Effect of rotation on stellar pulsations

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Issues

Accuracy on stellar parameters :

$> 50\%$ on the age $\sim 10\%$ on M, $\sim 6\%$ on R,

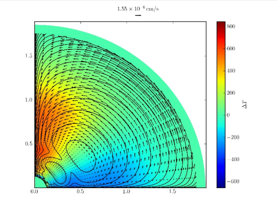
→ Sources of uncertainties : stellar interiors description

Stellar Physics

Rotation → dynamical processes :

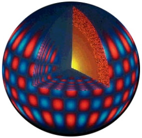
Angular momentum transport

⇒ Chemical elements transport, stellar winds, ...



⇒ Need for a new generation of stellar models

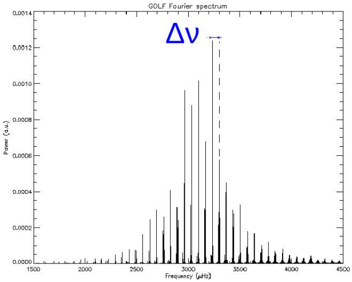
Mean : Asteroseismology



Study of the eigenmodes of a resonant cavity (the star)

Asteroseismic approach :

The sun' spectrum



(SOHO-Golf)

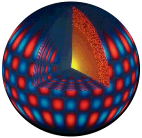
$\Rightarrow$ Mean sound speed :

$$\Delta\nu = \left[2\int_0^R \frac{dr}{c}\right]^{-1}$$

regularities $\Rightarrow$ seismic diagnostics

$\Rightarrow$ constraints on the internal structure

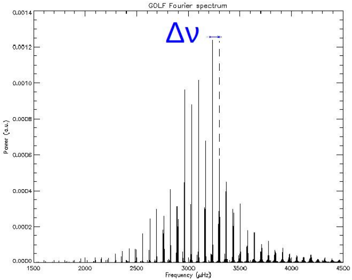
Mean : Asteroseismology



Study of the eigenmodes of a resonant cavity (the star)

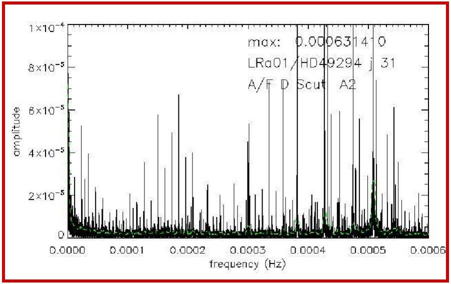
What about rotating stars ?

The sun' spectrum



(SOHO-Golf)

a δ Scuti spectrum



(CoRoT)

1. Introduction
2. The perturbative approach
3. Differential rotation in **Solar-like stars** $\epsilon, \mu \ll 1$
4. The 2D non perturbative approach : **ACOR**
5. Pulsation models of rapidly rotating stars $\epsilon \sim 1$
6. Rotational splitting in **Red giants** $\mu \sim 1$

Stellar pulsations without rotation

Formally : hydrodynamics equations perturbed + boundary conditions

$\Rightarrow$ pulsations eigenmodes ξ_r and eigenfrequencies σ

Separability in r and $(\theta, \varphi) \Rightarrow \xi_r = \tilde{\xi}_{r,n}(r)\, Y_\ell^m(\theta, \varphi)\, e^{i\sigma t}$

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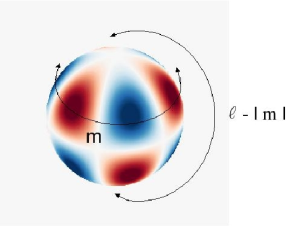
Angular distribution

3 quantum numbers : n, ℓ et m

- m : azimutal order $\in [-\ell : +\ell]$
  $|\, m \,|$ nodal meridians
- ℓ : angular degree
  $\ell - |\, m \,|$ nodal parallels

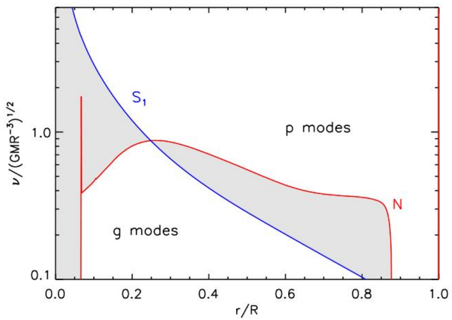
frequencies $\sigma_{n,\ell}$ are $(2\ell + 1)$ times **degenerate**



$\ell = 4,\ m = 3$

Stellar pulsations without rotation

Radial distribution



- Brunt-Vaissala frequency :

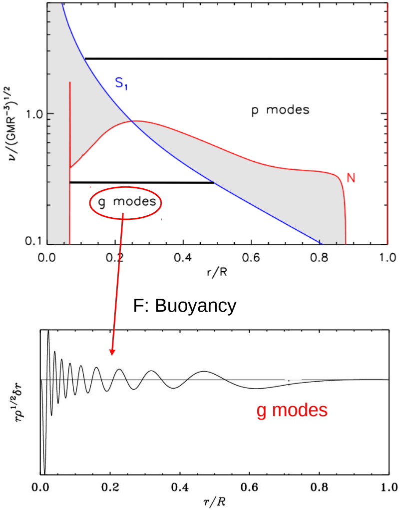
$$N^2 = g\left(\Gamma_1 \frac{d\ln P}{d\ln r} - \frac{d\ln\rho}{d\ln r}\right)$$

