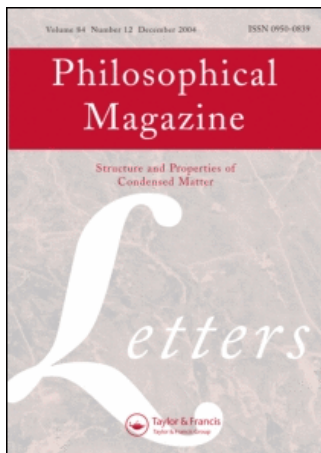


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Many body exchange effects close to the s-wave Feshbach resonance in two-component Fermi systems: is a triplet superfluid possible?

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We suggest that the exchange fluctuations close to a Feshbach resonance in a two-component Fermi gas can result in an effective p-wave attractive interaction. On the BCS side of a Feshbach resonance, the magnitude of this effective interaction is comparable to the s-wave interaction, therefore leading to a possible spin-triplet superfluid in the range of temperatures of actual experiments. We also show that the particle–hole exchange fluctuations introduce an effective scattering length which does not diverge, as the standard mean-field one does. Finally, using the effective interaction quantities we are able to model the molecular binding energy on the BEC side of the resonance.

In the atomic Fermi gases such as ^{40}K and ^6Li , the use of Feshbach resonances has opened the possibility of exploring the very interesting limit for which the mean-field approximation predicts a smooth crossover from BEC to BCS pairing as one goes through the resonance. At low energies, the interatomic interaction is very well described by the s-wave scattering length, a_s . Moreover, no direct interactions are possible in the triplet channel. In fact, higher-order expansions in the scattering length are suppressed at very low temperatures and the symmetry of the wave function, due to Pauli exclusion, does not allow s-wave scattering for fermionic atoms in the same spin channel.¶

Although the scattering length in the two-body problem is diverging, it is instructive to consider the possibility of pairing in the higher-order scattering channels due to exchange fluctuations. It is also not clear whether atomic systems behave as Fermi liquids (FL), or how similar they are with high T_c superconductors (HTSC) or any other strongly correlated systems.

In this Letter, we want to show two things. Firstly, that it is possible to build a Fermi liquid theory (FLT) in the atomic Fermi gases, particularly in the

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¶For the sake of clarity, throughout the paper, triplet pairing corresponds to pairing between particles in the same spin channel and s-wave to pairing in different spin channels.

BCS region. This formalism explains the basic features of these gases like the scattering lengths, and possibly, the binding energies in the strongly interacting regime, which is not accessible by simple perturbation theory. Secondly, we show that important contributions can arise in higher-order momentum channels. The resulting triplet pairing is comparable, in magnitude, to the s-wave one, and the correspondent triplet superfluid transition temperature is within experimental reach. Since the triplet interaction can only occur through the fluctuations induced by the strong interactions with the other spin channel, we focus our discussion primarily on the induced term. Its contribution may give an instability of the Fermi sea for (quasi-)particles with equal spins, and leads to a possible transition to a triplet superfluid. We will keep the discussion quite general, since our approach may be of interest to other fields. Then, we specify to the cold atom physics case, as we proceed.

In a Fermi liquid at sufficiently low temperatures ($T_c < T \ll T_F$, where T_c is the BCS transition temperature and T_F is the Fermi temperature), it was shown in [1, 2] that it is possible to separate the Fermi liquid parameters, $f_{pp'}$, which by construction do not contain any zero sound terms [3, 4], into two sets of terms in the limit when $p \sim p'$ (here, we assume the general notation $q = (\mathbf{q}, \omega)$ and $q = |\mathbf{q}|$). One term is the direct interaction of the quasiparticles (QP), and the other is the crossed term of the particle-hole contribution. Dropping for the moment the spin indices, the Fermi liquid parameters are

$$-f_{pp'} = 2\pi i Z_p Z_{p'} \lim_{\omega \rightarrow 0} \lim_{q \rightarrow 0} \Gamma^0 \left(p + \frac{q}{2}, p - \frac{q}{2}; p - p' \right) + \sum_{\mathbf{p}_1, \mathbf{p}_2} f_{pp_1} \left(\frac{\delta n_{\mathbf{p}_1}}{\delta U_{\mathbf{p}_2}} \right) f_{\mathbf{p}_2 p'} \quad (1)$$

