Effect of rotation on stellar pulsations

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Accuracy on stellar parameters:
- > 50% on the age
- \( \sim 10\% \) on M,
- \( \sim 6\% \) on R,

Sources of uncertainties: stellar interiors description

Stellar Physics
Rotation \( \rightarrow \) dynamical processes:
Angular momentum transport
\( \Rightarrow \) Chemical elements transport, stellar winds, ...

Need for a new generation of stellar models
Mean : Asteroseismology

Study of the eigenmodes of a resonant cavity (the star)

Asteroseismic approach :

The sun’ spectrum

⇒ Mean sound speed :

$$
\Delta \nu = \left[ 2 \int_0^R \frac{dr}{c} \right]^{-1}
$$

regularities ⇒ seismic diagnostics

⇒ constraints on the internal structure

(SOHO-Golf)
Mean: Asteroseismology

Study of the eigenmodes of a resonant cavity (the star)

What about rotating stars?

The sun’s spectrum

\[ \Delta \nu \]

(SOHO-Golf)

a \( \delta \) Scuti spectrum

(CoRoT)
1 Introduction

2 The perturbative approach

3 Differential rotation in Solar-like stars $\epsilon, \mu \ll 1$

4 The 2D non perturbative approach: ACOR

5 Pulsation models of rapidly rotating stars $\epsilon \sim 1$

6 Rotational splitting in Red giants $\mu \sim 1$
Stellar pulsations without rotation

Formally: hydrodynamics equations perturbed + boundary conditions

\[ \Rightarrow \text{pulsations eigenmodes } \xi_r \text{ and eigenfrequencies } \sigma \]

**Separability** in \( r \) and \((\theta, \varphi)\) \[ \xi_r = \tilde{\xi}_{r,n}(r) \ Y^m_\ell(\theta, \varphi) \ e^{i \sigma t} \]
Stellar pulsations without rotation

Formally: hydrodynamics equations perturbed + boundary conditions

⇒ pulsations eigenmodes $\xi_r$ and eigenfrequencies $\sigma$

Separability in $r$ and $(\theta, \varphi)$ ⇒ $\xi_r = \tilde{\xi}_{r,n}(r) Y^m_\ell(\theta, \varphi) e^{i\sigma t}$

Angular distribution

3 quantum numbers: $n$, $\ell$ et $m$

- $m$: azimuthal order $\in [-\ell : +\ell]$
  - $|m|$: nodal meridians
- $\ell$: angular degree
  - $\ell - |m|$: nodal parallels

frequencies $\sigma_{n,\ell}$ are $(2\ell + 1)$ times degenerate

$\ell = 4$, $m = 3$
Stellar pulsations without rotation

Radial distribution

- Brunt-Vaissala frequency:
  \[ N^2 = g \left( \Gamma_1 \frac{d \ln P}{d \ln r} - \frac{d \ln \rho}{d \ln r} \right) \]

- Lamb frequency:
  \[ S_{\ell}^2 = \ell (\ell + 1) \frac{c_s^2}{r^2} \]
Stellar pulsations without rotation

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Stellar pulsations without rotation

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Impact of rotation

→ Centrifugal force distorts the resonant cavity

⇒ Bi-dimensional structure
Impact of rotation

→ Centrifugal force distorts the resonant cavity

⇒ Bi-dimensional structure

→ Coriolis force modifies the modes dynamics:

\[
\frac{\partial \mathbf{v}'}{\partial t} + (\mathbf{v}_0 \cdot \nabla) \mathbf{v}' + 2\Omega \times \mathbf{v}' + (\mathbf{v}' \cdot \nabla)\mathbf{v}_0 = -\frac{1}{\rho_0} \nabla \rho' - \nabla \Phi' + \frac{\rho'}{\rho_0^2} \rho_0
\]

→ Non-separability of the system

⇒ Expansion on spherical harmonics serie

\[
\xi_r = \sum_{\ell \geq |m|} \tilde{\xi}_{r,n,\ell}(r) Y_{\ell}^m(\theta, \varphi) e^{i\sigma t}
\]
Impact of rotation