- Lamb frequency :

$${S_\ell}^2 = \frac{\ell(\ell+1)\,c_s^2}{r^2}$$

Stellar pulsations without rotation

Radial distribution



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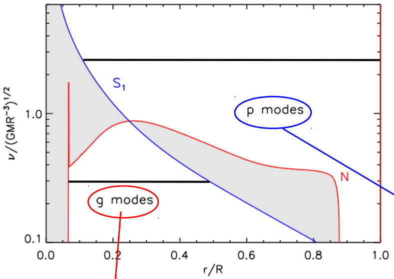

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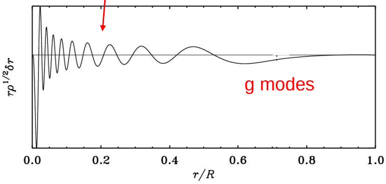
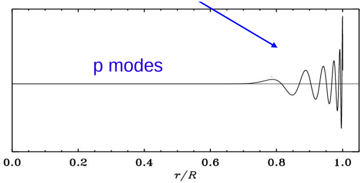
F: Buoyancy

F: Pressure force





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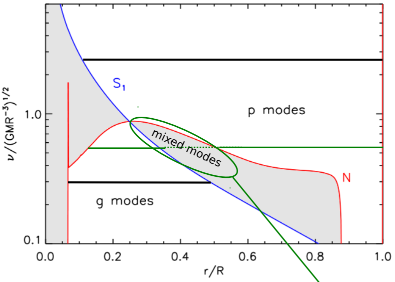

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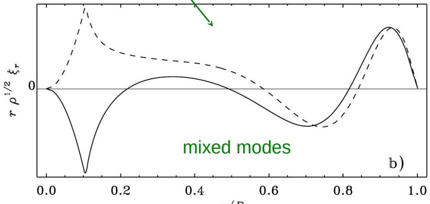
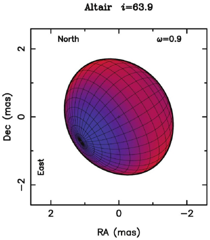


Impact of rotation

→ Centrifugal force distorts the resonant cavity

⇒ Bi-dimensional structure



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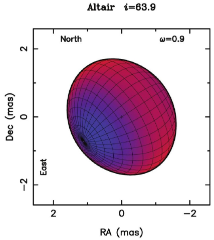


→ Centrifugal force distorts the resonant cavity

⇒ Bi-dimensional structure

→ Coriolis force modifies the modes dynamics :

$$\frac{\partial \mathbf{v}'}{\partial t} + (\mathbf{v_0} \cdot \nabla)\,\mathbf{v}' + 2\Omega \times \mathbf{v}' + (\mathbf{v}' \cdot \nabla)\mathbf{v_0} = -\frac{1}{\rho_0}\nabla p' - \nabla\Phi' + \frac{\rho'}{\rho_0^2}p_0$$

→ Non-separability of the system

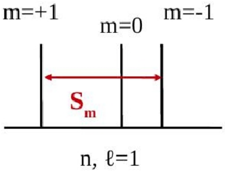
⇒ Expansion on spherical harmonics serie

$$\xi_r = \sum_{\ell \geq |m|} \tilde{\xi}_{r,n,\ell}(r)\, Y_\ell^m(\theta,\varphi)\, e^{i\sigma t}$$

Impact of rotation

- Lift of degeneracy

$\Rightarrow$ Observable : $S_{n,\ell,m}$ splitting



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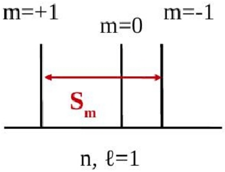
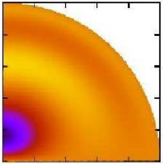
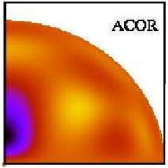


- Coupling of spherical harmonics

$\Omega = 0$



$\Omega \neq 0$



⇒ Problem of mode identification

Slow rotation / Rapid rotation

- Structural effects (centrifugal force) :

$$\epsilon = \frac{\Omega}{\Omega_k} \text{ where } \Omega_k = \sqrt{\frac{GM}{R^3}}$$

- Dynamical effects (Coriolis force) :

$$\mu = \frac{P_{osc}}{P_{rot}} = \frac{\Omega}{\omega}$$

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$\rightarrow$ $\epsilon,\ \mu \ll 1$
p modes in **Solar-like stars**

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p modes in **Solar-like stars**

→ $\epsilon \lesssim 1$
p modes in δ**Scuti stars**

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p modes in **Solar-like stars**

→ $\epsilon \lesssim 1$
p modes in δ**Scuti stars**

→ $\mu \lesssim 1$
g modes in **red giants**

Different approaches

The perturbative approach $\epsilon,\ \mu \ll 1$

- **Stellar model :** Spherical
- **Pulsations :**

$$\omega_{n,\ell,m} = \omega_{n,\ell}^{\Omega=0} + (\delta\omega)_{n,\ell,m}, \quad \text{with} \quad (\delta\omega) \ll \omega$$

The direct approach $\epsilon,\ \mu \sim 1$

- **Stellar model :** 2D distorted
- **Pulsations :** direct integration of the eigensystem

Formalism

$$\left(\mathcal{L} + \delta\mathcal{L}^{rot}\right)\xi = \omega^2\,\xi$$

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$\delta\mathcal{L}^{rot}$ due to rotation, $\delta\mathcal{L}^{rot} \ll \mathcal{L}$: Coriolis force, ...

