Chapter 5

Discussion[1]

A main purpose of this dissertation has been to develop a theoretical understanding of time-distance helioseismology. In particular we studied the sensitivity of travel times to subsurface sound speed perturbations, first in the single-source approximation and then in a distributed-source model. We also looked in detail at the validity of the Born approximation, which allows us to move beyond ray theory (the standard approach in time-distance helioseismology). The next three subsections give discussions of these three main topics. The remainder of this chapter is devoted to discussions of the observational work in this dissertation and a short section on possible extensions of the work that has been presented in this dissertation.

Single Source Kernels

We have shown an example single-source based travel-time sensitivity kernel in the Born approximation. This kernel shows that travel times depend on perturbations away from the geometrical ray, and in fact are insensitive to perturbations lying directly on the ray path. This is in direct contrast with ray theory.

We have demonstrated that the Born approximation agrees with normal mode perturbation theory for the case of spherically symmetric sound speed perturbations, though this is certainly not the most general statement that can be made about the connection between the two. We have shown, for an example forward calculation,

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[1] *Parts of this discussion are from the ApJ papers by Birch & Kosovichev (1998a, 2000); Birch et al. (2001a) and Gizon & Birch (2002).*

that for perturbations with large spatial scale the Born approximation and the ray approximation give the same result, while they differ significantly for small scale perturbations. The disagreement between the two is largest at the lower turning point of the ray and very near the surface.

Numerical Test of the Born and Ray approximations

Using a simple toy problem in a three dimensional Cartesian geometry with a homogeneous background sound speed, we have shown that for perturbations with radii much larger than the first Fresnel zone the Born and the first order ray approximations are equivalent. For smaller scale perturbations the ray approximation can overestimate travel times by orders of magnitude, while the Born approximation gives travel times that are of the correct order of magnitude. The Born approximation becomes inaccurate for perturbations where non-linear, in sound speed perturbation, effects are important (Hung et al., 2001). In addition, the Born approximation is inaccurate in cases where the diffracted wave is dominant.

A diffracted wave has not yet been observed with time-distance helioseismology. Confusion between direct and diffracted wave travel times is, therefore, not an important issue now but may be in the future. Of more importance are the implications of this work to the interpretation of ray based inversions. We have seen that ray theory overestimates travel times for small scale perturbations. As a result, inversions based on the ray approximation may underestimate small scale perturbations, as only very weak small scale perturbations are required to reproduce the observed travel times. The real perturbations inside the Sun are likely to be stronger than those inferred using ray theory. Inversions, however, involve complicated averages of observed travel times and are regularized. As a result the details of the effect of the inaccuracy of the ray approximation on ray based inversions are not yet clear. Born approximation kernels more accurately represent the true sensitivity of travel times to small scale perturbations; inversions done with these kernels can be expected to provide more accurate views of the solar interior.

Distributed Source Kernels

We now have a general recipe (section 3.4.1) for solving the linear forward problem, i.e. computing travel-time sensitivity kernels. This recipe is based on a physical description of the observed wave field. The kernels give the linear dependence of travel-time perturbations on perturbations to a solar model and they take account of the details of the measurement procedure. The sensitivity kernels depend on the background solar model, on the filtering and fitting of the data, and on position on the solar disk (through the line of sight).

In section 3.4.2 we have shown how to compute the 2D sensitivity of travel-time perturbations to source and damping inhomogeneities for surface gravity waves. This example was important as it shows that kernels can be obtained, using our recipe, once the physics of the model is fully specified. In particular the source spectrum and the details of the observation procedure need to be specified at the start of the problem and appear explicitly in the expression for the travel-time kernels.

The model with random excitation sources reveals some important details in the sensitivity kernels that are not accounted for in the single source model. In particular, the single-source kernels show only ellipse shaped features, while the distributed-source kernels show both hyperbola and ellipse shaped features. Computations of kernels in the single-source picture are as difficult, both analytically and numerically, as kernels in the distributed-source picture.