where Z_{p_i} are the residues of the single particle Green's functions at the pole of the QP, $n_{\mathbf{p}}$ is the Fermi distribution function, U is some interaction, and $\delta n_{\mathbf{p}_1} / \delta U_{\mathbf{p}_2}$ is related to the response function and can be obtained from the QP transport equation [5, 6]. Restoring the spin indices, we denote the first term by $d_{pp'}^{\sigma\sigma'}$ and the second term by $I_{pp'}^{\sigma\sigma'}$. The many-body effects in the QP interaction are therefore separated into two contributions:

$$f_{pp'}^{\sigma\sigma'} = d_{pp'}^{\sigma\sigma'} + I_{pp'}^{\sigma\sigma'} \quad (2)$$

where $d_{pp'}^{\sigma\sigma'}$, the direct term, includes only the diagrams which are not particle-hole reducible and is equivalent to the T-matrix in the particle-particle channel. The induced term, $I_{pp'}^{\sigma\sigma'}$, has contributions from the exchange of virtual collective excitations among the quasiparticles, i.e. density, spin-density, current, spin-current fluctuations to name a few (see [1, 2] for full details). The implicit assumption, as it is for all Fermi liquid theories, is that all the relevant processes occur on the Fermi surface. Consider, now, any quantity, say $f_\ell^{s,a}$. This is related to its counterpart $f_{pp'}^{\sigma\sigma'}$ by the definition

$$F_{pp'}^{\sigma\sigma'} = N(0) f_{pp'}^{\sigma\sigma'} = \sum_\ell (F_\ell^s + \sigma \cdot \sigma' F_\ell^a) P_\ell(\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}'), \quad (3)$$

where $F_\ell^{s,a} = N(0) f_\ell^{s,a}$, $N(0)$ is the density of states at the Fermi surface. The superscript s(a) indicates the symmetric (antisymmetric) contribution with respect

to the spin, the subscript ℓ indicates the Legendre component, and P_ℓ is the Legendre polynomial. Then, by expanding the Bethe–Salpeter equation for the QP interactions in the limit $\omega/|\mathbf{q}| \rightarrow 0$ in a rotationally invariant system into Legendre polynomials, it can be shown that [5, 6]

$$A_\ell^{s,a} = N(0)a_\ell^{s,a} = \frac{F_\ell^{s,a}}{1 + F_\ell^{s,a}/(2\ell + 1)}. \quad (4)$$

Here, $A_\ell^{s(a)} = N(0)a_\ell^{s(a)}$ is the symmetric (antisymmetric) Legendre components of the scattering amplitudes of the quasiparticles. Note that these scattering amplitudes differ from the bare scattering amplitudes, since they contain the many-body effects of the theory through the QP interactions f . Given that equation (4) is a non-perturbative result, it remains valid even when F_ℓ diverges, since the A_ℓ remain finite. The only approximation at this point has been in assuming a Fermi liquid and the low energy and momenta limits. From [1, 2], it follows that

$$F_{\mathbf{pp}'}^s = D_{\mathbf{pp}'}^s + \frac{1}{2} \frac{F_0^s \chi_0(q') F_0^s}{1 + F_0^s \chi_0(q')} + \frac{3}{2} \frac{F_0^a \chi_0(q') F_0^a}{1 + F_0^a \chi_0(q')}, \quad (5)$$

$$F_{\mathbf{pp}'}^a = D_{\mathbf{pp}'}^a + \frac{1}{2} \frac{F_0^s \chi_0(q') F_0^s}{1 + F_0^s \chi_0(q')} - \frac{1}{2} \frac{F_0^a \chi_0(q') F_0^a}{1 + F_0^a \chi_0(q')}, \quad (6)$$