Formalism

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$$\xi_n \,=\, \xi_{0,n} \,+ \sum_{\mathrm{i,\, i \neq n}} c_{n,i}\xi_{0,i} \quad \omega_{n,\ell,m} = \omega_{0,n,\ell} + \sum_{k=1,3} \mathrm{T_k}\, \Omega^{\mathrm{k}}$$

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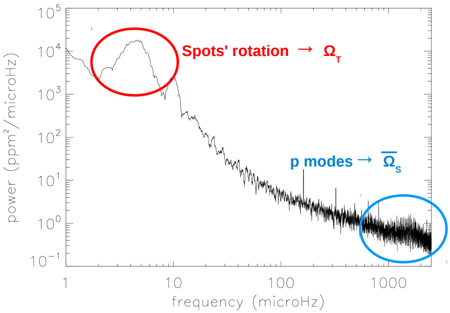
Differential rotation in solarlike stars

HD 181420



for some CoRoT targets

→ Spots'signature ⇒ Ω_T

$\neq$

→ Splitting ⇒ $\bar{\Omega}_S$

(Barban et al. 2009, Garcia et al. 2009)

Is the disagreement due to differential rotation in latitude ?

Differential rotation in solarlike stars

HD 181420, Solar like , slow rotator ($v \sin i \sim 18 \mathrm{km.s}^{-1}$)

Only a rotation profile differential in depth and in latitude is compatible with the CoRoT data

Rotation profile

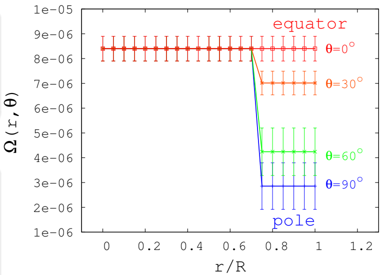
for $\mathrm{r} \leq \mathrm{r_{cz}}$, $\Omega(\mathrm{r}, \theta) = \Omega_0$

$\mathrm{r} \geq \mathrm{r_{cz}}$, $\Omega(\mathrm{r}, \theta) = \Omega_0 - \Delta\Omega \cos^2 \theta$

Perturbative methods give :

$$\mathrm{S}_{\ell=1} = \frac{\Omega_0}{\Omega_\mathrm{k}} \left(\beta - \frac{\Delta\Omega}{\Omega_0} \frac{1}{5} \beta_\mathrm{cz} \right)$$

$\Rightarrow$ determination of Ω_0 and $\Delta\Omega$



$\rightarrow$ Ouazzani & Goupil A&A 2012a

ACOR : Adiabatic Code of Oscillation including Rotation

Direct computations of linear adiabatic non-radial pulsations

Aims

- Rapid rotation
- for non-barotropic models $\Omega(\mathrm{r}, \theta)$
- at evolved stages
- adapted for massive computations

$\Rightarrow$ Requirements

- 2D, non-perturbative
- no hypothese on Ω
- adapted radial treatment
- numerically optimized

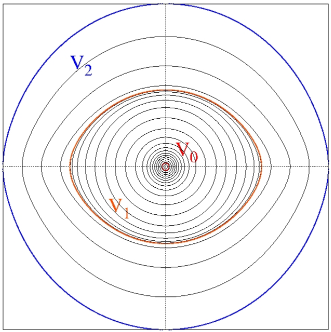
Adapted coordinate system

Multi-domain approach

new pseudo-radial coordinate : ζ

- $\zeta \to r$ at the center,
- ζ matches an isobar at the edge of the convective core,
- ζ matches the surface at $\zeta = 1$,
- additional domain V_2 : where $\zeta \to r$ at $\zeta = 2$.

2 $M_\odot$ star $\Omega \simeq 80\% \, \Omega_k$



$\Rightarrow$ Simplifies the boundary conditions

Numerical method

Radial discretization

Finite differencies of the 5^{th} order over two consecutive layers (Scuflaire 2008)

- Accurate to the 5^{th} order
- Numerically very stable

Inverse iteration method

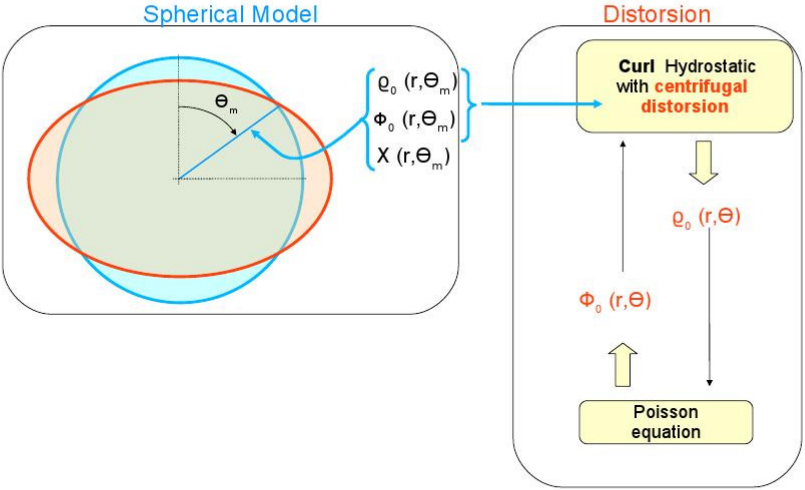
$$\mathcal{A}\,\mathcal{Y} = \delta\sigma\,\mathcal{B}\,\mathcal{Y}$$

- Resolution of the eigenvalue problem by the inverse iteration algorithm (Dupret 2003)
- Iteration $\sigma_0 = \sigma_0 + \delta\sigma$ until convergence

The code has been validated for polytropic models by comparison with Reese et al. 2006

→ Ouazzani, Dupret & Reese A&A 2012b

Distorsion of a spherical model (Roxburgh 2006)



