INVERSION OF F-MODE TRAVEL TIMES FOR FLOWS

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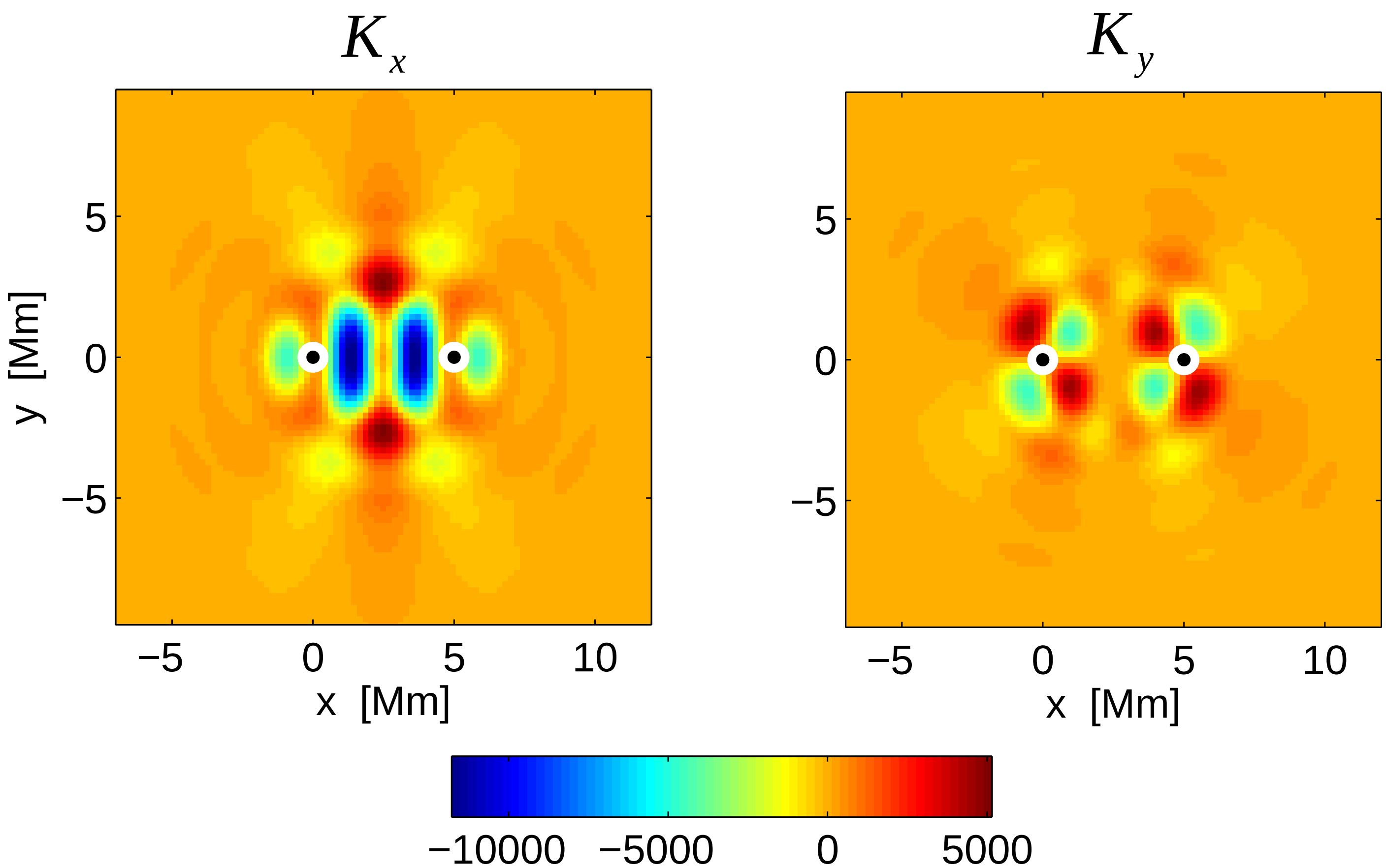
INTRODUCTION

The goal is to measure flows on the sun at the highest spatial resolution possible to learn about convection. The f-mode wavelength is 5 Mm at a frequency of 3 mHz. To probe scales below this wavelength we need to perform a careful inversion of the traveltimes.

