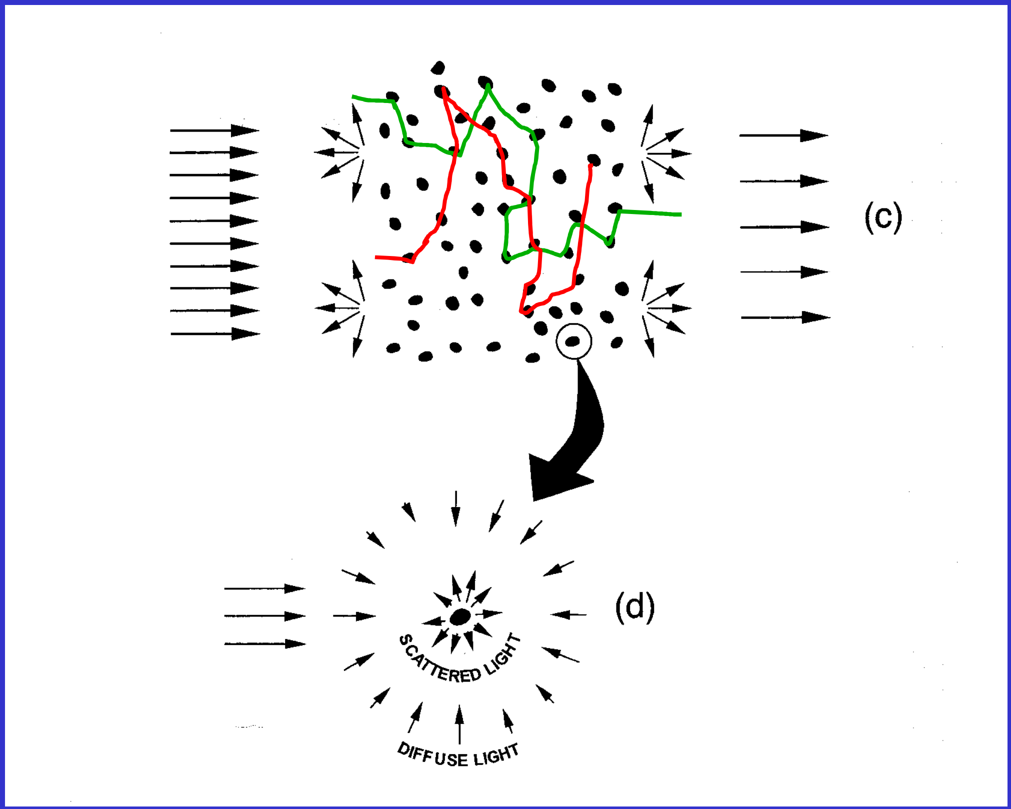
For a larger cluster we must take into account the higher scattering orders.

To calculate the Stokes parameters we must form the expressions

$$I = \langle E_{par} E^*_{par} \rangle + \langle E_{perp} E^*_{perp} \rangle$$
$$Q = \langle E_{par} E^*_{par} \rangle - \langle E_{perp} E^*_{perp} \rangle$$
$$U = \langle E_{par} E^*_{perp} \rangle + \langle E_{perp} E^*_{par} \rangle$$
$$V = \langle E_{par} E^*_{perp} \rangle - \langle E_{perp} E^*_{par} \rangle$$

From the linear sum of the electric fields we get bilinear sums for the Stokes parameters in which each term of eq. (1) is combined with all other terms.