- Lift of degeneracy

⇒ Observable: $S_{n, \ell, m}$ splitting
Impact of rotation

- Lift of degeneracy
  - Observable: $S_{n,\ell,m}$ splitting

- Coupling of spherical harmonics

  $\Omega = 0$

  $\Omega \neq 0$

  $m=+1$
  $m=0$
  $m=-1$

  $S_{m}$
  $n, \ell=1$

  $\Rightarrow$ Problem of mode identification
Slow rotation / Rapid rotation

* Structural effects
  (centrifugal force):

\[ \epsilon = \frac{\Omega}{\Omega_k} \quad \text{where} \quad \Omega_k = \sqrt{\frac{GM}{R^3}} \]

* Dynamical effects
  (Coriolis force):

\[ \mu = \frac{P_{\text{osc}}}{P_{\text{rot}}} = \frac{\Omega}{\omega} \]
Slow rotation / Rapid rotation

- **Structural effects** (centrifugal force):
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- **Dynamical effects** (Coriolis force):
  \[ \mu = \frac{P_{osc}}{P_{rot}} = \frac{\Omega}{\omega} \]

\[ \rightarrow \epsilon, \mu \ll 1 \]

p modes in **Solar-like stars**

p modes in \( \delta \) Scuti stars

\( \epsilon \approx 1 \)

\( \mu \approx 1 \)

p modes in **red giants**
Slow rotation / Rapid rotation

- Structural effects (centrifugal force):
  \[ \epsilon = \frac{\Omega}{\Omega_k} \text{ where } \Omega_k = \sqrt{\frac{GM}{R^3}} \]

  \[ \rightarrow \epsilon, \mu \ll 1 \]

  p modes in Solar-like stars

- Dynamical effects (Coriolis force):
  \[ \mu = \frac{P_{osc}}{P_{rot}} = \frac{\Omega}{\omega} \]

  \[ \rightarrow \epsilon \lesssim 1 \]

  p modes in \( \delta \)Scuti stars
Slow rotation / Rapid rotation

- Structural effects (centrifugal force):
  \[ \epsilon = \frac{\Omega}{\Omega_k} \text{ where } \Omega_k = \sqrt{\frac{GM}{R^3}} \]

- Dynamical effects (Coriolis force):
  \[ \mu = \frac{P_{osc}}{P_{rot}} = \frac{\Omega}{\omega} \]

\[ \rightarrow \epsilon, \mu \ll 1 \]
  p modes in Solar-like stars

\[ \rightarrow \epsilon \lesssim 1 \]
  p modes in δScuti stars

\[ \rightarrow \mu \lesssim 1 \]
  g modes in red giants
Different approaches

The perturbative approach

- **Stellar model**: Spherical
- **Pulsations**:

  \[ \omega_{n,\ell,m} = \omega_{n,\ell}^{\Omega=0} + (\delta\omega)_{n,\ell,m}, \text{ with } (\delta\omega) \ll \omega \]

\[ \epsilon, \mu \ll 1 \]

The direct approach

- **Stellar model**: 2D distorted
- **Pulsations**: direct integration of the eigensystem

\[ \epsilon, \mu \sim 1 \]
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Formalism

\[(\mathcal{L} + \delta \mathcal{L}^{\text{rot}}) \xi = \omega^2 \xi\]
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\[(\mathcal{L} + \delta \mathcal{L}^{\text{rot}}) \xi = \omega^2 \xi\]

\[\mathcal{L} : \text{Pulsation operator without rotation} : \mathcal{L} \quad \xi_{\Omega=0} = \omega_0^2 \quad \xi_{\Omega=0}\]
Formalism

\[(\mathcal{L} + \delta\mathcal{L}^{\text{rot}}) \xi = \omega^2 \xi\]

\[\mathcal{L} : \text{Pulsation operator without rotation} : \mathcal{L} \quad \xi_{\Omega=0} = \omega_0^2 \quad \xi_{\Omega=0}\]
\[\delta\mathcal{L}^{\text{rot}} \text{ due to rotation, } \delta\mathcal{L}^{\text{rot}} \ll \mathcal{L} : \text{Coriolis force, ...}\]
Formalism

\[(\mathcal{L} + \delta\mathcal{L}^{\text{rot}}) \, \xi = \omega^2 \, \xi\]