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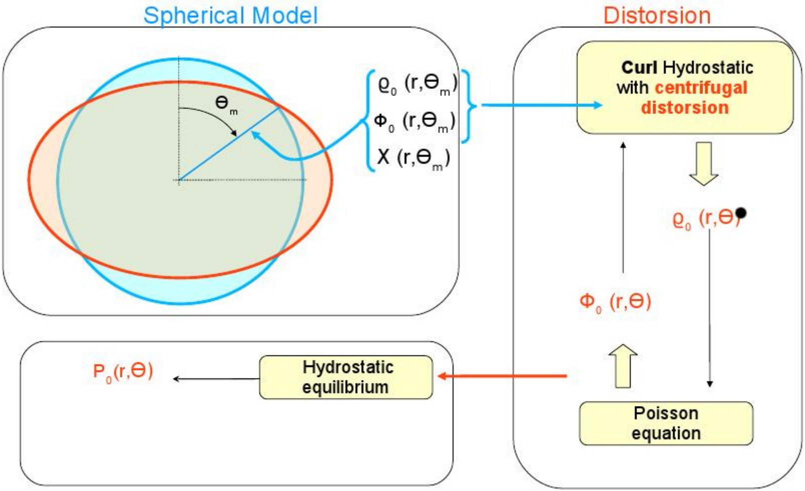

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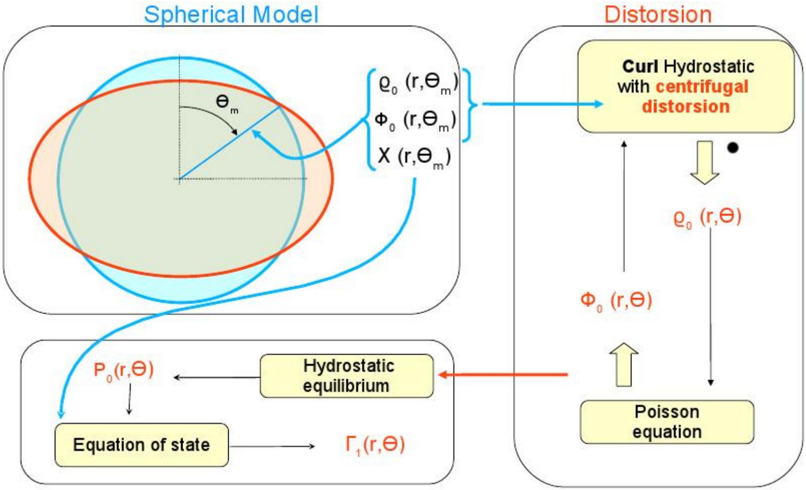
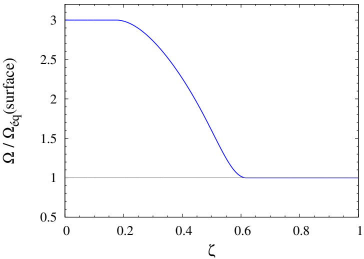


$\Rightarrow$ 2D acoustic structure

The spherical model : 2 $M_\odot$ non barotropic $\Omega = 80\% \, \Omega_k$

- Computed from a spherical model of 2 $M_\odot$ et 2.4 $R_\odot$,
- Initial composition : $X = 0.72$ et $Z = 0.02$,
- evolved until $X_c = 0.35$
- $\Omega_{center} / \Omega_{surface} = 3$ assumed

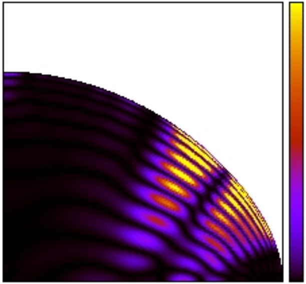
STAROX (OPAL) Roxburgh (2008)



$\Omega =$ constant on an isobar

Island modes

kinetic energy in a meridional plane



- Counterparts of low degree acoustic modes in the rapidly rotating case
- Caracterised by 2 quantic numbers $\tilde{n}$ and $\tilde{\ell}$
- Good visibility factor
- Show regularities

Explanation for regularities observed in δScuti stars ?

Island modes

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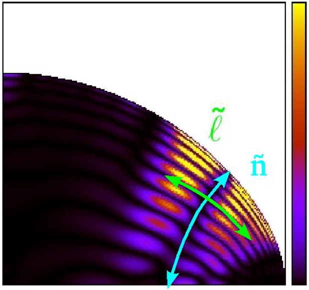
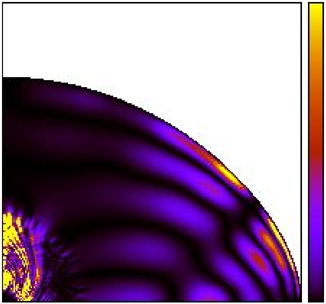


- Counterparts of low degree acoustic modes in the rapidly rotating case
- Probe outer layers
- Caracterised by 2 quantic numbers $\tilde{n}$ and $\tilde{\ell}$
- Good visibility factor
- Show regularities

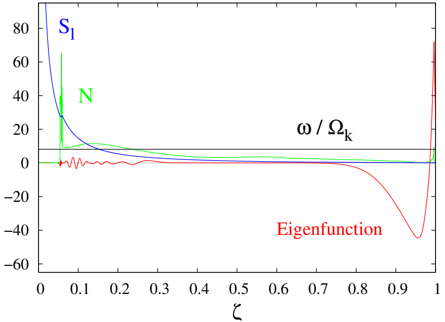
Explanation for regularities observed in δScuti stars ?

