Improved estimates of solar and stellar oscillation parameters

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Abstract

Quantitative helio- and asteroseismology require extremely precise measurements of the frequencies, amplitudes, and lifetimes of the global modes of stellar oscillation. Unfortunately continuous observations are rarely available. The Fourier analysis of such gapped time series is not straightforward. In the Fourier domain, the signal is convolved with the Fourier transform of the observation window. This implies, in particular, that Fourier amplitudes at different frequencies are correlated. We have derived and implemented maximum likelihood estimators of stellar oscillation parameters, which explicitly take such frequency correlations into account. Using Monte-Carlo simulations of stochastically excited solar-like oscillations, we find that our new fitting method retrieves oscillation parameters with less bias and greater precision.

1. Introduction

In Fourier space, the observed signal, $\hat{y}_i = \hat{y}(\omega_i)$, is given by the convolution of the observation window function, $\hat{w}_j$, with the unconvolved signal $\hat{x}_j$ (see Fig. 1). For a section of the observed signal starting at frequency ω_q and ending at frequency ω_{q+M-1} we have

$$y_i = \sum_{j=0}^{M+2p-1} W_{ij} x_j \quad i = 0, 1 \ldots, M-1, \qquad (1)$$

where $y_i = \hat{y}_{q+i}$ with $i = 0, 1, \ldots, M-1$, $x_i = \hat{x}_{q-p+i}$ with $i = 0, 1, \ldots, M+2p-1$, and the $W_{ij} = \hat{w}_{i-j+p}$ are the elements of an $M \times (M+2p)$ rectangular window matrix with rank M. The integer p refers to the cutoff frequency ω_p beyond which the observation window is assumed to have no significant power. Assuming that the unconvolved signal is stationary in the time domain, the x_j are $M+2p$ independent complex random variables in the Fourier domain. The M observed y_i, however, are correlated because of the convolution with the observation window, according to Eq. (1).

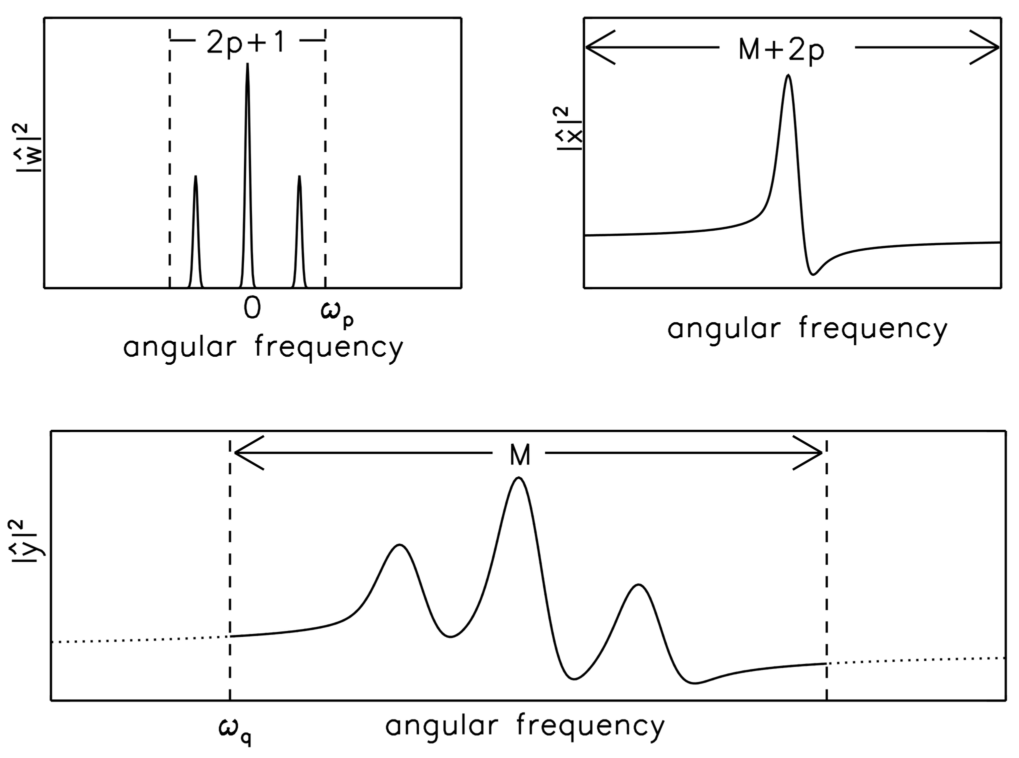

Figure 1: *Schematic illustration of the convolution of the unconvolved signal (top right) with the observation window (top left) to obtain the observed signal (bottom). For simplicity, only the power of the different quantities is shown.*

