Cross-spectral analysis of solar oscillation time series

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Abstract. Spatial leakage is an unavoidable artifact in the extraction of solar oscillation modes by spheric harmonic decomposition from the wavefield observed on the solar surface. The determination of solar frequencies by spectral analysis is therefore greatly affected by spatial leakage. Here we show in which way spatial leakage also influences the cross-spectra between different solar oscillation modes. Simulations show that spatial leakage induces significant coherences between oscillations of degree l and $l+2$ with low azimuthal order m.

1. Introduction

Solar oscillation signals are extracted from observations of the solar oscillation field visible on the solar surface by spherical harmonic decomposition. However, the separation into single oscillation modes is not perfect as observations are only possible from the front side of the sun. The spherical harmonics, however, form an orthogonal basis only on the full surface of the sphere. This results in spatial leakage, i.e. single solar oscillation modes contain linear contributions from other oscillation modes. This leakage effect is well known in helioseismology as it greatly affects the accurate determination of solar frequencies of global solar oscillations [1, 2].

However, an interesting topic in helioseismology is the investigation of couplings between modes, as they reflect mode interactions by physical processes in the solar interior [3]. Perturbation analysis of solar models has shown that large scale velocity fields may cause couplings between solar oscillation modes [3–6]. These couplings should result in stationary or time-dependent relationships among the modes [7, 8]. For the issue of detecting couplings, cross-spectral analysis is a promising approach, as couplings of first order, i.e. linear interactions between modes, would contribute to the cross-spectrum between modes.

Here we investigate and discuss the influence of leakage on cross-spectra of time series from solar oscillations. We use simulated stochastic velocity fields and real observations of low degree solar oscillations from the SOHO/MDI instrument.

2. Methods

Analogously to the auto-spectrum $S_X(\omega) = \langle f_X(\omega) f_X^*(\omega) \rangle$, the cross-spectrum $CS_{XY}(\omega)$ of two processes X, Y can be defined by their fourier-transforms $f_X(\omega)$, $f_Y(\omega)$ as

$$CS_{XY}(\omega) = \langle f_X(\omega) f_Y^*(\omega) \rangle \,, \qquad (1)$$

where ω denotes the frequency of interest and $\langle . \rangle$ the expectation value. An important property derived from the cross-spectrum is the coherence-spectrum. It is obtained by normalizing the modulus of the cross-spectrum with the auto-spectra S_X, S_Y of X and Y

$$Coh(\omega) = \frac{|CS_{XY}(\omega)|}{\sqrt{S_X(\omega) S_Y(\omega)}}. \quad (2)$$

In the case of a pure linear relationship between X and Y, the coherence equals one. However, independencies and non-linear relations between X and Y, observation noise, and other non-common additive influences on X or Y diminish the coherence. The cross-spectrum is complex-valued. Its phase $\Phi(\omega)$ is related to the functional relationship between X and Y and denoted as the phase-spectrum

$$CS_{XY}(\omega) = |CS_{XY}(\omega)| \exp(\Phi(\omega)). \quad (3)$$

For finite time series, coherence- and phase-spectra have to be estimated and a critical value can be determined for statistically testing the null hypothesis of zero coherence

$$\sigma_{crit} = \sqrt{1 - \alpha^{\frac{2}{\nu - 2}}}, \quad (4)$$

for ν degrees of freedom of the cross-spectra and a given significance level α. This critical value permits testing whether a given coherence at some frequency ω reflects a linear relationship between both processes or not [9, 10].

3. Results

In the following we present coherence- and phase-spectra estimated for simulated and observed solar oscillation data for different combinations of harmonic degree $l = 0, ..., 99$ and azimuthal order m.

3.1. Simulation Results

We simulated solar oscillation time series of distinct modes l, m using a stochastic radial velocity field $v(\theta, \phi, t)$. This field

$$v(\theta, \phi, t) = \sum_{l,m} \alpha_{l,m}(t) Y_l^m(\theta, \phi) \quad (5)$$

was generated by a superposition of selected modes l, m which were excited independently of each other by independent standard Gaussian processes $\alpha_{l,m}(t) \sim N(0, 1)$. Thereby, $N(0, 1)$ denotes the standard Gaussian distribution with expectation 0 and variance 1 and $Y_l^m(\theta, \phi)$ the spherical harmonic function of degree l and order m. As the single processes $\alpha_{l,m}(t)$ are independent by construction, the coherence is zero between them.