We perform a 2-D optimally localized averaging (OLA) inversion for flows using f-mode travel times. The inversion procedure is fully-consistent, in that we use f-mode wave kernels, the model error-covariance matrix, and traveltimes that are measured using the same definition of traveltimes with which the kernels are calculated.

Since this is a somewhat preliminary study, we only use kernels calculated for one separation distance, 5 Mm, and only measure traveltimes for this distance as well. Even this truncation of the problem proves to be formidable computationally.

We present the various kernels, a discussion of the error matrix, and preliminary results of the inversion.



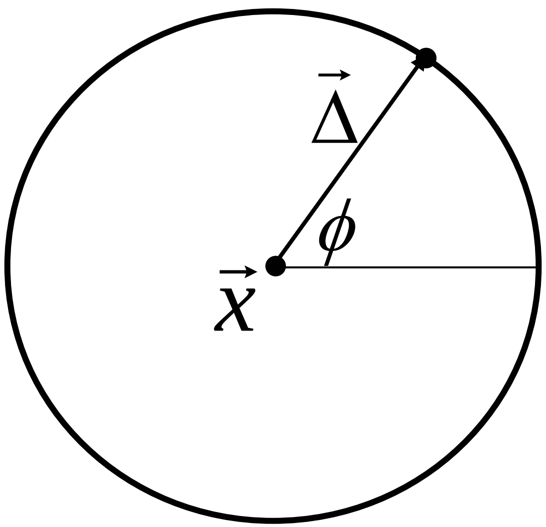
Example of point-to-point kernels. F-mode sensitivity kernels for flows for a separation between the observation points of 5 Mm. The observation points are given by the black dots. K_x is sensitive to flows in the ***x***-direction, and K_y is sensitive to flows in the ***y***-direction. These point-to-point kernels are then averaged over annuli and quadrants to produce the kernels used in the inversion. Units of the color scale are s²/Mm³. The relation between travel-time differences and the kernels is

$$\tau_{diff}(x_1, x_2) = \iint d\vec{r}\,\vec{K}(\vec{x}_1, \vec{x}_2, \vec{r}) \cdot \vec{U}(\vec{r})$$