Mixed modes

kinetic energy in a meridional plane



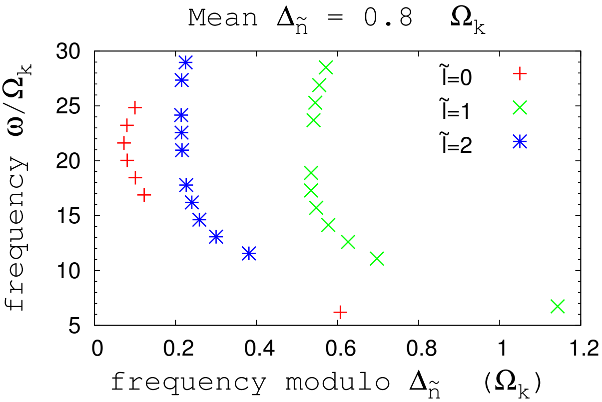
Propagation diagram

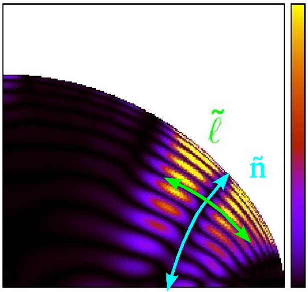


- Probe both the core and the envelop !
- Good visibility factor

Island modes regularity

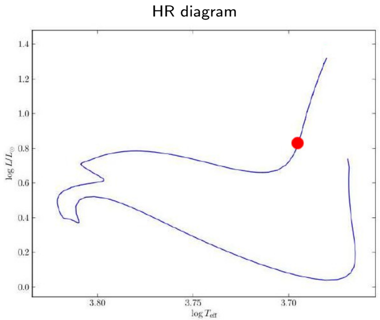
$$\Delta_{\tilde{n}} = \omega_{\tilde{n},\tilde{\ell},m} - \omega_{\tilde{n}-1,\tilde{\ell},m}$$

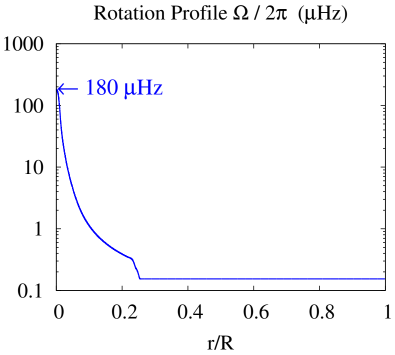




Asteroseismology of red giants : motivations

Test for angular momentum distribution along evolution
$\rightarrow$ mixed dipolar modes





detectable frequency range : $\nu_{\rm max} \pm 3\Delta\nu = 230 - 350\,\mu$Hz

Which treatment for rotational effects on red giants pulsations ?

Treatments of the pulsation-rotation interaction

Solar-like stars on the MS : linear treatment (Ledoux 1951) :

$$\mathrm{S_m} = \frac{\sigma_{\mathrm{n},\ell,-\mathrm{m}} - \sigma_{\mathrm{n},\ell,\mathrm{m}}}{2} = \frac{\mathrm{m}}{2\pi} \int \mathrm{K_{n,\ell,m}(r)}\, \Omega(\mathrm{r})\, \rho_0\, \mathrm{r}^2 \mathrm{dr}$$

$\Rightarrow$ Symmetrical and linear splittings

Treatments of the pulsation-rotation interaction

Solar-like stars on the MS : linear treatment (Ledoux 1951) :

$$S_m = \frac{\sigma_{n,\ell,-m} - \sigma_{n,\ell,m}}{2} = \frac{m}{2\pi} \int K_{n,\ell,m}(r)\, \Omega(r)\, \rho_0\, r^2 dr$$

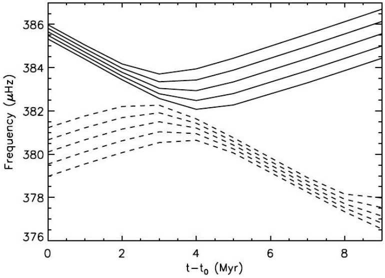
$\Rightarrow$ Symmetrical and linear splittings

Sub-giant stars :

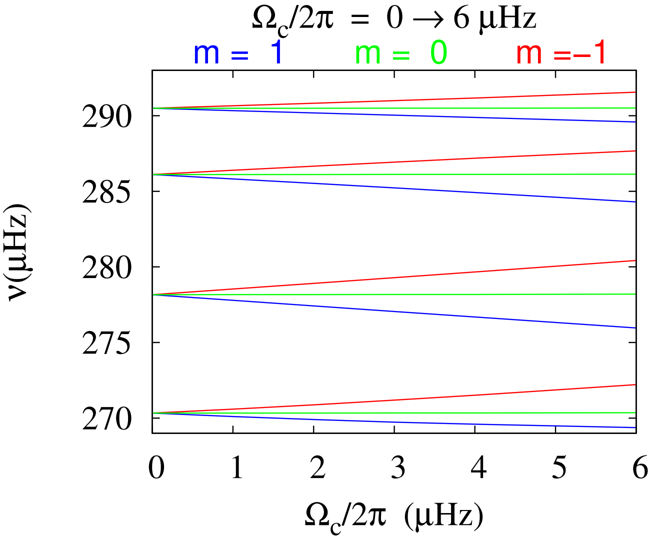
Asymmetry of mixed modes splittings

(Deheuvels, Ouazzani et al. 2012)

$\Rightarrow$ linear treatment not valid

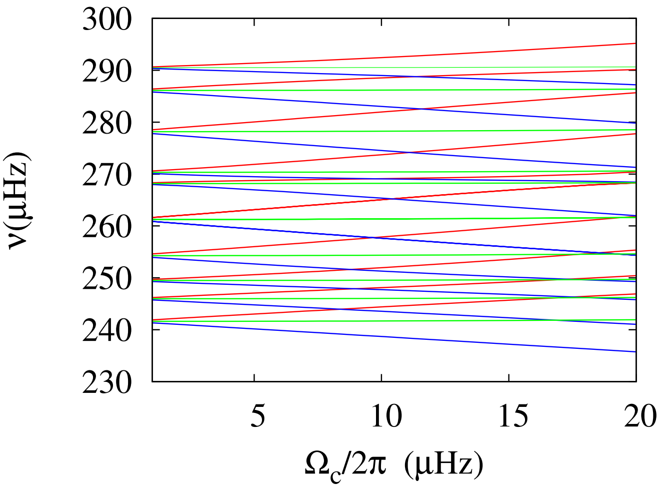


Is the linear treatment valid ?



