Overshoot at the base of the solar convection zone

What can we learn from numerical simulations?

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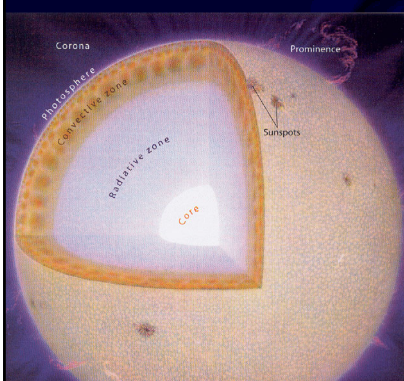
Outline

- What is overshoot?
- Why is overshoot interesting?
- Overshoot modeling
  different approaches ↔ different results
- What causes the discrepancies?
- What can we expect for solar overshoot?

What is overshoot?



- outermost 30% solar convection zone
- innermost 70% radiation zone
  $\delta = \nabla - \nabla_{ad} < 0$
  $\nabla = d \ln T / d \ln P$
- transition towards radiation zone no solid boundary

Penetration of convective motions

↓

Change of stratification

↓

Change of convective and radiative energy flux

Thermally relaxed state:
- momentum balance
- energy flux balance

Why is overshoot interesting?

Dynamo theory:
- Generation of strong toroidal magnetic field in solar tachocline
- Suppression of buoyancy instabilities require subadiabatic stratification
- $\beta\,|\delta|$ enters stability criteria, where
  $\beta = p_{gas} / p_{mag} \approx 10^5 \quad (10\,T)$

Indirect observations of convection by helioseismology:

- Measurements of flow fields near surface (local helioseismology)
- Measurements of differential rotation and meridonal circulation (local / global)
- Measurements of overshoot (global)
  change of soundspeed owing to stratification change

Overshoot modeling

different approaches ↔ different results ?

- van Ballegooijen (1982)
  convective rolls
- Schmitt, Rosner & Bohm (1984)
  convective plumes
- Pidatella & Stix (1986)
  non-local mixing length theory
  (Shaviv & Salpeter)

All 3 approaches got similar results!

Mixing-length theory:

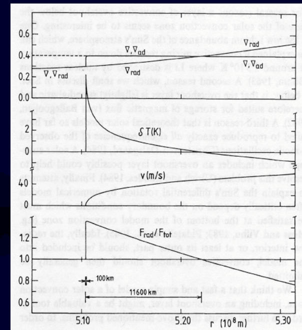


Fig. 1. Temperature gradients, ∇, ∇_{ad} and ∇_{rad}, temperature excess of convective "bubbles", convection velocities and the ratio, F_{rad}/F_{tot}, of the radiative and total energy fluxes, as functions of the distance, r, from the Sun's center. Case $l/H = \alpha = 1.25$

Pidatella & Stix (1986)

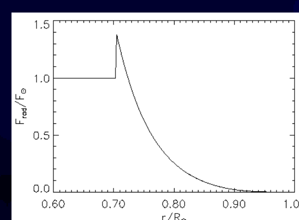
- low Mach number flow
  Ma ≈ 10^{-4}
- nearly adiabatic overshoot region
  $\delta \approx -10^{-6}$
- steep transition towards radiation zone
- depth of overshoot ≈ 0.2 — 0.3 Hp

Problems with this class of models:

- Helioseismology sets strong constraints:
  d < 0.05 … 0.1 Hp
  for models with sharp transition
- Radiative heating problem for magnetic flux storage:
  (Fan & Fisher 1996, Rempel 2002)

  ⇒ $\delta < -10^{-4}$ required for flux storage

Radiative heating problem:



- outward decrease of F_{rad} leads to heating
- quasistatic upward drift of flux tubes v ~ 1 / |δ|

Possible solutions:

- overshoot is more subadiabatic
- overshoot becomes more subadiabatic owing to suppression of convective motions

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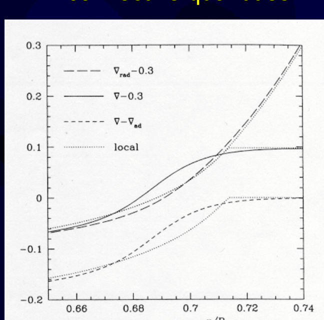
- Xiong & Deng (2001)
  non-local model for correlation functions of convective quantities



