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LINE PROFILES OF FUNDAMENTAL MODES OF SOLAR OSCILLATION

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Abstract. I present MDI-SOHO measurements of f-mode line profile asymmetry at high spatial wavenumbers. The f-mode line asymmetry is pronounced in the degree range 600-1200 and has opposite signs in velocity and intensity power spectra.

1. Introduction

The asymmetry of solar p-mode line profiles (Duvall et al., 1993) finds its origin, in part, in the localisation in depth of the mode excitation mechanism (Gabriel, 1992); p modes are preferentially excited by turbulent convection near the top of the convection zone, just below the photosphere. According to the classical interpretation of Duvall et al. (1993), line asymmetry is the result of interference between the outward acoustic wave generated by a buried source and the corresponding wave that passes through the acoustic cavity. Due to the interaction of waves with surface convection, the degree and sense of p-mode line asymmetry is different in velocity and intensity power spectra (e.g. Nigam & Kosovichev, 1998a). Furthermore, line asymmetry depends on the radiation pattern of turbulent convection (e.g. Kumar & Basu, 2000).

Here we study high-degree f-mode line profiles, which have received less attention than p-mode line profiles. Solar f modes, with a dispersion relation nearly identical to $\omega^2 = gk$, where $g = 274$ m/s^2 is the surface gravitational acceleration, are traditionally classified as surface-gravity waves (Gough, 1993). The physics of f modes, however, remains unclear. Questions have be raised about the outer boundary condition, thermal and compressibility effects, and the nature of the excitation mechanism. The variation of power with frequency has been observed to be quite similar for f- and p-mode ridges (Fernandes, 1992). This single observation suggests a similar excitation mechanism for f and p modes. Although f modes cannot be driven by entropy fluctuations (Goldreich, 1994), they can be excited by a turbulent flow.

A practical definition of line asymmetry is provided in Section 2. In Section 3 we use high-resolution MDI-SOHO Doppler and intensity images to measure the line asymmetries of high-degree f modes. In Section 4 we discuss a highly simplified toy model to examine under which conditions surface-wave line asymmetry may occur.

2. Line asymmetry parameter

The computation of the velocity power spectrum of solar oscillations at fixed horizontal wavenumber, k, reduces to solving an equation for the temporal Fourier transform of the vertical component of velocity, w, of the form

$$\mathcal{L}_\omega \; w(\omega, z) = S(\omega, z), \tag{1}$$

where $\mathcal{L}_\omega$ denotes a differential operator that encapsulates the physics of wave propagation and $S(\omega, z)$ is a stochastic source term due to forcing by turbulent convection. The solution of this problem can be expressed in terms of the response at z to an impulsive source at z', i.e. the Green's function defined by

$$\mathcal{L}_\omega \; G_\omega(z|z') = \delta(z - z'). \tag{2}$$

Under the assumption that sources are spatially uncorrelated,

$$\overline{S(\omega, z) S^*(\omega, z')} = Q(\omega, z) \delta(z - z'), \tag{3}$$

the expectation value of the power spectrum of ψ reads

$$P_{\rm vel}(\omega) = \int |G_\omega(z_{\rm o}|z)|^2 Q(\omega, z) \, {\rm d}z. \tag{4}$$

In this expression, $z_{\rm o}$ denotes the height at which observations are made, or about 200 km above the photosphere in the case of MDI observations. If the source function S peaks at a particular height $z_{\rm s}$ then the velocity power spectrum is roughly proportional to $|G_\omega(z_{\rm o}|z_{\rm s})|^2$. On the other hand, if the source function takes the form $S = \partial_z^n S_n$, where S_n is highly localised in depth, then the relevant quantity to consider is $|\partial^n G_\omega(z_{\rm o}|z_{\rm s})/\partial z_{\rm s}^n|^2$. Thus to each type of radiation pattern corresponds a particular spectrum. The computation of the intensity power spectrum requires, in addition, a formal relationship between wave displacement and light-flux perturbations.