2. The old fitting method

Stellar oscillation parameters are estimated by maximizing the joint probability density function (pdf) of the random vector $\mathbf{y}$, evaluated at the observed sample data. So far, this joint pdf is often estimated to be the product of the pdf's of the y_i, as if the frequency bins were uncorrelated (see e.g. Anderson et al. 1990). This assumption is not strictly correct when there are gaps in the data.

3. The new fitting method

Our improved fitting method is closely related to the work of Gabriel (1994). The data correlations are explicitly taken into account. It can be shown that the joint pdf of the observed signal $\mathbf{y} = (y_0, y_1, \ldots, y_M)^T$ is then

$$p_{\mathbf{y}}(\mathbf{y}) = \frac{\exp(-\|C^\dagger(\mathbf{y} - W\mathbf{d})\|^2)}{\pi^M (\det \Lambda)^2}. \qquad (2)$$

- $C^\dagger$ is the Moore-Penrose generalized inverse of $C = WD$
- $D = \mathrm{diag}(\sigma_0, \sigma_1, \ldots, \sigma_{M+2p-1})$, where σ_i is the standard deviation of the stochastic component of the signal at frequency ν_i
- $\Lambda = \mathrm{diag}(\lambda_0, \lambda_1, \ldots, \lambda_M)$, where the λ_i are the singular values of the matrix C
- $\mathbf{d} = (d_0, d_1, \ldots, d_{M+2p-1})^T$ describes the deterministic component of the unconvolved signal

This expression for the joint pdf, which takes into account the effect of the correlations in the observed data, leads to correct maximum likelihood estimators, as we now show using simulated data.

4. Monte-Carlo simulations

An observed time series is a realisation of a random process, either because of noise and/or the stochastic nature of stellar oscillations. Here we wish to test the maximum likelihood estimator derived above by fitting many realizations of the same random process to reconstruct the distributions of the inferred parameters and conclude about the bias and the dispersion of the estimator.

In the next section, we consider 1000 realisations of a single mode of solar-like oscillation on top of a white noise background. In section 6, we consider a pure sine wave (in the time domain) plus white noise (500 realisations). To simulate the observations, the unconvolved data are convolved with a fixed observation window function (power shown in Figure 2). It is a typical window for a single ground-based site and a total observation duration $T = 16$ days; it has a main periodicity of 24 hours, a duty cycle of 30%, and some randomness in the end time of each observation block.

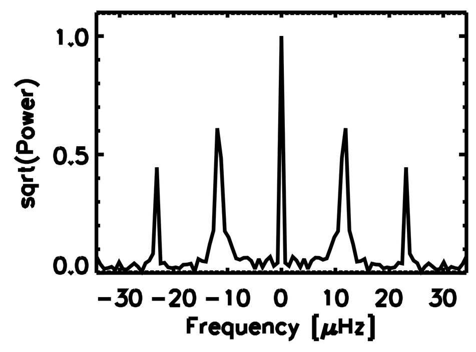

Figure 2: *Square root of the power spectrum of the observation window used for the Monte-Carlo simulations.*

5. Solar-like oscillations

In this example, we consider one mode of solar-like oscillation with a FWHM of $3.2\ \mu$Hz, some degree of asymmetry, a signal-to-noise ratio of 6 (defined in the power spectrum). The window is as defined above. Figure 3 shows the power of one realization, the old fit (blue), the new fit (red), and the expectation value of the power spectrum (green). In this particular case, the new fit is significantly better than the old one.