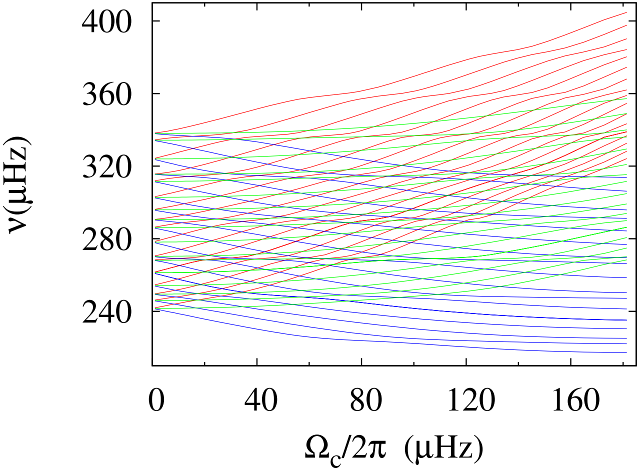
Is the linear treatment valid ?

$$\Omega_c/2\pi = 0 \rightarrow 20 \ \mu Hz$$



Is the linear treatment valid ?

$\Omega_c/2\pi = 0 \rightarrow 180\ \mu Hz$



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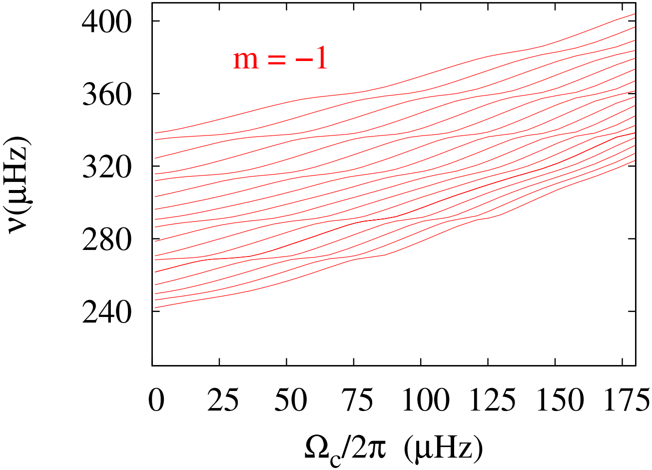

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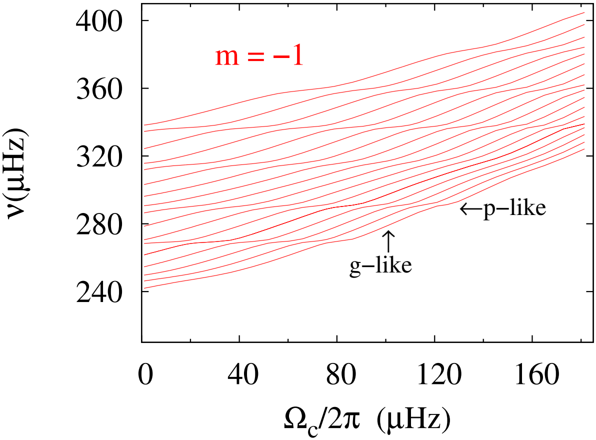
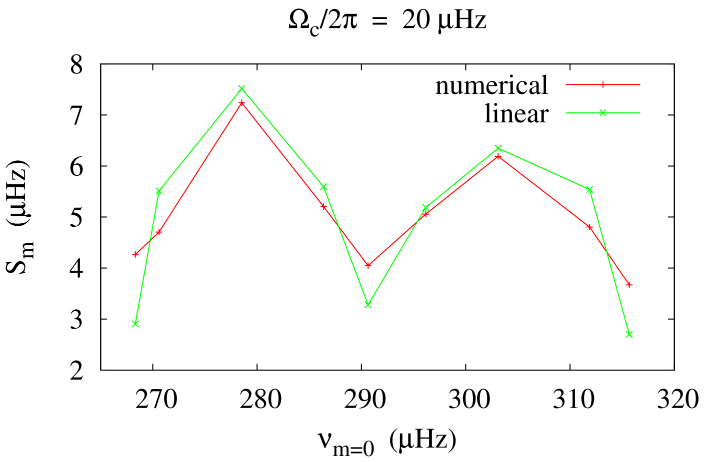
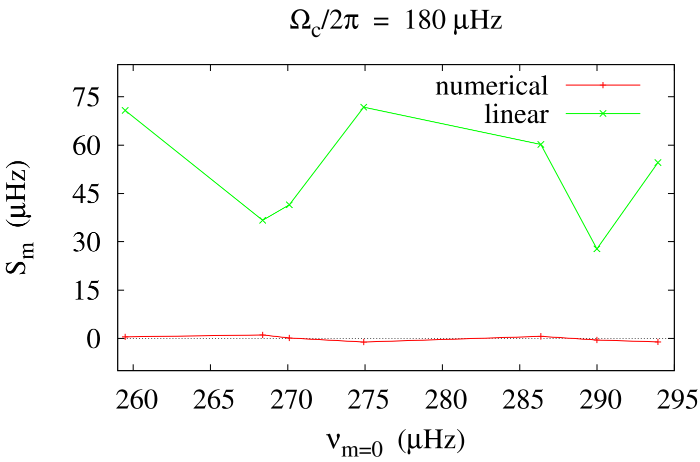


⇒ The concept of rotational splitting is not relevant

Misinterpretation of the pseudo-splitting ?