where $q'^2 = |\mathbf{p} - \mathbf{p}'|^2 = k_F^2(1 - \cos\theta_L)$ and $\cos\theta_L = \hat{\mathbf{p}} \cdot \hat{\mathbf{p}'}$ is the Landau angle, and $\chi_0(q')$ is the density–density correlation (Lindhard) functions (see [1, 2]). Including $\ell \geq 1$ terms is straightforward in this model, but only leads to small corrections to the results. For the direct term D in the low temperature limit, the particle–particle T-matrix is proportional to the bare s-wave scattering length a_s . Since D is then angle independent, it contributes only in the $\ell = 0$ momentum channel, given by $D_0^s = -D_0^a = -N(0)U/2$, where $U = 4\pi\hbar^2 a_s/m$ is some on-site interaction. The direct interaction is antisymmetric and obeys the Pauli principle $D_0^{\uparrow(\downarrow\downarrow)} = 0$. We purposely neglect the remaining diagrams in the particle–particle channel, since we are mostly interested in the induced interaction driven by the exchange of collective excitations between the quasi-particles. In fact, at these temperatures, this is the only way a triplet interaction can arise in the same spin channel. We observe that the form of the direct term depends on the model used, whereas the induced term does not. Still, we emphasize that the resulting scattering lengths, calculated through the Bethe–Salpeter equation, have the correct symmetries and conserve the Pauli principle through the Landau sum rule, given by $\sum_\ell (A_\ell^s + A_\ell^a) = 0$. This is not the case for the random phase approximation (RPA). Looking at the diagrams in figure 1 may help understand the differences. The RPA lack of the exchange terms in the particle–hole channel implies an inconsistent treatment of the QP interactions, since the scattering amplitudes are not properly antisymmetrized. The consequences of this will appear clear below.

We now apply the above Fermi-liquid formalism to the specific case of the cold atomic Fermi gases, in particular to ^{40}K . In these systems, the scattering length

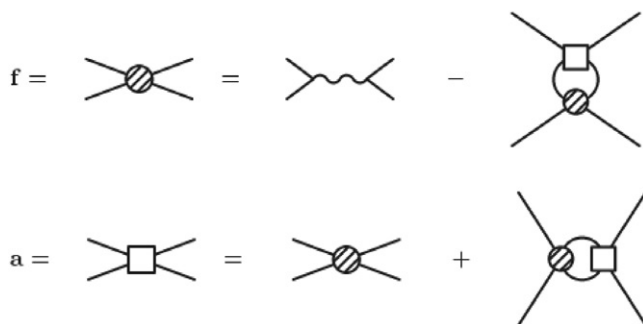


Figure 1. Schematic diagrammatic relation between the Landau parameters and the scattering amplitudes defined in equations (2), (4), (5) and (6). The first term on the RHS of f is the direct interaction d , and the second is the induced term I . Notice that when neglecting the induced terms, this description reduces to the RPA.

can be varied by tuning the system close to a magnetic Feshbach resonance [7]. The s-wave scattering length a_s (denoted in the figures as a_s^{bare}) varies as

$$a_s = a_{bg} \left(1 - \frac{\Delta B}{B - B_0} \right), \quad (7)$$

where B_0 denotes the magnetic field value of the Feshbach resonance, ΔB is the width of the resonance, and a_{bg} is the background scattering length. Since most of the experimental systems deal with broad resonances only, the contribution of the molecules from the closed channel can be neglected [8].

We solve self-consistently equations (5) and (6) by varying the direct interaction a_s and use equation (4) to obtain the scattering amplitudes $A_\ell^{s,a}$ for $\ell = 0, 1$. We then use these scattering amplitudes to construct the singlet and triplet pairing amplitudes in the well-known s-p approximation [5, 6], and call this the effective singlet scattering length a_s^{eff} . In figure 2 we show the results for the bare (equation 7, dashed line) and effective (thick line) s-wave scattering lengths calculated in this model for ^{40}K on both sides of the Feshbach resonance $B = B_0$. The most important feature is that the effective scattering length does not diverge as the resonance is approached, while a_s does diverge. Our results are very close in slope and magnitude to the experimental values. On the other end, far from B_0 , the effective and mean-field scattering lengths are comparable. We note that the presence of a strong p-wave interaction would influence the background scattering length a_{bg} . However, since in that channel there is no resonance, the many-body effects will give a negligible contribution and we can safely assume the background scattering length to be constant.