What is the simplest sufficient condition on the Green's function to produce asymmetrical line profiles? Suppose that in the neighbourhood of the complex resonant frequency $\omega_0 - {\rm i}\Gamma/2$, the function G_ω is asymptotically equivalent to

$$G_\omega(z_{\rm o}|z) \sim {\rm const} \times \frac{\sigma - \omega_1}{\sigma - \omega_0}, \qquad (5)$$

where ω_1 is a real frequency and $\sigma = \omega + {\rm i}\Gamma/2$. Then, the substitutions

$$X = \frac{\omega - \omega_0}{\Gamma/2} \quad \text{and} \quad B = \frac{\Gamma/2}{\omega_0 - \omega_1} \qquad (6)$$

imply

$$|G_\omega|^2 \propto \frac{(1 + BX)^2 + B^2}{1 + X^2}. \qquad (7)$$

The above formula is precisely Nigam's formula for asymmetric line profiles, which was derived in full detail for model p modes (Nigam & Kosovichev, 1998b). The frequency ω_1 depends implicitly on the source location. Following the terminology of Rast & Bogdan (2000), line shapes are controlled by the relative position of valleys ($\omega = \omega_1$) and peaks ($\omega = \omega_0$) in the power spectrum. A large amount of line asymmetry requires that valleys and peaks be close to each other.

Nigam & Kosovichev (1998b) called B the "asymmetry parameter". Here, however, we prefer to reserve the term for the quantity

$$\chi = \frac{B\omega_0}{\Gamma/2} = \frac{\omega_0}{\omega_0 - \omega_1}. \qquad (8)$$

This definition is perhaps more appropriate as χ does not depend on the damping rate, which is quite irrelevant in the context of line asymmetry. Note that, as χ tends to zero, the line profile becomes Lorentzian with a full-width at half maximum given by the damping rate Γ.

3. Observations

We used 17 hours of intensity and line-of-sight velocity images obtained on 27 January 1997 by the Michelson Doppler Imager in its high-resolution mode at a cadence of 60 s (Scherrer et al., 1995). After correction for rotation, power spectra were averaged over the direction of wave propagation.

The intensity and velocity power spectra are plotted in Figure 1 versus degree, $l = kR_\odot$, and $(\omega - \sqrt{gk})/2\pi$. The degree resolution is 20.7 and the frequency resolution is 16.27 μHz. For each value of l, the f-mode power was normalised to its maximum value. At low wavenumbers and low frequencies the signal-to-noise ratio is poor: the f mode can hardly be seen below $l \sim 250$ in velocity and $l \sim 500$ in intensity. A quick examination of the contours of equal (normalised) power reveals that the velocity line profiles are asymmetric near degree 700, with more power in the low frequency wing. Line asymmetry is less obvious in the intensity power spectrum. Figure 2 displays the velocity and intensity power spectra averaged over the degree range 879-940 as a function of frequency, showing both the f and p_1 modes. Line asymmetry is clearly seen to be different in intensity and velocity power spectra, while the sign of the asymmetry is the same for f and p_1 at these wavenumbers.

We measured the asymmetry parameter χ from fits to the observed velocity and intensity f-mode line profiles using equation (7). The background signal (solar noise plus p-mode wings) was modelled by a smooth fourth-order polynomial. Nonlinear least-square fits were performed over a frequency interval of about four line-widths centered on the frequency $\sqrt{gk}/2\pi$. Figure 3 shows the asymmetry parameter measured by the fits. Results are only shown for the degree range 600-1200. In this range, the resonant frequency ω_0 is measured to be the same in the velocity and intensity power spectra (within standard error bars). The velocity and intensity line-widths are also within error bars of each other, and consistent with the measurements of Duvall, Kosovichev & Murawski (1998).

The formal errors on χ are small, as can be judged in Figure 3 from the smooth variation with wavenumber. Systematic errors, on the other hand, are difficult to estimate. In particular, an error could be introduced by an inaccurate model of the noise background.