The example we have presented is a simplified model for the solar f mode. Improvements to the model would include stratification, spherical geometry, compressibility, and a physical model of excitation and damping. In particular, in a compressible medium the effect of the conversion of p modes into f modes by scattering could be computed. Despite these limitations, we believe that our 2D example kernels can be useful in studying solar problems using time-distance helioseismology. The kernels may be interpreted as depth averages over the first few Mm below the photosphere of the three-dimensional solar kernels (Duvall & Gizon, 2000).

Woodard (1997) performed an analysis of the effect of localized damping on travel times for acoustic waves; this analysis showed that for a model sunspot, with radius 10 Mm, the travel-time difference is of order -1 minute, in the case where $\mathbf{1}$

is located at the center of the sunspot and **2** is a distance 10 Mm away. For the same geometry the kernel K^{γ}_{diff}, which we have computed, predicts a positive travel-time difference of 1 s for a 50% increase in damping rate. These two apparently conflicting results are, however, for different types of waves and quite different models for the effect of damping inhomogeneities. The damping perturbation employed by Woodard (1997) can be understood in terms of a reduction in source strength for sources located behind the sunspot from the observation points, as scattering by the damping inhomogeneties was neglected. There remains work to be done on this subject. For example, it is known that absorption by magnetic structures is a strong function of frequency (Braun et al., 1988; Bogdan et al., 1993). This effect could be modeled by writing kernels for local changes to the exponent β in equation (3.97) for the damping rate. We plan to do a quantitative analysis of this problem in the future. Note that perturbations in sunspots are strong and that linear theory may not be accurate in this case (e.g. Cally & Bogdan, 1997)

The most significant obstacle to the computation of accurate travel-time kernels is our lack of a detailed understanding of turbulent convection. The excitation and damping of solar oscillations is due to convection and is thus extremely difficult to account for in the background model: approximations must be introduced. We employed a phenomenological model based on observed properties of solar convection. An important constraint on the zero-order solar model is that it must produce a k-ω diagram compatible with observations. A further complication introduced by turbulence is that, in principle, it demands a theory for wave propagation through random media, i.e. a treatment of perturbations that vary on short temporal and spatial scales.

We have not addressed the computation of three dimensional travel time kernels in a spherical solar model. Preliminary efforts have shown that such a computation is feasible, but is demanding (section 3.2).

There are a number of less fundamental issues relating to the interpretation of travel times. We emphasize that the filter $\mathcal{F}$ includes the point spread function of the instrument, which is not always well known. It is unclear how an inaccurate estimate of the point spread function affects the interpretation of travel-time measurements. A straightforward issue is that cross-correlations are typically averaged over annuli or sectors of annuli (Duvall et al., 1997); this can easily be accounted for by averaging the point-to-point kernels described in this dissertation.

Despite all of the aforementioned difficulties, the approach described in section 3.4.1 is feasible, as we have seen in section 3.4.2. Gizon et al. (2000) have shown the same procedure to work with real f-mode data.

Differential Rotation

In section 4.2 we showed a procedure for measuring the near-surface rotation from p-mode frequency splittings. Slow near-pole rotation was neither predicted by simulations nor expected from theory. Unfortunately the results of this study cannot be compared to the predictions of the global simulations that have been done to date. The global simulations (e.g. Gilman & Miller, 1986; Miesch et al., 2000) do not attempt to resolve near-surface small-scale convection so the upper layer in the simulations is typically near $0.95R_\odot$. As our result gives only a radial average of the rotation rate it cannot be compared in a meaningful way with the local *f*-plane model of Brummell and colleagues (1998), which predicts that zonal velocity is nearly independent of depth at 75° latitude. This prediction could be tested via a comparison of the subsurface rotation rate calculated here with the observed surface rotation for the same time period.