Also shown in figure 2 is the result predicted by RPA (thin line), which clearly fails to capture the correct physics as the resonance is approached. The divergence of the RPA scattering length implies the emergence of two new ground states on either side of the Feshbach resonance. On the BEC side, this would correspond to the Stoner instability or the onset of ferromagnetism. On the BCS side, this would correspond to phase separation. We note that neither of these two instabilities has been observed experimentally.

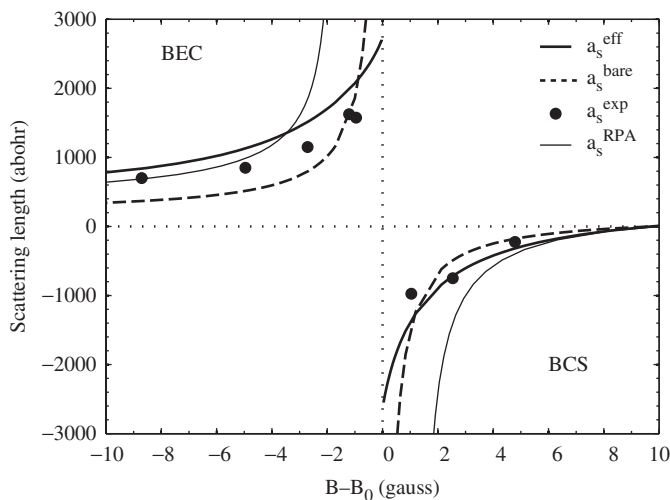


Figure 2. S-wave scattering lengths (units of Bohr radii) as a function of the magnetic field on both sides of the Feshbach resonance B_0 , in ^{40}K , using data from [9, 10]. The particular scattering length is denoted in the legend. The 50% error bars in the experimental data have been removed for clarity. We also plot the RPA scattering length for comparison.

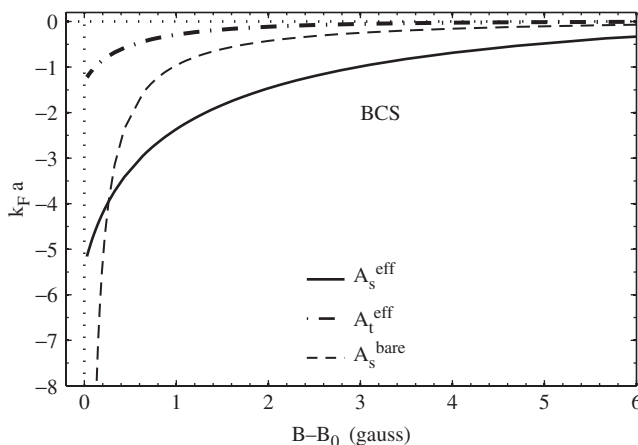


Figure 3. Scattering amplitudes on the (attractive) BCS side of the resonance B_0 . In our model, the effective (unitless) s-wave singlet $k_F a_s^{\text{eff}} = A_s^{\text{eff}}$ and triplet $k_F a_t^{\text{eff}} = A_t^{\text{eff}}$ amplitudes are finite and of similar magnitude at the resonance, while the bare interaction $k_F a_s = A_s^{\text{bare}}$ diverges, according to equation (7).

We now turn to the BCS side of the resonance where the scattering lengths are negative, and compare the effective triplet (a_t^{eff}) and effective singlet scattering lengths scaled by the Fermi momentum k_F . For magnetic fields near the resonance, the strength of the triplet and the singlet potentials are actually comparable in magnitude, as shown in figure 3. Therefore, although direct (triplet) pairing in the

same hyperfine state is suppressed initially at these low temperatures, the exchange of collective excitations upon approaching the resonance drives a substantial attractive interaction in the triplet channel through the induced interactions. Note that there is always attraction in the triplet channel on both sides of the resonance, independent of the sign of the bare interaction U . At low enough temperatures, it is not obvious that one can disregard the possibility of having a triplet superfluid near resonance (on the BCS side). In fact, below the triplet transition point, it seems quite reasonable that these two many body states will compete, as figure 3 suggests.

