

Introduction

Solar f-modes are very suitable for the study of flows in the first 2 Mm below the surface. They also have a favorably small wavelength (~ 5 Mm) which can be used to probe phenomena in detail such as supergranulation and flows around active regions.

We study the effect of flows on f-mode travel times. The forward problem consists of calculating sensitivity kernels which relate the travel times of the surface waves between two points on the sun to small amplitude flow perturbations that affect the travel times. Since the observed signal is the line of sight projection of the velocity on the solar surface, we take into account the line of sight in the calculations and show that it is not negligible.

The methods described here are quite general and are ultimately directed towards carrying out inversions using the data of the SOHO/MDI and SDO/HMI instruments.

The model

For the sake of simplicity, we adopt the following prescription for the model calculations:

- sphericity is ignored, use a plane-parallel geometry
- constant density + free surface
- horizontal and irrotational flow $U(x,y)$ that is independent of depth
- randomly distributed sources
- damping and sources **Doppler shifted** by the flow
- definition of travel times taken from (Gizon 2004)

As a consequence of these ingredients, the model is **2-dimensional**.

The treatment of the scattering of the waves by the perturbations is handled by using the first **Born approximation**, a single scattering assumption, which has been applied to calculate kernels before (Gizon 2002), and also has been verified to capture the main features of 'observed' travel-time kernels (Duvall Jr. 2006).

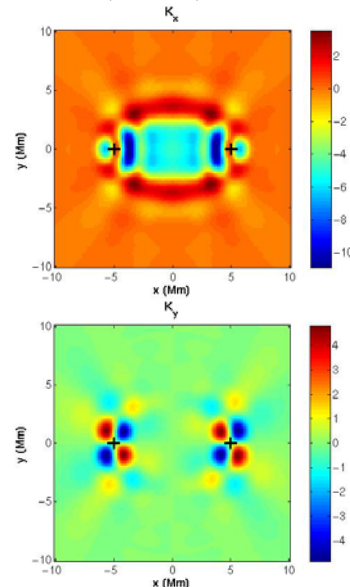
Sensitivity kernels are calculated on a small local area of the sun by considering two points x_1 and x_2 on the solar surface, separated by a distance $d = x_2 - x_1$. The travel time differences are defined as the time it takes for waves traveling from x_1 to x_2 , minus the time it takes for them to go from x_2 to x_1 .

The kernels

It can be shown that the relationship between the **travel time difference** between two points on the surface and a flow U can be approximated, in the limit of small U , by a linear equation such as

$$\Delta \tau_{diff}(\vec{d}) = \iint d\vec{r} \vec{U}(\vec{r}) \cdot \vec{K}(\vec{d}, \vec{r})$$

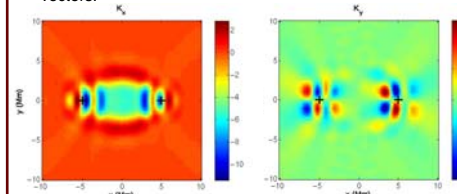
The integral is taken over all points r on the surface. The kernel K is a 2-d vector which is sensitive to flows in both of the x and y directions. Example of two sensitivity kernels to travel time differences are shown below, for the case of the line of sight normal to the surface (**disk center**).



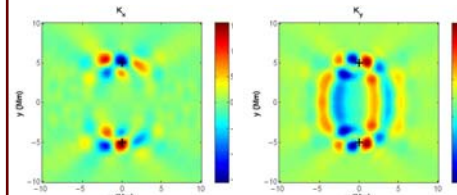
The observation points (crosses) in these plots have coordinates $x_1 = (-5, 0)$ Mm and $x_2 = (5, 0)$ Mm. The kernel K_x gives the sensitivity to a flow U_x in the x -direction, and K_y is sensitive to flows U_y in the y -direction. The units of the colorbar are $(km/s)^2 Mm^2$. Notice the ellipses and hyperbolae, the latter of which are a result of the randomly distributed sources. The integral of K_x over the grid is zero, which means that the travel time difference for flows in the y -direction is zero, while the integral of K_y is finite. Travel time difference kernels are the ones **most sensitive** to flows.

The effect of the line of sight

Any local region of the solar surface can be approximated by a plane tangent to the surface. The kernels depend on the orientation of the normal of this plane with respect to the direction to the observer (**line of sight**). Here are some examples of kernels computed for different line of sight vectors.

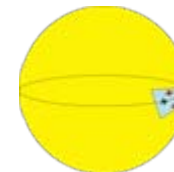


The kernels above correspond to a region near the solar equator and at **75 degrees west** of the central meridian. The two observation points lie on the equator and are separated by **10 Mm**.



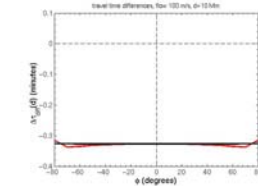
These two kernels are calculated for the same region on the disk as above, however the observation points now lie on the same meridian. Note that for K_x in the above figure, there is very little sensitivity along the ray path connecting the two observation points.

A sketch of the geometry for the above 4 kernels, where the **black crosses** correspond to the observation points of the top two kernels, and the **red crosses** to the points of the two kernels directly above.



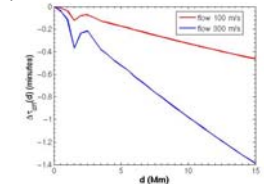
Travel times for a constant flow

For a constant flow, one may integrate the kernel K over space. This enables us to study the mean effect of the line of sight projection on travel times differences.



The above figure shows the travel time differences of kernels calculated at different longitudes along the equator for a flow $U=100$ m/s and $U = Ud/d$. The black line is for the case when x_1 and x_2 are on the equator, and the red denotes when x_1 and x_2 are on the same meridian. The travel time differences do not vary appreciably for smoothly varying flows, however, for real flows on the sun, it is clear that the effect of the line of sight should be taken into account.

Below, we plot travel time differences at disk center versus the separation distance d of the observation points. The dip around **2.5 Mm** corresponds to about $1/2$ of the wavelength of the f-modes, where near-field effects come into play.



Conclusions

- The effect of the line of sight is important for flow kernels, as is the orientation of the observation points with respect to the line of sight.
- Calculations of other types of kernels need to be extended to take this effect into account (Birch 2004).

References

- Gizon L. and Birch A.C., *ApJ*, **614**, 472 (2004).
- Gizon L. and Birch A.C., *ApJ*, **571**, 966 (2002).
- Duvall Jr. T.L., Birch A.C., Gizon L., in press (2006).
- Birch A.C., Kosovichev A.G., Duvall Jr. T.L., *ApJ*, **608**, 580 (2004).