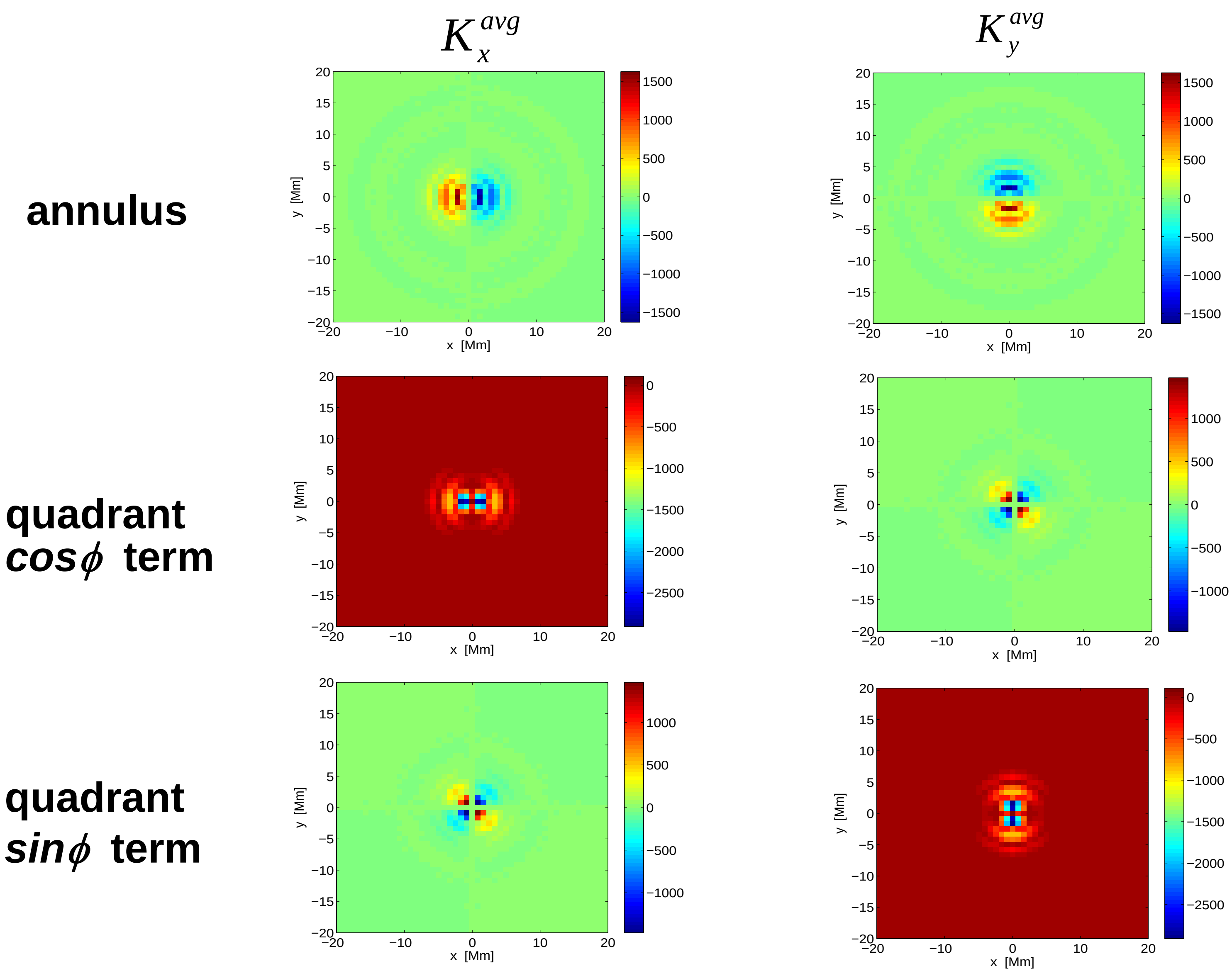
ANNULI AVERAGING OF TRAVELTIMES

The travel-time differences for the inversion are averaged over annuli and quadrants (after Duvall):

$$\begin{pmatrix} \bar{\tau}_{out-in}(\vec{x}) \\ \bar{\tau}_x(\vec{x}) \\ \bar{\tau}_y(\vec{x}) \end{pmatrix} = \frac{1}{2\pi} \int_0^{2\pi} d\phi \begin{pmatrix} 1 \\ \cos\phi \\ \sin\phi \end{pmatrix} \tau_{diff}(\vec{x}, \vec{x} + \vec{\Delta})$$



Since the traveltimes we use are averaged in this manner, we take the point to point kernels and average them in the same way.



Averaged point-to-point kernels. These kernels are produced by taking the ptp kernels and averaging them over an annulus or a quadrant, as with the traveltimes. The kernels in the left column are sensitive to flows in the ***x***-direction, and the ones in the right column are sensitive to flows in the ***y***-direction. Only these 6 kernels are used for this inversion.

Time-distance 2-D OLA inversion for flows

- Choose a target function that has a spatial scale and characteristics of the flows that are being inverted for. E.g., to invert for a flow in the ***x***-direction, the target may be a Gaussian of width 3 Mm in the ***x***-direction and of width 0 in the ***y***-direction.