It is important to compute the critical temperatures expected for the various pairing instabilities. The expression is very similar to the BCS one (see [11, 12]). The singlet transition temperatures with our effective scattering amplitude could be as large as $T_c^{\text{sing}} \sim 0.7 T_F$, while the triplet $T_c^{\text{trip}} \sim 0.2 T_F$, if we use T_F as the cut-off scale. Note that these critical temperatures are quite high and that this is due to the use of the high-energy cut off. Also, there are numerous indications that singlet transition temperatures are of the order of $0.2 T_F$. This introduces a proportionality relation which gives an (upper bound of the) triplet transition temperature, within our framework, of $\sim 0.05 T_F$. These temperatures are already obtainable in current experiments. We also mention that in the limit when the partial waves get very large, i.e., far away from the resonance, our approach gives the Gorkov and Melik-Barkhudarov critical temperature [13], since the particle-hole corrections become unimportant in this regime.

Lastly, we should remark that the present calculations hold for both equal populations or for very small polarizations, $m = (n^\uparrow - n^\downarrow)/n$, where $n^{\uparrow(\downarrow)}$ is the majority (minority) particle density, and n is the total density. For $m \ll n$, the corrections to the Fermi liquid parameters, which are quadratic in m , are, in fact, negligible. Recent experiments [14, 15] have opened up the possibility of exploring the triplet interactions of the system. Since the singlet BCS state is still stronger than the triplet one, in order to see the triplet transition, it is probably necessary to suppress the singlet superfluid. Indeed, even small polarizations, at low enough temperatures, might create the possibility of a triplet superfluid in that channel. More likely, the presence of an external field can establish a preferential direction and favour the triplet pairing, similar to the A_1 phase in ^3He . We emphasize, though, that the triplet pairing is not only interesting in the superfluid phase, but also in the normal one, as it will contribute to the properties of the system. In fact, its thermodynamic properties, which we will discuss somewhere else, can be largely affected and therefore provide the experimental tools to verify the Fermi liquid behaviour of the system close to resonance.

Up to this point, we have assumed that the corrections due to the particle-particle contribution are not relevant. In the normal phase, this assumption is plausible on the BCS side [3, 4], but on the BEC side can be justified only very close to the resonance and/or for temperatures $k_B T > E_b$ (the experimental data are taken at $T/T_F \sim 0.4$, although in this paper we are considering only the corrections due to the quantum fluctuations $T \ll T_F$), where E_b is the binding energy. Thus, deep into the BEC regime, our theory breaks down. On the other hand, we recover the bare scattering length as soon as we get far away from the resonance and therefore any FLT assumption is irrelevant.

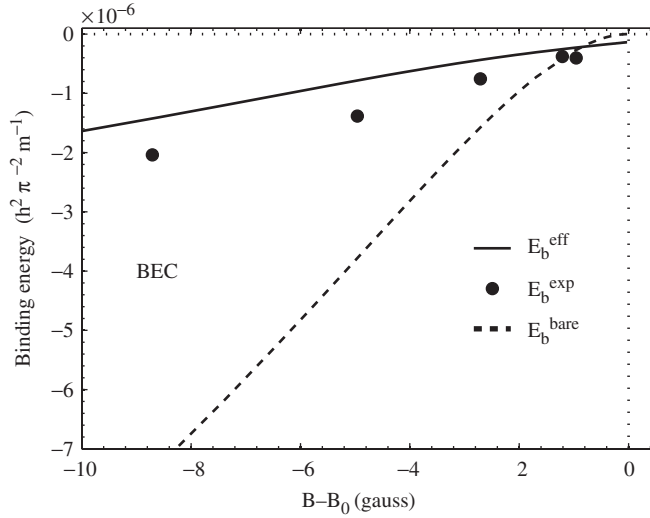


Figure 4. Binding energies on the BEC side in ^{40}K , compared with data taken from [9, 10]. The density used here is $n = 5.8 \times 10^{13} \text{cm}^{-3}$, with $\Delta B = 9.7 \text{ G}$ and the Feshbach resonance, marked above with the vertical line, occurs at $B_0 = 224.21 \text{ G}$. E_b^{eff} is the binding energy calculated from the effective scattering length in our theory, and E_b^{bare} is the bare (mean-field) binding energy.