\(\mathcal{L} : \) Pulsation operator without rotation : \(\mathcal{L} \quad \xi_{\Omega=0} = \omega_0^2 \quad \xi_{\Omega=0}\)

\(\delta\mathcal{L}^{\text{rot}} \) due to rotation, \(\delta\mathcal{L}^{\text{rot}} \ll \mathcal{L} : \) Coriolis force, ...

\[\xi_n = \xi_{0,n} + \sum_{i, i \neq n} c_{n,i} \xi_{0,i} \quad \omega_{n,\ell,m} = \omega_{0,n,\ell} + \sum_{k=1,3} T_k \Omega^k\]
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Differential rotation in solarlike stars

HD 181420

for some CoRoT targets

$\rightarrow$ Spots’s signature $\Rightarrow \Omega_T$

$\rightarrow$ Splitting $\Rightarrow \bar{\Omega}_S$


Is the disagreement due to differential rotation in latitude?
Differential rotation in solarlike stars

HD 181420, Solar like, slow rotator ($v \sin i \sim 18\, \text{km.s}^{-1}$)

Only a rotation profile differential in depth and in latitude is compatible with the CoRoT data

**Rotation profile**

for $r \leq r_{cz}$, $\Omega(r, \theta) = \Omega_0$

$r \geq r_{cz}$, $\Omega(r, \theta) = \Omega_0 - \Delta \Omega \cos^2 \theta$

**Perturbative methods give:**

$$S_{\ell=1} = \frac{\Omega_0}{\Omega_k} \left( \beta - \frac{\Delta \Omega}{\Omega_0} \frac{1}{5} \beta_{cz} \right)$$

$\Rightarrow$ determination of $\Omega_0$ and $\Delta \Omega$

→ Ouazzani & Goupil
A&A 2012a
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Direct computations of linear adiabatic non-radial pulsations

**Aims**
- Rapid rotation
- for non-barotropic models $\Omega(r, \theta)$
- at evolved stages
- adapted for massive computations

**Requirements**
- 2D, non-perturbative
- no hypothese on $\Omega$
- adapted radial treatment
- numerically optimized
Adapted coordinate system

Multi-domain approach
new pseudo-radial coordinate: $\zeta$

- $\zeta \rightarrow r$ at the center,
- $\zeta$ matches an isobar at the edge of the convective core,
- $\zeta$ matches the surface at $\zeta = 1$,
- additional domain $V_2$:
  where $\zeta \rightarrow r$ at $\zeta = 2$.

$2 \ M_\odot$ star $\Omega \simeq 80\% \Omega_k$

$\Rightarrow$ Simplifies the boundary conditions
Numerical method

Radial discretization
Finite differences of the 5\textsuperscript{th} order over two consecutive layers (Scuflaire 2008)
- Accurate to the 5\textsuperscript{th} order
- Numerically very stable

Inverse iteration method
\[ A \mathbf{Y} = \delta \sigma B \mathbf{Y} \]
- Resolution of the eigenvalue problem by the inverse iteration algorithm (Dupret 2003)
- Iteration \( \sigma_0 = \sigma_0 + \delta \sigma \) until convergence

The code has been validated for polytropic models by comparison with Reese et al. 2006
→ Ouazzani, Dupret & Reese A&A 2012b
Introduction

The perturbative approach

Differential rotation in Solar-like stars $\epsilon, \mu \ll 1$

The 2D non perturbative approach : ACOR

Pulsation models of rapidly rotating stars $\epsilon \sim 1$

Rotational splitting in Red giants $\mu \sim 1$
Distorsion of a spherical model (Roxburgh 2006)
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Curl Hydrostatic with centrifugal distortion

\( \rho_0 (r, \Theta) \)

\( \Phi_0 (r, \Theta) \)

\( P_0 (r, \Theta) \)

Equation of state

\( \Gamma_1 (r, \Theta) \)

Poisson equation

⇒ 2D acoustic structure
The spherical model: $2 \, M_\odot$ non barotropic $\Omega = 80\% \Omega_k$

- Computed from a spherical model of $2 \, M_\odot$ et $2.4 \, R_\odot$,
- Initial composition: $X = 0.72$ et $Z = 0.02$,
- Evolved until $X_c = 0.35$
- $\Omega_{\text{center}} / \Omega_{\text{surface}} = 3$ assumed