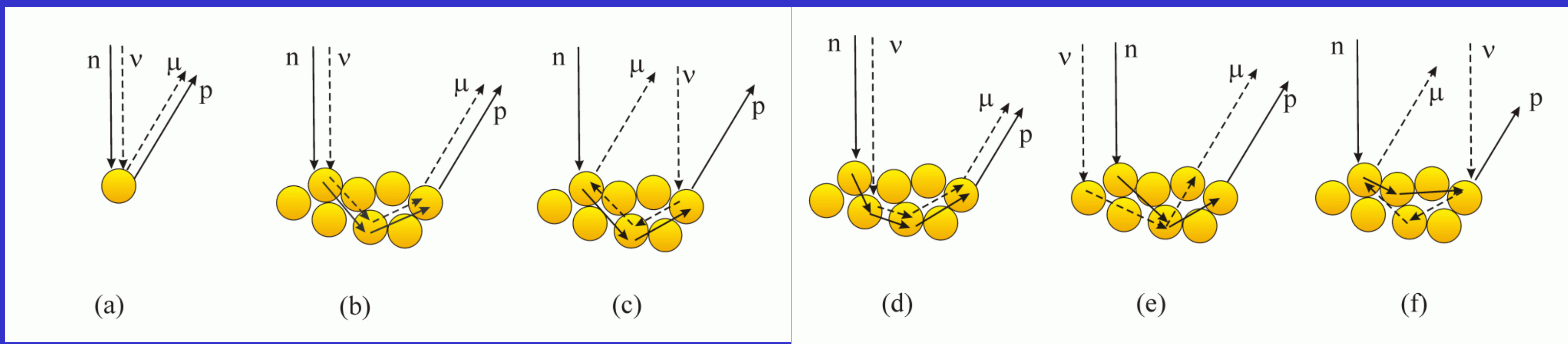
Consider pairs of paths through the cloud of particles and let them interfere.




Most of the paths be can considered uncorrelated, i.e. they will not interfere.

# Diagrammatic representation of scattering scenarios

important for dense media




2 waves interact (full and dotted lines).

*n, p* and *μ, ν* can have the values ±1, i.e. there are 4 components for the 4 Stokes parameters for each wave.

(a) Single scattering

(b) Ladder diagrams: incoherent (diffuse) scattering

(c) Cyclical diagram (coherent opposition effect)

(d) and (e): Generalization of (b) for the interference between different scattering orders (possible only in a dense medium with particles of size comparable to or less than the wavelength).

(f): Generalization of (c) for the interference of waves scattered by neighboring particles.

# Diagrammatic representation of scattering scenarios (continued)

Cases a-f: The later the letter in the ABC the denser the medium must be to cause an observable effect.

Sparse medium:
Only single scattering is important: Terms $<E_i E^*_k> = 0$ for $i \neq k$.

"Less sparse" medium:
Multiple scattering becomes important, but only ladder-type diagrams, and in the opposition range, the cyclical diagrams.

Dense medium:
If distance between scatterers ≤ λ, diagrams a – f are important.

# Outline of talk:

1. Transversal waves → Polarization must be taken into account! Characterization of polarization.

2. Scattering of electromagnetic radiation by an arbitrary, finite particle in the far-field zone

3. The optical theorem

4. Thermal radiation

5. Diagrammatic representation of scattering processes – scattering by random, discrete scatterers

6. Vector radiative transfer equation

Mishchenko, M. I.: “Vector radiative transfer equation for arbitrarily shaped and arbitrarily oriented particles: a microphysical derivation from statistical electro-magnetics”, Applied Optics 41 7114-7134, 2002.

To derive the VRTE, I had to make the following approximations.

- Assume that each particle is located in the far-field zones of all other particles and that the observation point is also located in the far-field zones of all the particles that form the scattering medium.
- Neglect all scattering paths that go through a particle two and more times (the Twersky approximation).
- Assume that the position and state of each particle are statistically independent of each other and of those of all other particles and that the spatial distribution of the particles throughout the medium is random and statistically uniform.
- Assume that the scattering medium is convex, which assured that a wave exiting the medium cannot reenter it.
- Assume that the number of particles $N$ that form the scattering medium is large and replace all factors of the type $(N-n)!/(N-n-k)!$ by $N^k$.
- Ignore all the diagrams with crossing connectors in the diagrammatic expansion of the dyadic correlation function (the ladder approximation).

*This is the end, thank you for your attention!*