Stellar Structure and Evolution
—Exercises—
Wolfgang Glatzel
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GEORG-AUGUST-UNIVERSITÄT
GÖTTINGEN

Part 1

Calculate the intensity of the radiation field at the observers position of the thermal emission of an interstellar cloud. Its diameter is negligible compared to its distance. Assume LTE and constant temperature within the cloud and consider the limit of long wavelenghts. Discuss the intensity as a function of the optical depth of the cloud. Consider the limit of small and large optical depth.

How does the intensity depend on frequency ν and temperature? What is obtained for an opacity κ of the form $\kappa \propto \nu^{-2}$? Sketch the result.

Hint: Consider the radiation transport equation for the geometry given. Integrate it with respect to the optical depth (make use of integrating factors) from the observer across the cloud. Assume that the cloud is not irradiated by an external source. Is there an intuitive interpretation of the result? Then make use of the assumption of LTE and constant temperature.

Part 2

1) On the basis of order of magnitude estimates prove that a relation between pressure and density together with the requirement of hydrostatic equilibrium implies a relation between mass M and radius R of a star.

Hint: Derivatives of the form $\frac{dy}{dM_r}$ may be estimated by y/M, the density by $\rho \propto M/R^3$. Assume a power law relation between pressure and density of the form $p \propto \rho^\gamma$.

What are the mass - radius relations for $\gamma = 4/3$ and $\gamma = 5/3$? Interpret the result.

2) A stellar model: Assume that pressure and density of the stellar matter are related by an equation of state of the form $p = K\rho^2$ (p: pressure, ρ: density, K: Constant). Calculate the density as a function of radius using the requirement of hydrostatic equilibrium.

Hint: Use Poisson's equation for the gravitational potential. For the solution of the problem use $r\rho$ as dependent variable rather than ρ.

Are all solutions physically meaningful? Are there free parameters? Sketch and interpret the solution.

Part 3

1) Assume that the stellar matter can be described by the equation of state of an ideal gas $p/\rho \propto T/\mu$ (p: pressure, ρ: density, T: temperature, μ: mean molecular weight). Use hydrostatic equilibrium, radiative energy transport and "mass conservation" to derive (by order of magnitude estimates) a relation between luminosity L and mass M of a star. Hint: derivatives of the form $\frac{dy}{dM_r}$ may be estimated by y/M. How does the energy generation influence the luminosity of a star?

In addition, take into account energy conservation (same kind of approximation) with an energy generation rate of the form $\varepsilon \propto \rho T^\nu$ and derive a relation between luminosity and effective temperature (use Stefan - Boltzmann's law).

For the pp - chain we have $\nu \approx 5.3$, for the CNO - cycle $\nu \approx 15.6$. Where in the Hertzsprung - Russell - diagram do you expect to find the objects described by the relation derived? For orientation: where are stars that have the same radius?

2) Apart from nuclear reactions accretion is a mechanism by which heat and observable radiation is generated in astrophysics. In this process a gravitating object collects matter on its surface. The gravitational potential energy thereby released is transformed into thermal energy. How big is the maximum radiation energy per accreted mass for an accreting object with given mass and radius, which can be generated in this process?

Express the result in units of c^2 (c: speed of light) for the following accreting objects: a) the earth. b) the sun, c) a giant, d) a white dwarf e) a neutron star, and f) a black hole. Compare with the maximum energy per unit mass that can be generated by nuclear processes.

Part 4

1) Stars are acoustic resonators. Their period of oscillation (pulsation) Π is determined by their mean density ($\rho \sim M/R^3$) approximately through the period - density - relation

$$\Pi\sqrt{\rho} = constant$$

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Prove this relation on the basis of order of magnitude estimates: Use the relation between period and wavelength of a sound wave. How big is the maximum wavelength of a standing sound wave in a star (order of magnitude)? The sound speed may be estimated using the condition of hydrostatic equilibrium.

2) The period - luminosity - relation for Cepheids ($\log \Pi \sim \log L$) is of fundamental importance for distance measurements in astrophysics. Show that such a relation exists, if the excitation of Cepheid pulsations is restricted to a narrow strip in the HRD, which is approximately represented by $\log L = \alpha \log T_{eff} + constant$ with constant α.

Hint: Use the period - density - relation together with the mass - luminosity - relation and Stefan - Boltzmann's law.