- Search for a linear combination of the kernels which matches the target function.

- Calculate the error covariance matrix which describes the correlations in the travel-time measurements due to stochastic noise.

- Find the best set of weights that balances the trade-off between the misfit of the kernels and target, and the errors.

- Convolve the weights with the travel-time measurements to infer the flows.

minimize:

$$\sum_j \left(W_i K_{ij} - T_{ij}\right)^2 + \beta W_i W_j E_{ij}$$

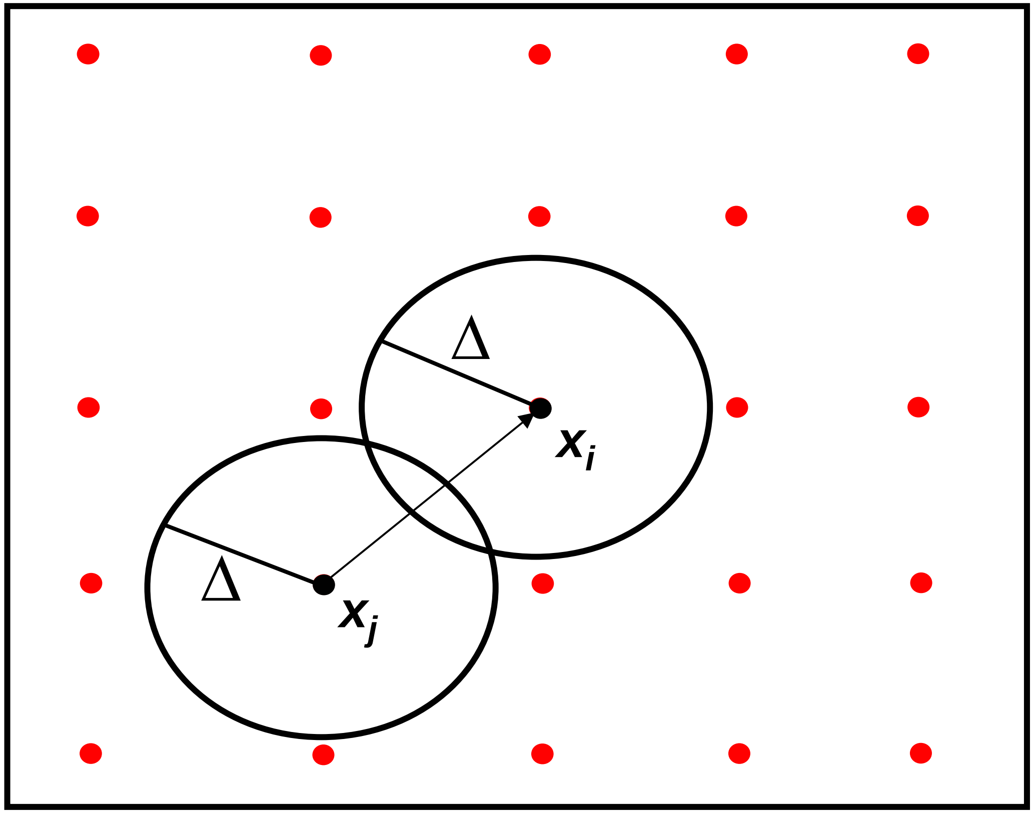
K	kernel matrix
T	target function
E	error covariance matrix
β	regularization parameter
W	weights

to get:

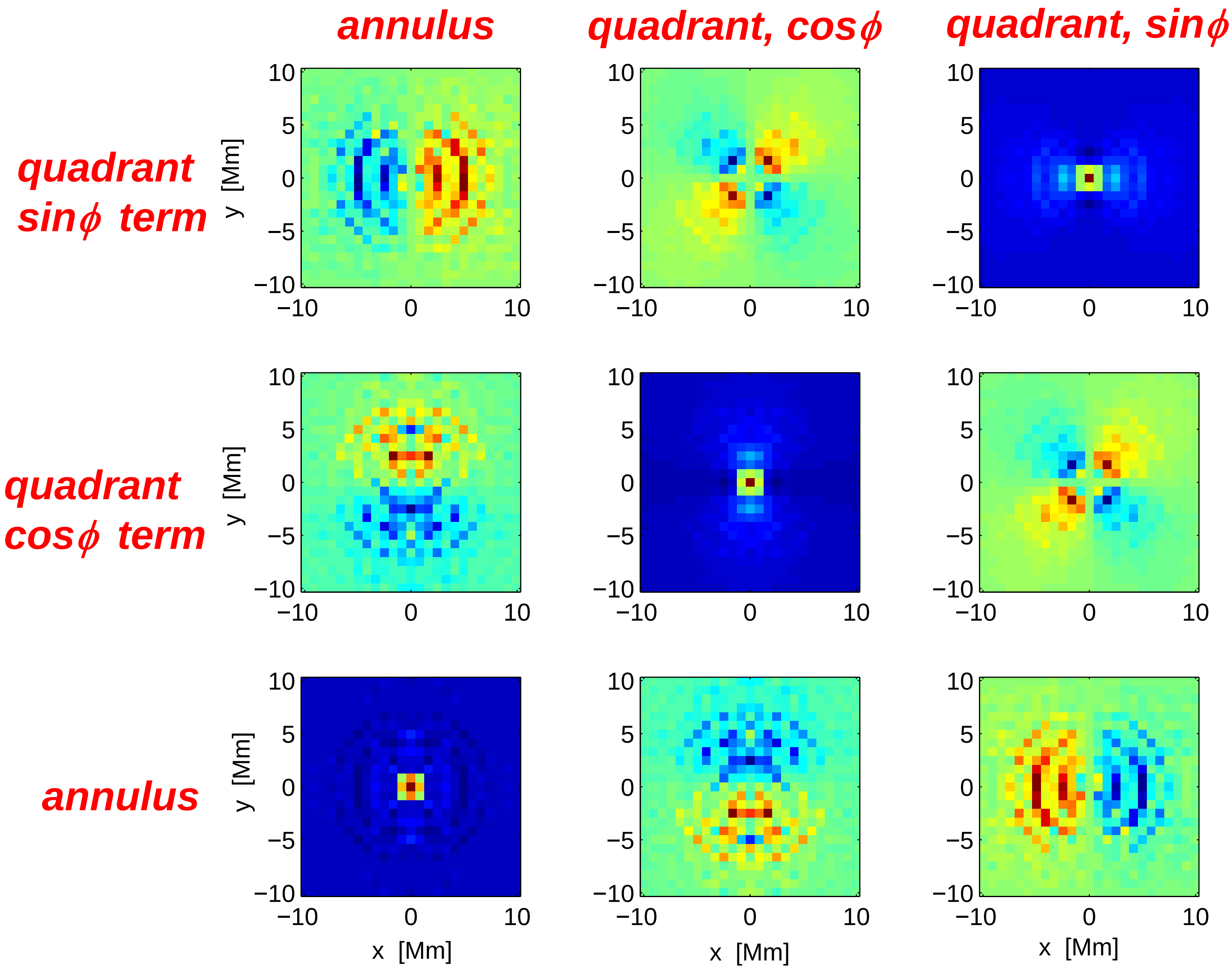
$$\left[K_{ij} K_{lj} + \beta E_{il}\right] W_l = K_{ij} T_{jl}$$

The final step is to then invert the matrix on the left hand side to find the weights, ***W.***