STAROX (OPAL) Roxburgh (2008)

\[
\begin{align*}
\Omega & = \text{constant on an isobar} \\
\end{align*}
\]
Island modes

kinetic energy in a meridional plane

- Counterparts of low degree acoustic modes in the rapidly rotating case
- Characterised by 2 quantic numbers $\tilde{n}$ and $\tilde{\ell}$
- Good visibility factor
- Show regularities

Explanation for regularities observed in $\delta$Scuti stars?
Island modes

kinetic energy in a meridional plane

- Counterparts of low degree acoustic modes in the rapidly rotating case
- Probe outer layers
- Characterised by 2 quantic numbers $\tilde{n}$ and $\tilde{\ell}$
- Good visibility factor
- Show regularities

Explanation for regularities observed in $\delta$Scuti stars?
Mixed modes

kinetic energy in a meridional plane

Propagation diagram

- Probe both the core and the envelop!
- Good visibility factor
Island modes regularity

\[ \Delta \tilde{n} = \omega_{\tilde{n}, \ell, m} - \omega_{\tilde{n} - 1, \ell, m} \]

Mean \( \Delta \tilde{n} = 0.8 \Omega_k \)
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Asteroseismology of red giants: motivations

Test for angular momentum distribution along evolution
→ mixed dipolar modes

detectable frequency range: \( \nu_{\text{max}} \pm 3 \Delta \nu = 230 - 350 \mu \text{Hz} \)

Which treatment for rotational effects on red giants pulsations?
Solar-like stars on the MS: linear treatment (Ledoux 1951):

\[ S_m = \frac{\sigma_{n,\ell,-m} - \sigma_{n,\ell,m}}{2} = \frac{m}{2\pi} \int K_{n,\ell,m}(r) \Omega(r) \rho_0 r^2 dr \]

⇒ Symmetrical and linear splittings
Treatments of the pulsation-rotation interaction

**Solar-like stars** on the MS: linear treatment (Ledoux 1951):

\[
S_m = \frac{\sigma_{n,\ell,-m} - \sigma_{n,\ell,m}}{2} = \frac{m}{2\pi} \int K_{n,\ell,m}(r) \Omega(r) \rho_0 r^2 dr
\]

⇒ Symmetrical and linear splittings

**Sub-giant stars:**

Asymmetry of mixed modes splittings

(Deheuvels, Ouazzani et al. 2012)

⇒ linear treatment not valid
Is the linear treatment valid?

\[ \frac{\Omega_c}{2\pi} = 0 \rightarrow 6 \mu \text{Hz} \]

\[ \Omega_c/2\pi = 0 \rightarrow 6 \mu \text{Hz} \]

\[ m = 1 \quad m = 0 \quad m = -1 \]
Is the linear treatment valid?

\[ \Omega_c/2\pi = 0 \rightarrow 20 \, \mu\text{Hz} \]
Is the linear treatment valid?

\[ \frac{\Omega_c}{2\pi} = 0 \rightarrow 180 \ \mu\text{Hz} \]
Is the linear treatment valid?

\[ \Omega_c/2\pi = 0 \rightarrow 180 \text{ } \mu\text{Hz} \]
Is the linear treatment valid?

\[ \frac{\Omega_c}{2\pi} = 0 \rightarrow 180 \, \mu\text{Hz} \]

\[ m = -1 \]

⇒ The concept of rotational splitting is not relevant
Misinterpretation of the pseudo-splitting?

\[ \Omega_c/2\pi = 20 \mu\text{Hz} \]

\( S_m (\mu\text{Hz}) \)

\( \nu_{m=0} (\mu\text{Hz}) \)

Numerical
Linear
Misinterpretation of the pseudo-splitting?

\[ \Omega_c/2\pi = 180 \, \mu\text{Hz} \]

\[ S_m (\mu\text{Hz}) \]

\[ \nu_{m=0} (\mu\text{Hz}) \]

⇒ Small pseudo-splittings compatible with rapid core rotation
Toward a new seismic diagnostic

Spectrum structure: 20 $\mu$Hz

$$\frac{\Omega_c}{2\pi} = 0 \rightarrow 180 \ \mu\text{Hz}$$
Toward a new seismic diagnostic