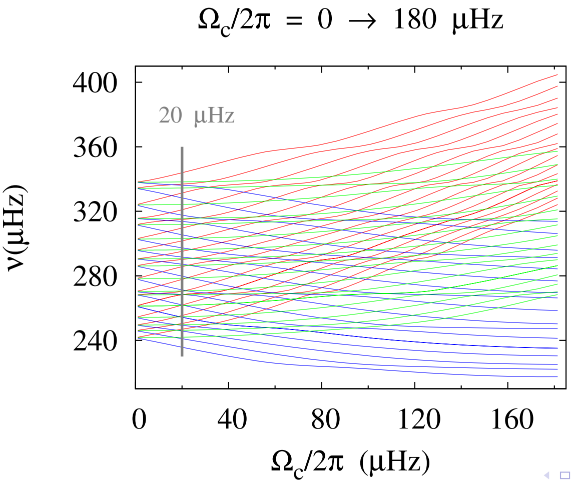
Misinterpretation of the pseudo-splitting ?



⇒ Small pseudo-splittings compatible with rapid core rotation

Toward a new seismic diagnostic

Spectrum structure : 20 μHz



Toward a new seismic diagnostic

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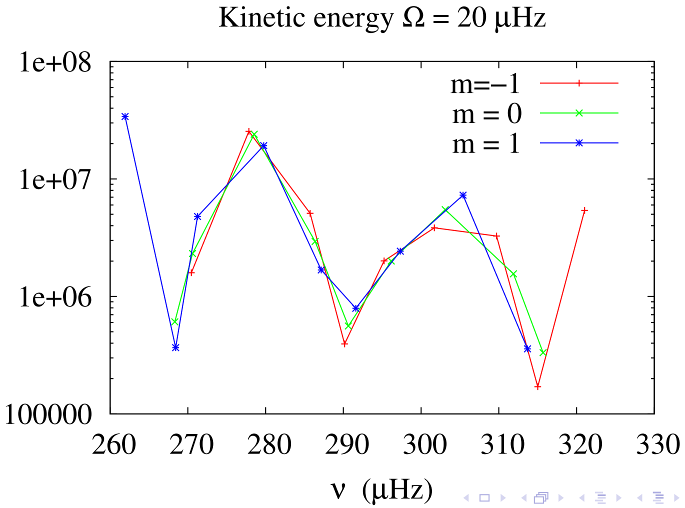
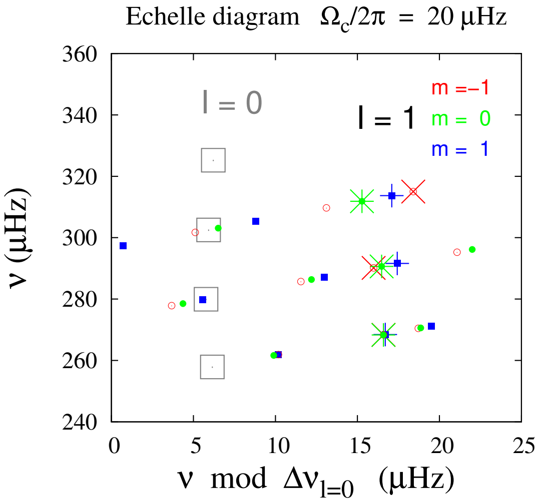
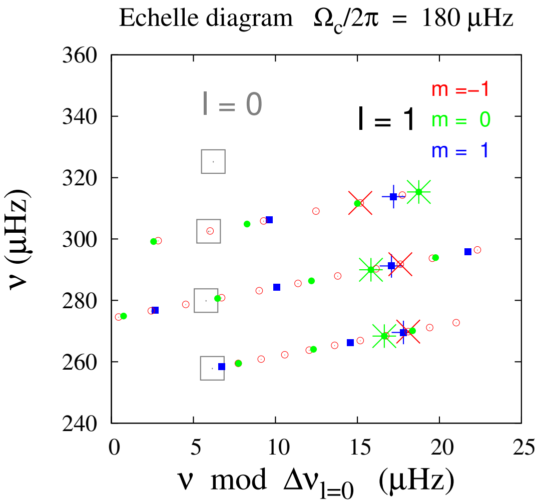