Schematic geometry for calculation of model error-covariance: E_{ij}^{cov}

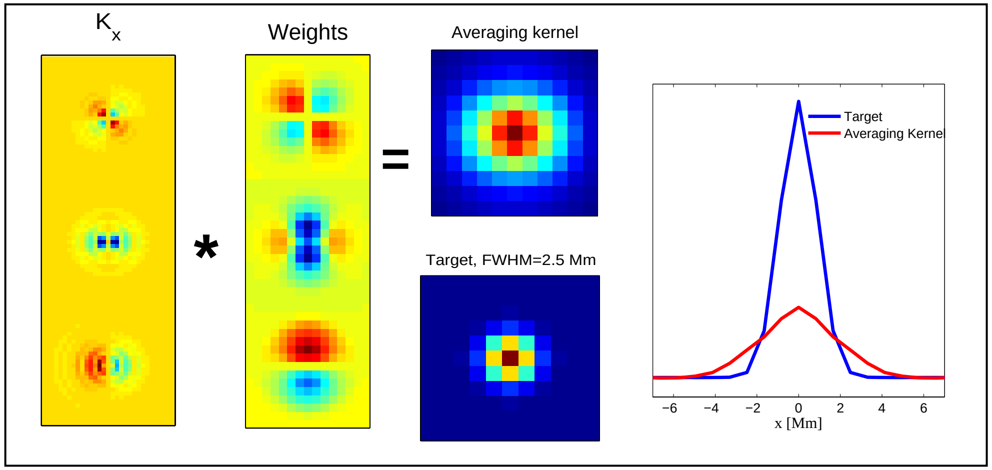


The circles denote combinations of the annuli and the quadrant traveltimes, and the centers are shifted around on an appropriate (*n x n*) grid to determine the correlations. In the full problem we would also correlate annuli over a range of different radii, but here the radii are equal. In this case we calculate a *3 x 3 x* n^2 matrix.

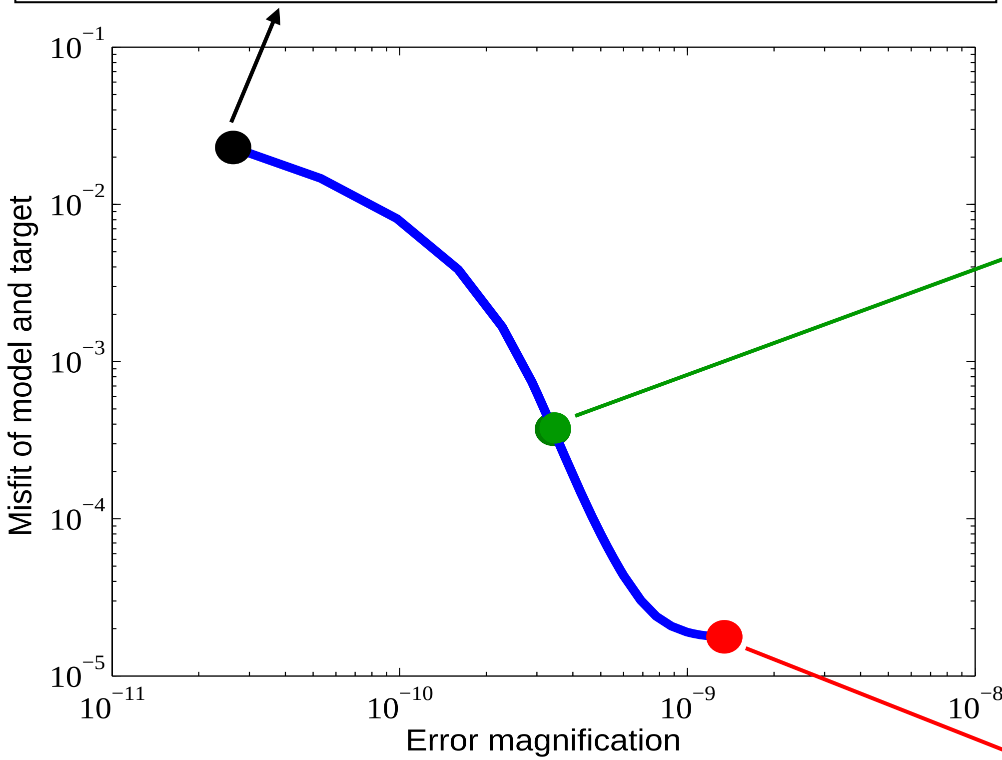


Model error covariance matrix. Each panel represents the correlation of one type of averaged traveltime with another, given by the red labels, as they are shifted around on the grid. The full error covariance matrix is constructed from these panels.