- depth ≈ 0.6 Hp
- smooth transition to radiation zone
- lower part of CZ already significantly subadiabatic

Numerical simulations:

- Hurlburt et al. (1994)
  2D simulations
  - nearly adiabatic overshoot (low stiffness)
  - strongly subadiabatic overshoot (large stiffness)
- Brummell et al. (2002)
  3D simulations
  - strongly subadiabatic overshoot

Overshoot depth about 1 … 2 Hp

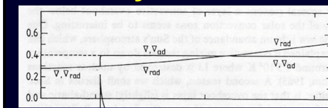
Solar parameters:

- Re ~ 10^{12}
- Pr ~ 10^{-8} (Pr = ν / κ)
- very small energy flux:
  $\eta = F / (\rho\, c_p T\, v_s) \sim 5\ 10^{-11}$
- highly stratified: $\rho_{bot} / \rho_{top} \sim 10^{6}$

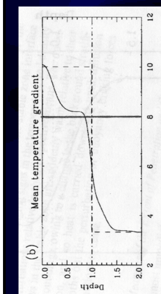
Numerical simulations:

- Re ~ 100 - 1000
- Pr ~ 1
- $\eta \sim 10^{-3} \ldots 10^{-4}$
- weakly stratified: $\rho_{bot} / \rho_{top} \sim$ 100 ... 1000

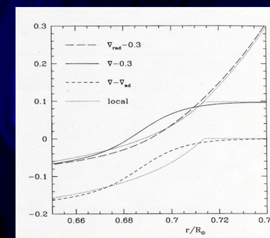
Summary: Overshoot models



Non-local mixing length (Pidatella & Stix 1985)



Turbulence theory (Xiong & Deng 2001)



Numerical simulation (Brummell et al. 2002)

What causes the discrepancies?

Two combined approaches:

- Numerical experiments closer to solar parameters:
  - low energy flux
  - smooth transition of rad. conductivity
- Semi-analytical model:
  - downflow dominated convection
  - low filling factor

Numerical model:

- Pr number of large scale flow is based on turb. viscosity and heat conductivity and is of O(1)
- Turbulent > radiative heat conductivity (ratio of 10^5 for sun)
- Decomposition of conductive heat flux:

$$F = -\kappa_{rad}\ grad\ \overline{T}\ -\ \kappa_{turb}\ grad\ T'$$

Advantages:

- Pr of convection independent of κ_{rad}
- Freedom to choose solar like profile for κ_{rad}
- Freedom to vary energy flux independent of Pr
- Possibility to choose very low energy flux

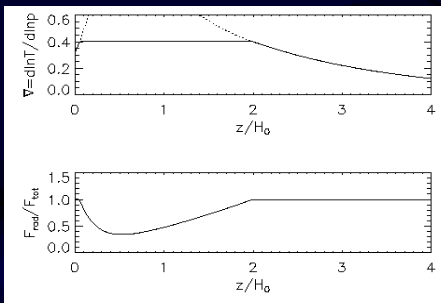
Problems:

- Very long thermal relaxation time owing to low energy flux
- Forced relaxation in the beginning of each simulation

$$\frac{\partial}{\partial t}\frac{d\overline{T}}{dz} = \lambda\left(1 - \frac{F_{conv} + F_{rad}}{F_{tot}}\right)$$

- Reduction of thermal relaxation time by a factor of 10 … 100

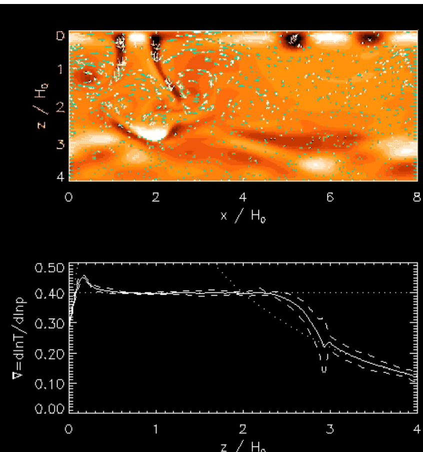
Initial state:



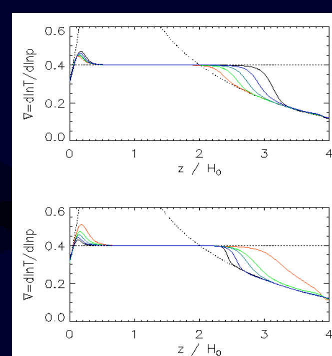
$\kappa \sim \rho_{0,}$, radiative boundary layer on top