It has been suggested that the sharp decrease in near surface angular velocity with latitude seen at 70° is due to torque from the fast solar wind (e.g. Schou et al., 1998). Gilman (1974), however, argued that because of mixing the solar convection zone should be rigid to the solar wind torque. This conclusion was based on the assumption that mixing would extend to the base of the convection zone. Recent studies (e.g. Schou et al., 2002) have shown that there is a shear layer extending to a depth of roughly $0.02R_\odot$ in the near pole-regions, perhaps this thin shear layer plays some role in limiting the effect of mixing. Another factor that may be important is that we now know that magnetic field from the polar regions fills the bulk of the heliosphere, as most low latitude field is closed (e.g. Schrijver & Zwaan, 2000). Apparently additional work is needed on both the theoretical and numerical simulation, as well as observational, approaches to the problem of polar rotation. Now that helioseismic inferences of near-pole rotation are available perhaps further progress in the theory of solar differential rotation can be made.

A further argument for a magnetic effect causing slow polar rotation is the solar cycle dependence of the high latitude rotation rate that is suggested by the inversions of the BBSO data. Solar cycle variations have been seen in the near-pole surface rotation rate as well (e.g. Snodgrass & Howard, 1985). Snodgrass & Howard (1985) reported an approximately 15 nHz peak to peak variation in the surface rotation rate averaged between 62° and 70° latitude, with much smaller time dependence at lower latitudes. This variation is much like what is seen in the in OLA inversions of the BBSO data. Recently Antia & Basu (2001) studied the temporal evolution of the near-pole rotation rate using six years of GONG data and five years of MDI data. At high latitudes they saw bands of faster and slower rotation moving poleward, resulting in variation in the near-pole rotation rate on the time scale of the solar cycle. These bands have been associated with the solar dynamo (e.g. Covas et al., 2000), which, at least indirectly, connects the near-pole rotation rate to the large scale magnetic field.

Search for Longitudinal Structure

In section 4.3 we showed the preliminary results of a search for longitudinal variations in the sound speed in the deep convection zone. This search was not conclusive, but allows us to put a rough upper limit on variations in the strength of the magnetic field in the tachocline of 10^6 gauss. Current models of rising toroidal flux tubes suggest the field strength should be 10-100 kG (Fan et al., 1994) in order for the flux tubes to arrive at the surface at the active latitudes. Gilman & Fox (1999) argue, based on a model of the instability of differential rotation and toroidal magnetic field, for field strengths of at least 60 kG in the tachocline. The estimate we have obtained is quite preliminary and based on the very simple model that the sole effect of magnetic field is to change the wave speed from the sound speed to the fast mode speed. More detailed models are clearly necessary.

5.1 Future Work

There remains a substantial amount of work to be done on the interpretation of time-distance data. While we have shown a general approach for the calculation of travel-time kernels this approach has not yet been applied to the computation of three-dimensional sensitivity kernels for p-mode travel times. P-mode kernels would be very helpful; essentially all inversion codes currently in use are based on the ray approximation. In principle inversions should be developed that take account not only of flows and sound speed perturbations but also of magnetic field, source strength, and damping perturbations.

It is not clear how much information other than travel times can be extracted from observed cross-correlations. Perhaps the amplitude and central frequency of the cross-correlation could also be used as inputs into inversion codes. Or perhaps the entire waveform could be used, in the spirit of the work by Woodard (2002), instead of reducing the observed cross-correlations to a small set of parameters.

A serious current problem that has not been addressed in this dissertation is the noise source in time-distance helioseismology. In particular, the statistics and source of the noise are essentially not known. There is speculation that realization noise is the dominant noise source, but this has not been conclusively demonstrated. A good understanding of the noise is necessary for accurate inversions of time-distance data.

The measurements of near-pole rotation described in the first section of chapter 4 could be extended to cover more time, to follow the temporal evolution of the rotation rate over longer time scales. The near-pole rotation rate could perhaps be used as a constraint on those dynamo models which allow a back-reaction of the magnetic field on the plasma.

The time-distance work begun in the second section of chapter 4 could be extended to produce synoptic charts for more rotations. Perhaps interesting patterns would emerge. Another interesting avenue of research would be to look further into the correlation between the surface magnetic field and the observed deep-focusing travel times. One approach would be to make spatially resolved maps of the deep-focusing travel times that could be compared more directly to the magnetic field synoptic charts.