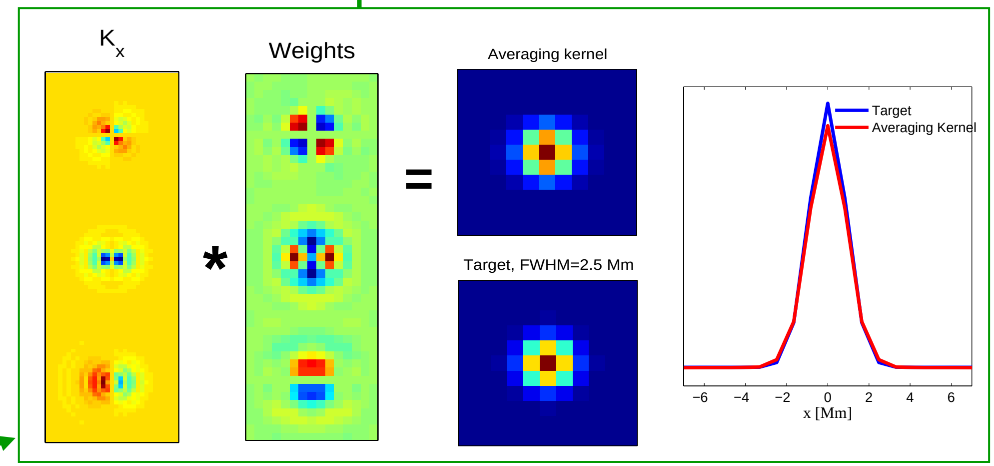
RESULTS



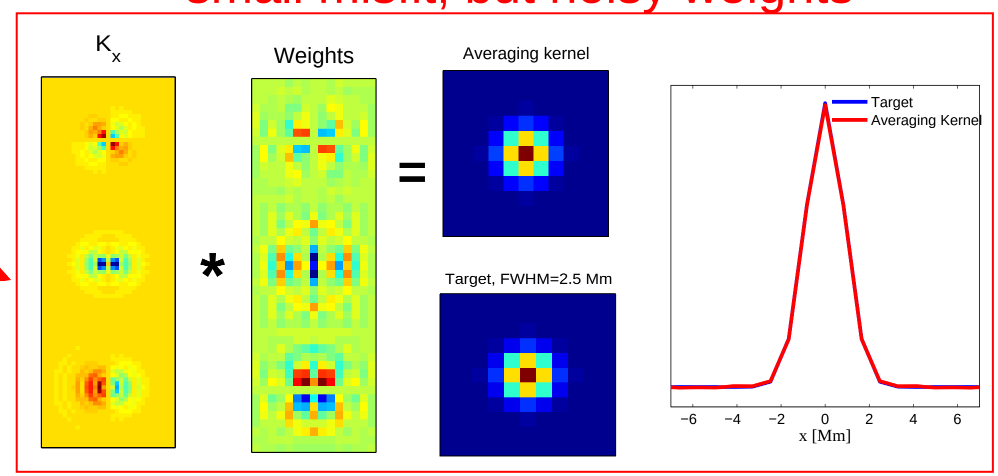
large misfit, smooth weights



optimal trade-off



small misfit, but noisy weights



L-curve. Find the point where the trade-off between the misfit and the error magnification is optimal.

FUTURE WORK

- Do the inversion with annuli of many radii.

- Use the real data, which we have already.

- Add more freedom by considering the point-to-point traveltimes.

References:

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