Thus, we might expect the theory to give a rough estimate of the scattering lengths in the intermediate region as well. We therefore compute the binding energy of the bound state in the open channel. It is calculated using the standard mean-field formula $E_b = -\hbar^2 / m a_s^2$, where a_s is the bare *s*-wave scattering length. Since we lack a better estimate of the corrections to this formula due to the many-body effects, we simply replace a_s with a_s^{eff} and m with $m^* = 1 + F_1^s/3$, the effective mass in FL theory. The results are shown in figure 4. The agreement with the experimental data is quite surprising, but can be explained in terms of an effective Hamiltonian, which, in the spirit of Landau's theory, progressively transforms the bare particles into quasiparticles and the bare scattering length becomes the effective one as the interaction increases. In this sense, one can adopt the same mean-field formula of the binding energy, and indeed, the effective binding energy profile reduces to the mean-field one in the weakly interacting regime. We also note that the Fermi liquid parameters represent only a part of the mean-field shift, since they do not contain the zero sound channel contribution. The full contribution is instead given by the full effective scattering length.

Engelbrecht *et al.* [16] calculated the energy gap equation in the weakly interacting limit. They correctly pointed out that a comparison of the gap with the full solution should show roughly the extent of the strongly interacting regime. It is also interesting to note that the BEC-BCS crossover behaviour is lost by using the weak-limit gap solution. In figure 5 we plot the energy gap corresponding to the weakly interacting BEC and BCS limits, but with our effective

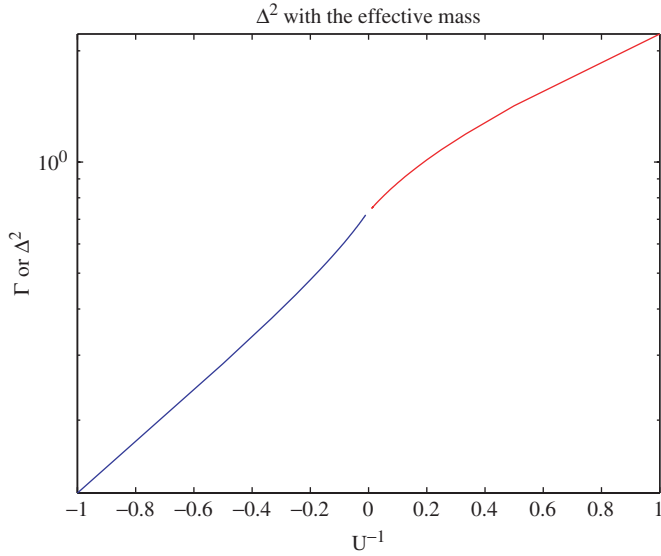


Figure 5. Gap squared using the weakly interacting limit equations as derived in [16], but replacing the effective mass and effective interaction from our theory to the bare ones. U is the bare interaction strength as defined in the text. Γ is related to the gap squared as in [8], although our units do not correspond to those of the reference.

scattering lengths and masses. It is clearly seen that the crossover is re-established in terms of the quasiparticles. This shows that although the bare particles are strongly interacting, the quasiparticles may not, and hints that the gas in the normal phase is probably behaving as a Fermi liquid, even very close to the resonance.

In conclusion, we have built a theory which takes into account the many body exchange effects in the quasiparticle–quasihole channel. This theory, contrary to the RPA, respects the Pauli principle and does not give spurious ground states. Inclusion of the exchange effects is therefore fundamental in obtaining the correct physics. We obtain a finite scattering amplitude as seen experimentally. We have also shown that a triplet superfluid is possible within the temperatures today achievable in cold atom traps and that triplet pairing should be taken into account when discussing the properties of the system close to resonance. Furthermore, it seems possible in this formulation to derive the basic properties on the BEC side, although one should include properly the presence of the bound state, which we have not. The strong agreement with experiments indicates that quasiparticles, not bare particles, are binding in the open channel. The good interpolation of the intermediate interacting region between the BCS and BEC sides is probably due to a careful account of the particle–hole contributions in the theory. Finally, we remark that this approach, since it is not restricted to the dilute gases, can be applied to other systems.

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