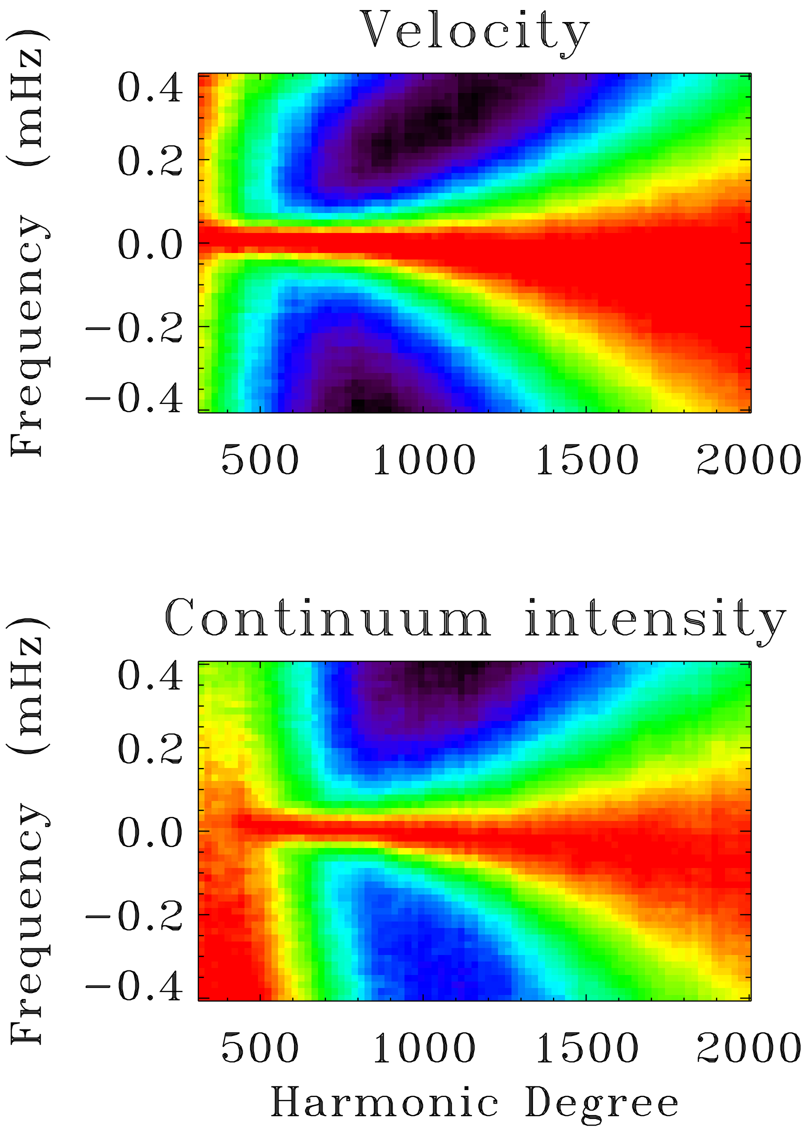

Figure 1: F-mode power spectra computed from 17-hour time series of high-resolution MDI Dopplergrams and intensity images. The vertical coordinate is the temporal cyclic frequency minus the resonant frequency of the pure f mode, $\sqrt{gk}/2\pi$. At each value of $l = kR_\odot$, the power was normalized to its maximum value in the f-mode ridge. The colors are values of equal normalized power (see *electronic version of the paper*). Line asymmetry is clearly visible in the velocity power spectrum for degrees near 700.

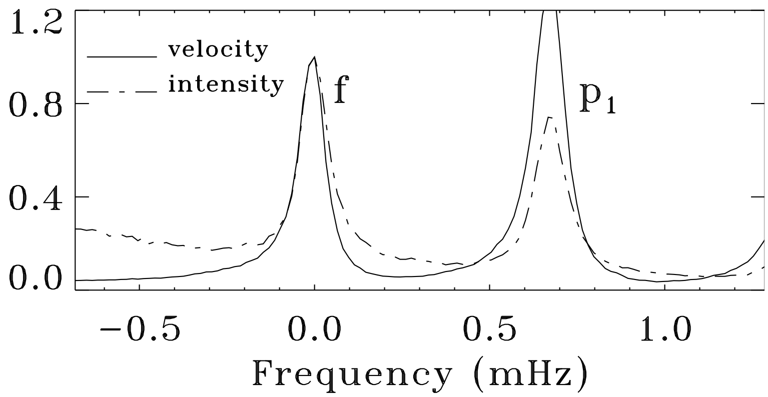

Figure 2: MDI power spectra averaged over the degree range 879-940. The horizontal coordinate is the temporal cyclic frequency minus $\sqrt{gk}/2\pi$. The solid and dashed lines are for the velocity and intensity data respectively. For comparison, power spectra were normalised by the values at maximum f-mode power.