Spectrum structure: $20 \, \mu\text{Hz}$

Kinetic energy $\Omega = 20 \, \mu\text{Hz}$

$m = -1$

$m = 0$

$m = 1$
Large separation : 20 $\mu$Hz

Echelle diagram $\Omega_c/2\pi = 20 \mu$Hz

\begin{align*}
&\nu \mod \Delta\nu_{l=0} \quad (\mu\text{Hz}) \\
&\nu (\mu\text{Hz})
\end{align*}
Large separation: 180 \( \mu \text{Hz} \)

Echelle diagram: \( \Omega_c/2\pi = 180 \mu \text{Hz} \)

\( \nu \mod \Delta \nu_{l=0} (\mu \text{Hz}) \)

\( \nu (\mu \text{Hz}) \)

\( l = 0 \)

\( m = -1 \)

\( m = 0 \)

\( m = 1 \)

\( l = 1 \)

⇒ Large separation conserved at high rotation
Toward a new seismic diagnostic

Mode density: 180 $\mu$Hz

Kinetic energy $\Omega = 180 \, \mu$Hz

$\Rightarrow$ Mode density of the spectrum depends on $m$ the phenomenon increases with rotation
Toward a new seismic diagnostic

Mode density: 180 $\mu$Hz

$$\frac{\Omega_c}{2\pi} = 0 \rightarrow 180 \ \mu\text{Hz}$$
Toward a new seismic diagnostic

Period spacing $\Omega_c/2\pi = 20 \mu Hz$

- $l = 1$, $m = 1$
- $m = 0$
- $m = -1$
Toward a new seismic diagnostic

Period spacing \( \Omega_c/2\pi = 180 \mu \text{Hz} \)

\( l = 1 \)  \( m = 1 \)
\( m = 0 \)
\( m = -1 \)

⇒ Period spacings of \( \neq m \) as a seismic diagnostic of rotation
Conclusions

**Red giants**
Linear splitting not valid for the inversion of $\Omega$ profile in RGs
★ rotation not as slow as predicted (Beck et al 2012)
→ new seismic diagnostics of rotation: period spacing of $\neq m$

**Solar-like stars**
UHP asteroseismic mission $\Rightarrow$ accurate individual splittings
★ constrain for differential rotation profile in solar-like stars

**Rapid rotators**
Non-perturbative 2D computations $\Rightarrow$ island modes regularities
★ these regularities explain HD 174936? (Garcia-Hernandez 2009)
→ statistical analysis of synthetic spectra
Formalism
\[
(\mathcal{L} + \delta \mathcal{L}^{rot}) \xi = \omega^2 \xi
\]

1\textsuperscript{st} order

\[\omega = \omega_0 + m \Omega (1 - C_{n,\ell})\]

Effect of Coriolis force mostly

→ multiplets components \textbf{equally spaced}
Formalism

\[(\mathcal{L} + \delta \mathcal{L}^{\text{rot}}) \xi = \omega^2 \xi\]

**1\textsuperscript{st} order**

\[
\omega = \omega_0 + m \Omega (1 - C_{n,\ell})
\]

Effect of Coriolis force mostly

→ multiplets components equally spaced

Ledoux 1951

\[
\begin{array}{c}
m = +1 \quad 0 \quad -1 \\
S_m\end{array}
\]

\[\ell = 1\]

**2\textsuperscript{nd} order**

\[
\omega = \omega_1 + \frac{\Omega^2}{\omega_0} (D_{1,n,\ell} + m^2 D_{2,n,\ell})
\]

Effect of centrifugal force mostly

→ multiplets components not equally spaced

Dziembowski & Goode 1992

\[
\begin{array}{c}
m = +1 \quad 0 \quad -1 \\
S_m\end{array}
\]

\[\ell = 1\]
Validity of the perturbative approach?

Model of 1.3 $M_\odot$ ZAMS distorted as $P_2(\cos \theta)$, $p$ modes pulsations

second order, triplets $l=1$

Comparison: perturbative 2$^{nd}$ order calculations
non-perturbative calculations
CoRoT: short run
$\rightarrow$ 12 km.s$^{-1}$
long run $\rightarrow$ 5 km.s$^{-1}$