The single solar oscillations were re-estimated from the stochastic velocity field $v(\theta, \phi, t)$ using an incomplete spherical harmonic decomposition according to [11]. The decomposition incorporated the projection onto the line of sight and the spatially discret observation on a square grid, here 301×301 pixel of equal size. Thus, we obtained time series of single solar oscillation modes which were now mixed with adjacent modes due to leakage. Coherence-spectra between different modes of these time series were estimated and compared with a critical value at significance level $\alpha = 0.01$.

We observe significant coherence between simulated solar oscillations modes (l, m) and $(l \pm 2, m)$ while between modes (l, m) and $(l \pm 1, m)$ coherence is not significant. Exemplarily, coherence- and phase-spectra from $(l = 55, m = 0)$ with $(l = 55 \pm 1, m = 0)$ and $(l = 55, m = 0)$ with $(l = 55 \pm 2, m = 0)$ are shown in Figure 1. The critical value is indicated by the horizontal line. In the case of significant coherence, we observe a constant phase $\Phi(\omega)$ of $-\pi$. In case of non-significant coherence we observe a random phase.

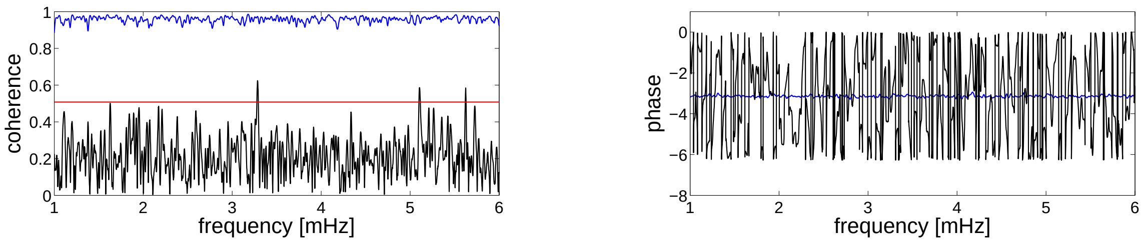

Figure 1. Coherence- and phase-spectra from simulated data between modes $l = 55$ and $l = 56$ (black), and $l = 55$ and $l = 57$ (blue), $m = 0$. The horizontal red line marks the 99% significance level for significant coherence. For this simulation the modes $l = 0, ..., 99$, $m = 0$ were excited.

The dependence of coherence on the azimuthal order m is shown for $l = 55$ and $l = 57$ in Figure 2. Significant coherence is only observed for low azimuthal degrees m.

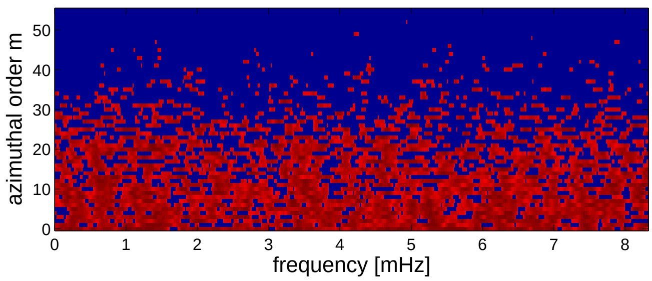

Figure 2. Dependence of coherence between modes $l = 55$ and $l = 57$ on azimuthal order m for simulated data. For this simulation the modes $l = 55, 56, 57$ and $m = 0, ..., l$ were excited. Only significant values are plotted (red).

3.2. Sample Results

We investigated a consecutive sequence of 300 hours duration (covering the period from 2000 December 31 through 2001 January 13) for solar oscillations of degree $l = 0, ..., 99$ obtained from the 'medium-l' program of the MDI/SOHO instrument. This sequence contained only minor gaps, which we filled by linear interpolation.

We observe significant coherences for MDI/SOHO-data between modes with (l, m) and $(l \pm 2, m)$, whereas between (l, m) and $(l \pm 1, m)$ coherence is not significant (Figure 3). Significant coherences are mainly observed along the ridges defined by the radial order n and not in the background. For increasing azimuthal order m, the coherence vanishes (Figure 4). The significant coherence close to one strongly indicates a linear relationship between modes (l, m) and $(l \pm 2, m)$.