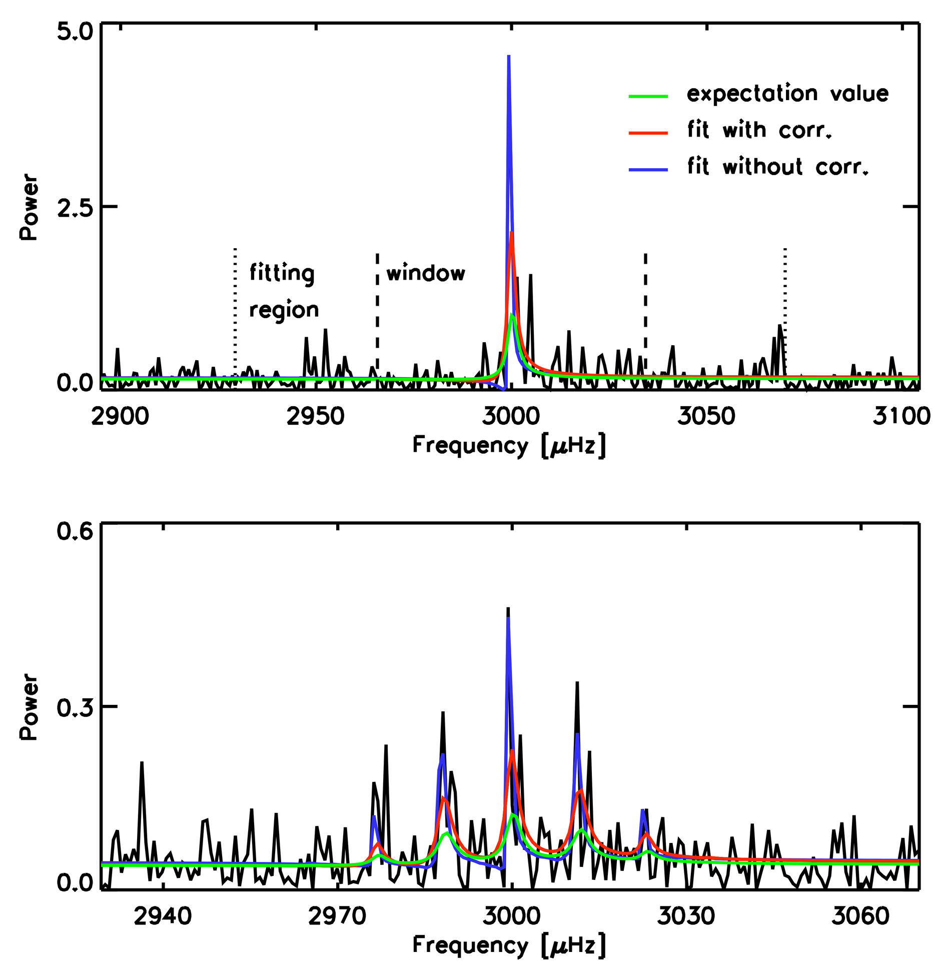

Figure 3: *Realisation of a power spectrum of a solar-like oscillation mode with the corresponding fit with the new fitting method (red) and the old one (black). The green line shows the expectation value. The top panel shows the signal of an uninterrupted time series, the bottom panel shows the observed signal after the convolution with the observation window ($T = 16$ days, 30% duty cycle).*