Large separation : 20 μHz



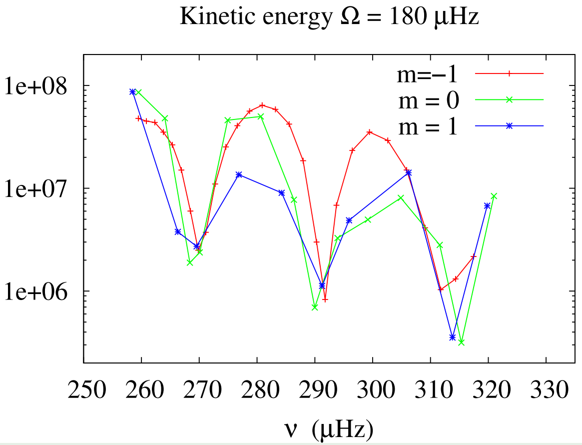
Large separation : 180 μHz



⇒ Large separation conserved at high rotation

Toward a new seismic diagnostic

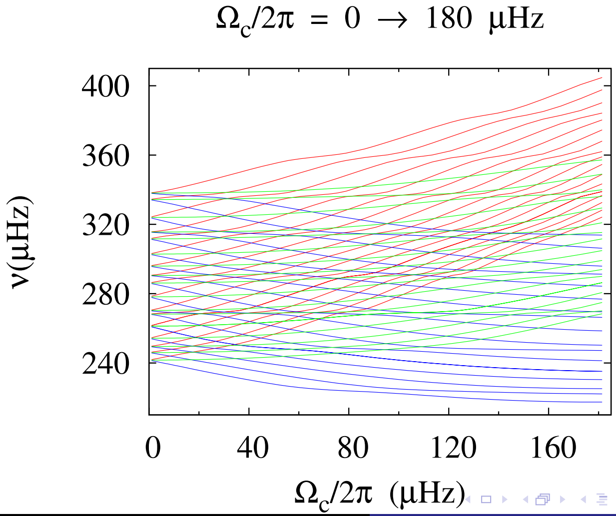
Mode density : 180 μHz



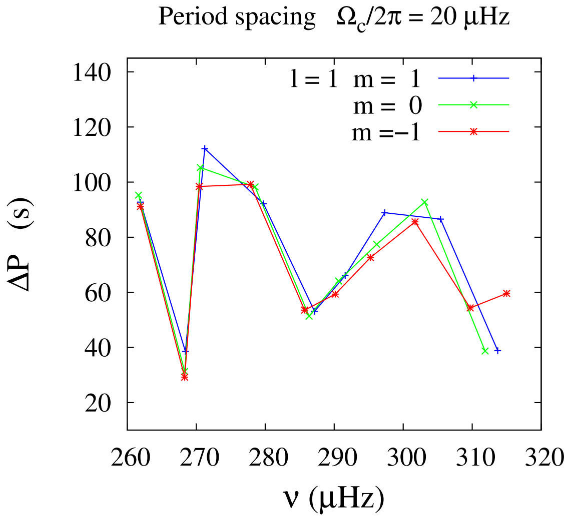
⇒ **Mode density of the spectrum depends on m**
the phenomenon increases with rotation

Toward a new seismic diagnostic

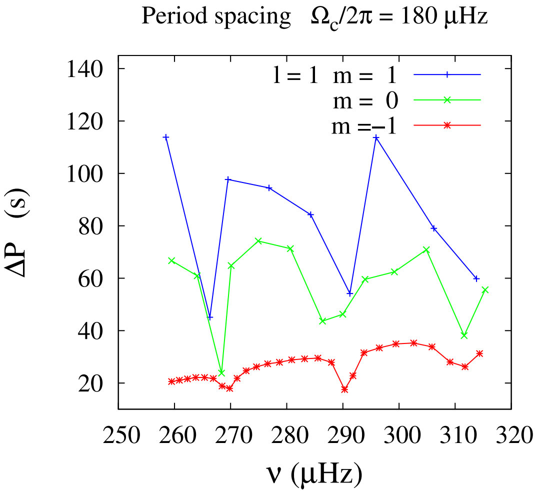
Mode density : 180 μHz



Toward a new seismic diagnostic



Toward a new seismic diagnostic



⇒ Period spacings of $\neq$ m as a seismic diagnostic of rotation

Conclusions

Red giants

Linear splitting not valid for the inversion of Ω profile in RGs

$\star$ rotation not as slow as predicted (Beck et al 2012)

$\rightarrow$ new seismic diagnostics of rotation : period spacing of $\neq m$

Solar-like stars

UHP asteroseismic mission $\Rightarrow$ accurate individual splittings

$\star$ constrain for differential rotation profile in solar-like stars

Rapid rotators

Non-perturbative 2D computations $\Rightarrow$ island modes regularities

$\star$ these regularities explain HD 174936 ? (Garcia-Hernandez 2009)

$\rightarrow$ statistical analysis of synthetic spectra

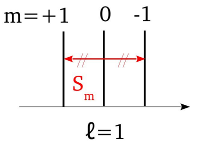
Formalism $\left(\mathcal{L} + \delta\mathcal{L}^{rot}\right)\,\xi = \omega^2\,\xi$

1^{st} order — Ledoux 1951

$$\omega = \omega_0 + m\,\Omega\,(1 - \mathrm{C}_{\mathrm{n},\ell})$$

Effect of Coriolis force mostly

$\rightarrow$ multiplets componants **equally spaced**



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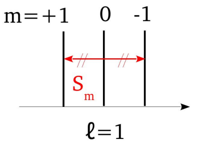
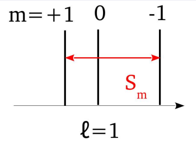


2^{nd} order — Dziembowski & Goode 1992

$$\omega = \omega_1 + \frac{\Omega^2}{\omega_0}\left(D_{1,n,\ell} + m^2 D_{2,n,\ell}\right)$$

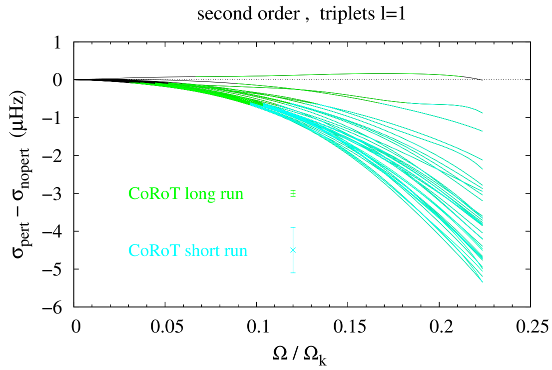
Effect of centrifugal force mostly

→ multiplets componants **not equally spaced**



Validity of the perturbative approach ?

Model of 1.3 $M_\odot$ ZAMS distorted as $P_2(\cos\theta)$, p modes pulsations



Comparison : perturbative 2^{nd} order calculations
non-perturbative calculations CoRoT : short run $\rightarrow$ 12 $\mathrm{km.s^{-1}}$ long run $\rightarrow$ 5 $\mathrm{km.s^{-1}}$