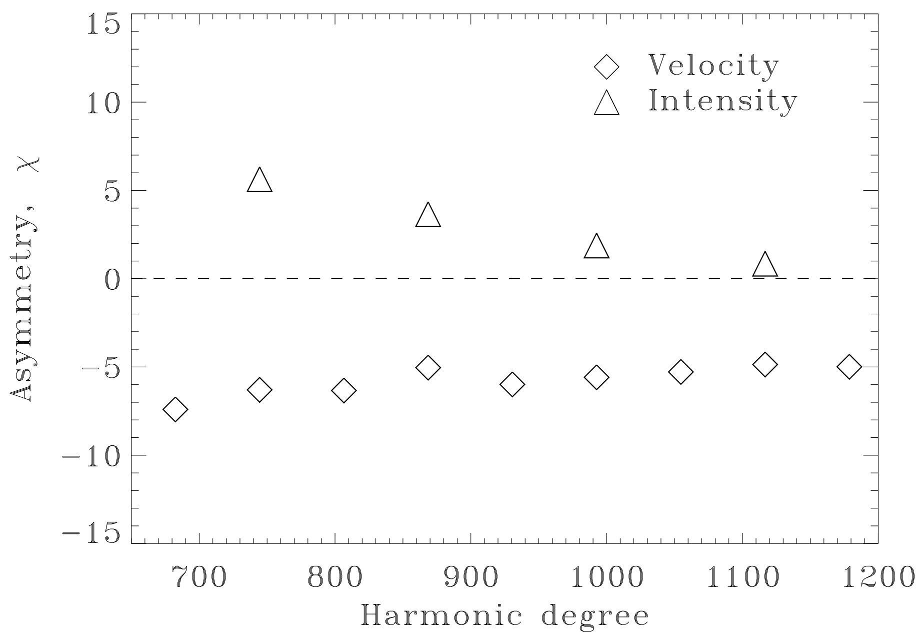

Figure 3: F-mode line asymmetry, χ, measured from velocity (diamonds) and intensity (triangles) MDI power spectra.

4. Discussion

We measured the line asymmetry of high-degree f modes from MDI power spectra. The asymmetry parameter, as defined in Section 2, is negative for velocity data and positive for intensity data. It is more pronounced in velocity ($\chi \sim -5$) than in intensity data. Although these conclusions are quite solid, the simple fitting procedure that we used could be improved in many ways. For example, the whole k-ω diagram could be fitted all at once (e.g. Meunier & Jefferies, 2000) to better capture the functional form of the noise background and the contribution from the wings of neighbouring p modes.

One may ask if the mechanism responsible for f-mode line asymmetry can be described in simple physical terms, as is done for p modes. An argument based on wave interference (Duvall et al., 1993) has little value in the case of f modes, which do not propagate in the vertical direction. Is it at all conceivable that line asymmetry may occur from combinations of exponential wave functions? To answer this question, we consider the propagation of a surface wave at the interface $z = z_*$ between two media with different constant densities ($\rho_1 > \rho_2$), forced at height z_s by a vertical momentum impulse. Resonance occurs at $\omega_0 = \sqrt{\alpha g k}$, where $\alpha = (\rho_1 - \rho_2)/(\rho_1 + \rho_2)$. The Green's function for the vertical wave velocity is such that line asymmetry is only significant when the source is situated above the interface, i.e. $z_* < z_\mathrm{s} < z_\mathrm{o}$. In particular, in the limit of a large density discontinuity ($\alpha \rightarrow 1^-$), the asymmetry parameter measured in the velocity power spectrum is given by

$$\chi \simeq \frac{2}{1-\alpha} \left(e^{2k(z_\mathrm{s}-z_*)} - 1 \right) \quad \text{for} \quad z_* < z_\mathrm{s} < z_\mathrm{o} \,. \tag{9}$$

Thus $\chi > 0$ in this case. It can be shown, however, that the sign of χ becomes negative when the Green's function is replaced by its first derivative with respect to source position. This toy model is not intended to approximate the Sun, but it has the merit of demonstrating that line asymmetry can occur even for waves that do not propagate in the vertical direction, such as the f mode.

A proper explanation for f-mode line asymmetry must include the entire wave propagation diagram (e.g. Rast & Bogdan, 2000) and must take into account a background component correlated with the oscillations in both velocity and intensity measurements (Jefferies et al., 2003). Notice that the ratio in mode power between f and p_1, which depends strongly on the observation type (velocity or intensity, see Fig. 2), is likely to contain valuable information about the excitation mechanism of solar oscillations.

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