Setup:

- 2D - simulations (3D shows no sig. difference)
- Box size: 8 H_0 x 4 H_0
- Resolution: 512 x 256
- Viscosity: $\mu = H_0\rho_0 v_0 / 3000$
- Constant gravity (g=1)
- Pr: 0.25 … 4.0 by changing κ_{turb} !
- $\eta = F / (\rho \, c_p T \, v_s)$: 10^{-6} … 10^{-4}
- MHD - code of M.P. Rast



Results:



Change of Pr
4.0 … 0.25

Change of η
10^{-6} … 10^{-4}

All models show nearly adiabatic overshoot

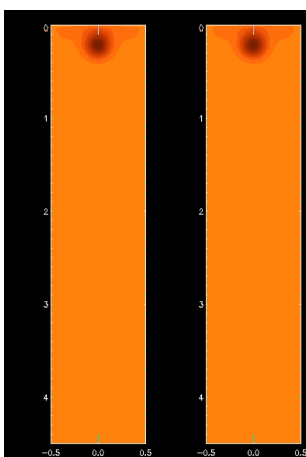
Influence of Pr on solution:

CZ: Change of structure and coherence of downflow plumes

OS: Deceleration of fluid parcels

> Reduction of buoyancy breaking of a single plume?

> Thermal adjustment of mean stratification?



Pr =1

Pr = 0.25 Pr = 4.0

Influence of η on solution:

- Change of Ma number of flow
- Change of effective Re of flow

Both effects difficult to separate in numerical simulation

- easy to change flux by a factor of 100
- difficult to change viscosity by a factor of 100

⇒ semi-analytical model to disentangle influences

Semi-analytic CZ + Overshoot model:

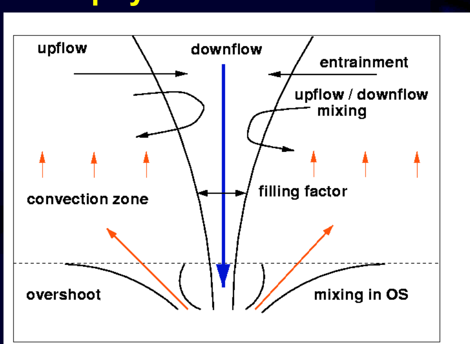
Assumptions:

- Convection driven by downflows
- Upflow passive (mass conservation)

⇒ 1D model describing downflow properties

⇒ Adjustment of mean stratification until momentum and energy balance fulfilled

Basic physics:



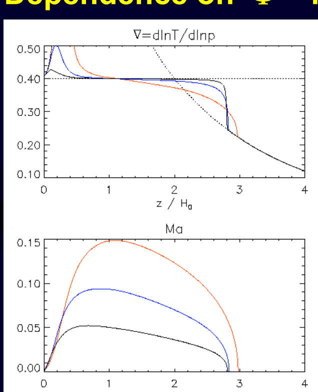
Downflow: buoyancy driving, mixing of momentum and enthalpy with upflow region

Free parameters (functions of z):

- filling factor of downflow
  - → massflux of downflow
  - → entrainment
- upflow / downflow mixing in CZ
- mixing in overshoot region
- total energy flux (can be absorbed into ff)

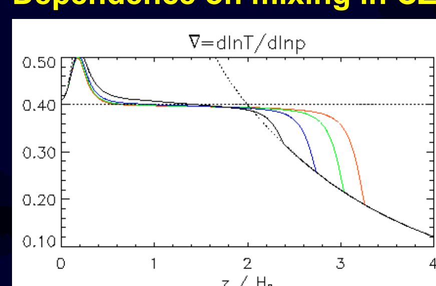
$$-\frac{ff}{F_{tot}}\rho\, v\left(c_p\left(T_d-T_u\right)+\frac{v^2}{2}\right)-\frac{\kappa}{F_{tot}}\frac{d\overline{T}}{dz}=1$$

Dependence on Φ = ff / F_{tot}:



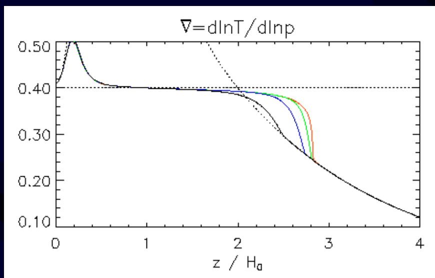
- largest influence on shape of overshoot
  - large Φ → step
  - small Φ → rounded
- significant change of Ma, but only little change of OS depth
- rounded profiles for solar OS if: ff ~ 10^{-7}

Dependence on mixing in CZ:



- large influence on depth of OS
- little influence on shape of OS
- change of superadiabaticity of CZ

Dependence on mixing in OS:



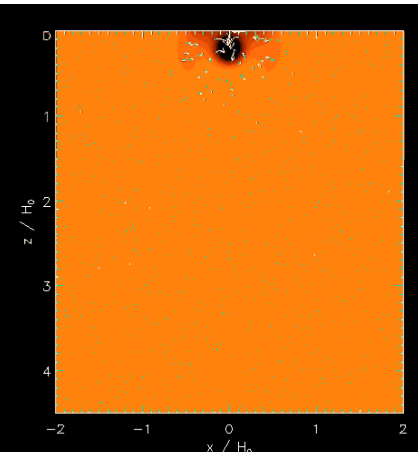
- large influence on depth of OS
- large influence on sharpness of transition region (only for moderate values of Φ (< 1000)

Conclusions:

- Basic overshoot properties for <u>downflow dominated</u> convection can be understood in terms of a highly simplified CZ-OS model
- Structure of overshoot determined by:
  - > Downflow filling factor at base of CZ (step function ↔ rounded profile)
  - > Mixing properties in CZ (mainly depth)
  - > Mixing properties in overshoot region (mainly depth, steepness of transition region)

Solar values:

- Filling factor:
  10^{-7} (no entrainment) . . . 0.1 (MLT)
- Mixing in CZ and OS: unknown
- Do downflows reach base of CZ?
  - > num. convection simulations: YES
  - > single downflow simulations (M.P. Rast): probably NOT



What are the next steps?

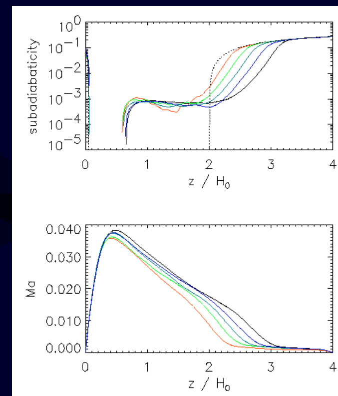
- 3D overshooting convection simulations
  > maybe in 20 years
- Single plume simulations:
  > don't solve full problem!
  (energy transport, plume interaction)
- Is there a possible step in between?

Approach from both sides:

- 3D experiments
  - > artificial reduction of downflow coherence by screen in convection zone
  - > artificial boundary condition preferring large scale motions
- Single downflow convection:
  > inclusion of energy flux
- Do merging plumes increase stability?

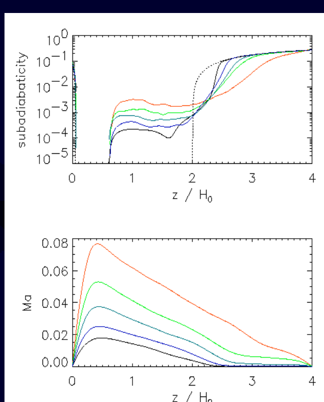
THE END

Change of Pr:



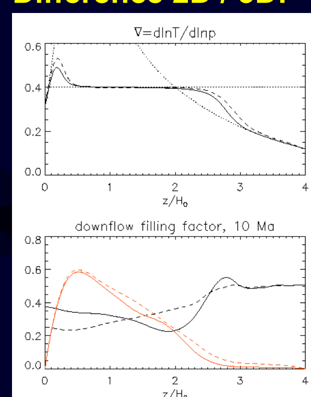
- increase of depth with decreasing Pr
- slight increase of Ma with decreasing Pr
- structure of CZ nearly unaltered
  > 50% of CZ subadiabatic

Change of η:



- increase of depth with total energy flux increase of Ma in CZ
- transition region steeper at low η
- CZ more subadiabatic at high η

Difference 2D / 3D:



---160 x 160 versus
—160 x 160 x 160
with same parameters

Main differences:

- profile of filling factor
- 3D: less deep OS
- 3D: more sub-adiabatic CZ