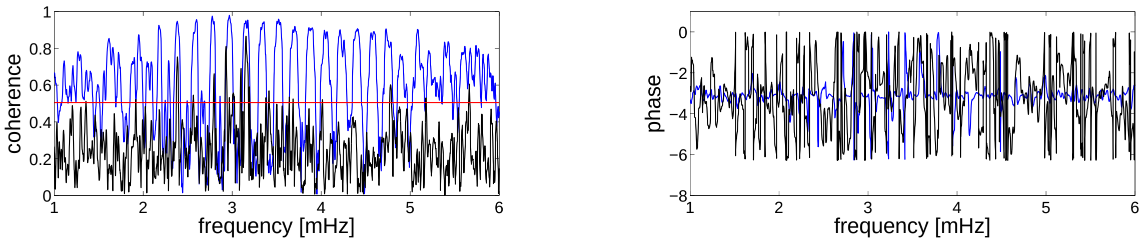

Figure 3. Coherence- and phase spectra between modes $l = 55$ and $l = 56$ (black), and $l = 55$ and $l = 57$ (blue), $m = 0$ from SOHO/MDI-data. The critical value for significant coherence is shown as a red line.

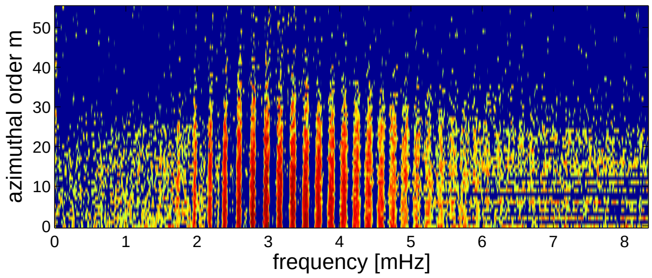

Figure 4. Dependence of coherence-spectra on azimuthal order m and radial order n between degree $l = 55$ and $l = 57$ form MDI/SOHO-data.

3.3. Comparison of results

A comparison between coherence-spectra from simulated data and MDI/SOHO-data shows the same dependency on degree l (Figure 5). Further, the comparison of Figure 2 and Figure 4 reveals the same dependence of coherence on the azimuthal order m. The phase-spectra from simulated signals as well as observed signals with significant coherence are shown in Figure 1 and Figure 3. For observed and simulated signals a constant phase is found. This indicates that probably the same kind of functional linear relationships underly this finding. As the original modes of the simulated velocity-field are independent, the coherence must be caused by spatial leakage from incomplete spherical harmonic decomposition.

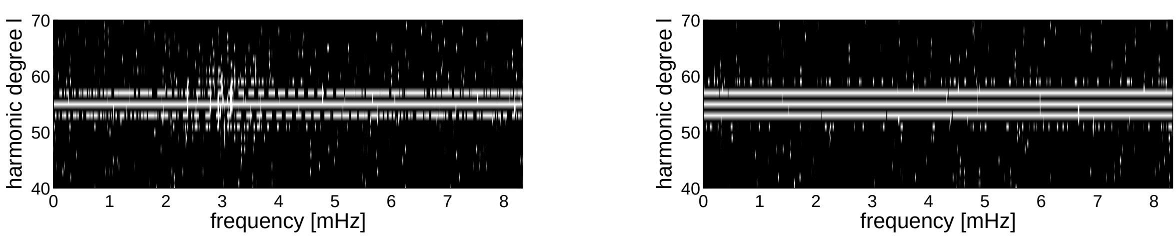

Figure 5. Comparison of coherence between $l = 55$ and $l = 40, ..., 70$ for time series of SOHO/MDI data (left) and simulated data (right). Only significant values of coherence are shown (white).

4. Discussion

We investigated the effect of spatial leakage on the cross-spectrum of time series from solar oscillations. We simulated an incomplete spherical harmonic decomposition of independent Gaussian time series to study the effect of leakage and compared these results to findings in real observations of solar oscillations on the Sun. Our simulations showed that spatial leakage induces significant coherences between oscillations of degree l and $l \pm 2$ with equal m. The same behavior of coherence-spectra with regard to mode degree was observed in real solar data. We conclude that for distinct combinations of the harmonic degrees the leakage effect dominates the coherence-spectra and linear couplings of solar modes cannot be identified between these modes using cross-spectral analysis. However, for other combinations of oscillation modes the leakage effect is negligible. Thus, for investigations of linear couplings between modes by cross-spectral analysis, we suggest to use a simple stochastic velocity-field model to test in advance for significant leakage contributions to the cross-spectra.

In future studies we will investigate how far cross-spectral analysis may contribute to determine and eliminate leakage effects in spectral analysis by estimating the leakage matrix.

Acknowledgments

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