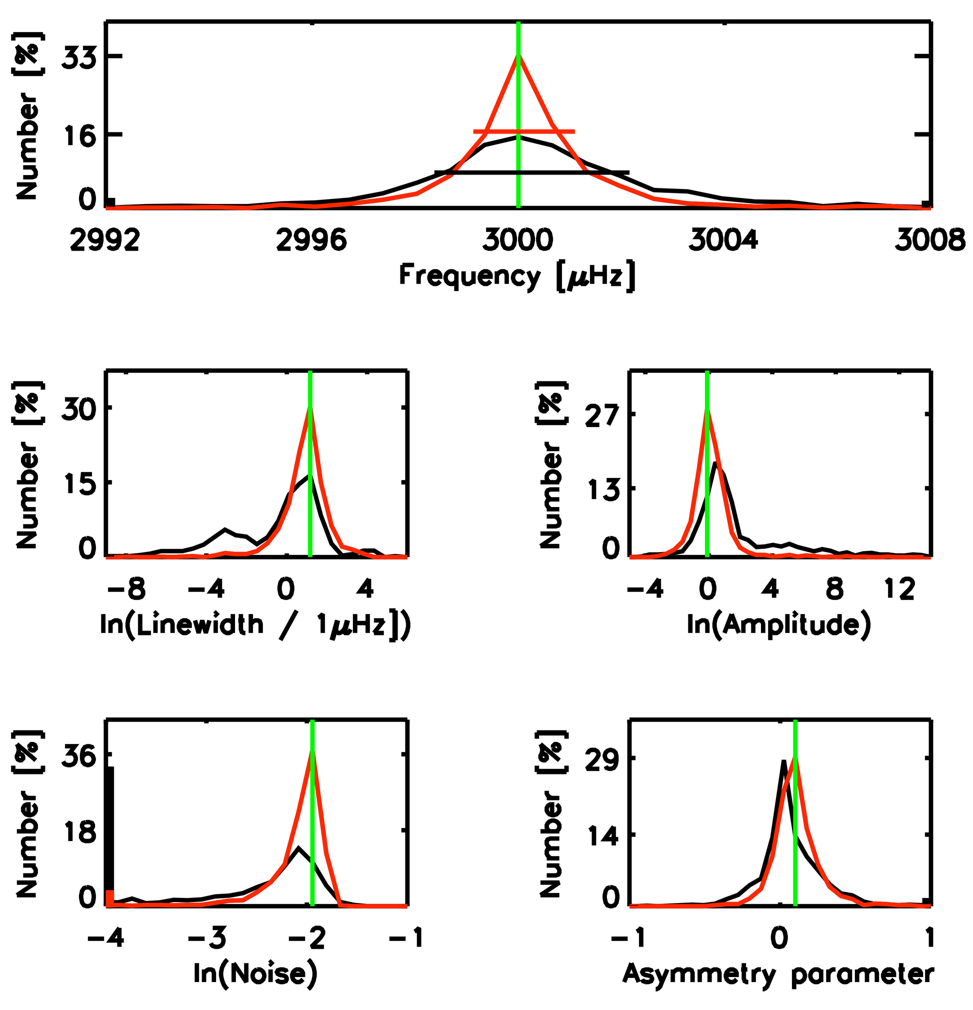

Figure 4: *Distributions of the mode parameters for a set of 1000 realisations with $S/N = 6$. The red line corresponds to the new fitting method, the black one shows the results with the old method, the green line indicates the input parameter (window: $T = 16$ days, 30% duty cycle).*

Using 1000 such realisations, we can study the distributions of the inferred parameters (Fig. 4). The new fit is clearly superior. Estimates are less biased, the dispersion on the frequency estimate is divided by two compared to the old fitting method, and the number of outliers for linewidth, amplitude and noise estimates goes down.

6. Deterministic oscillations

Here we consider the case of a pure sine wave (in the time domain) plus white noise (500 realisations). We find that the estimators for all the mode parameters (frequency, amplitude, phase, noise level) are unbiased and have nearly the same dispersion for both the old and the new fitting methods (see Fig. 5). The new fitting method does not provide any significant improvement, regardless of the signal-to-noise ratio.

It is worth mentioning that the mode frequency can be measured with very high precision, even for low signal-to-noise ratios. In Figure 5, for example, the frequency dispersion is smaller than $1/T$ by a factor of 5 ('super-resolution') and it is comparable with a theoretical estimate for the error of the frequency (Cuypers 1987).

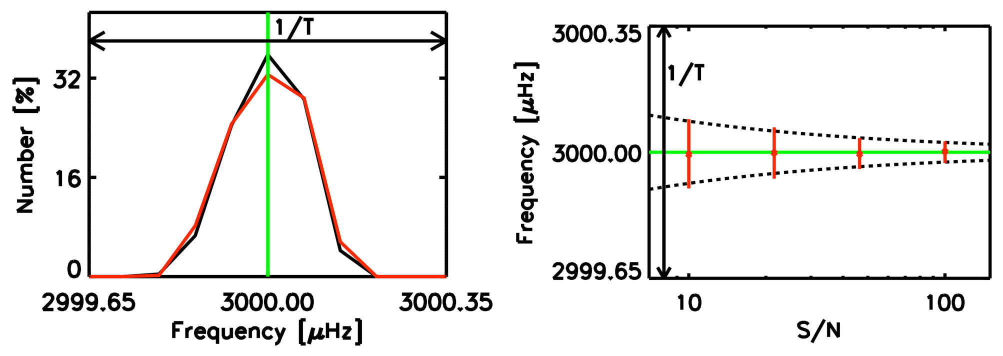

Figure 5: *Left: Distribution of inferred frequencies in the case of a deterministic harmonic signal plus a white noise background (500 realisations). The signal to noise ratio is 21. Right: The median and the dispersion of the frequency estimates is plotted as a function of signal-to-noise ratio. The dashed line represents the theoretical estimate for the frequency error (Cuypers 1987). [red: new method, black: old method, green: input mode frequency; observation window: $T = 16$ days, 30% duty cycle].*

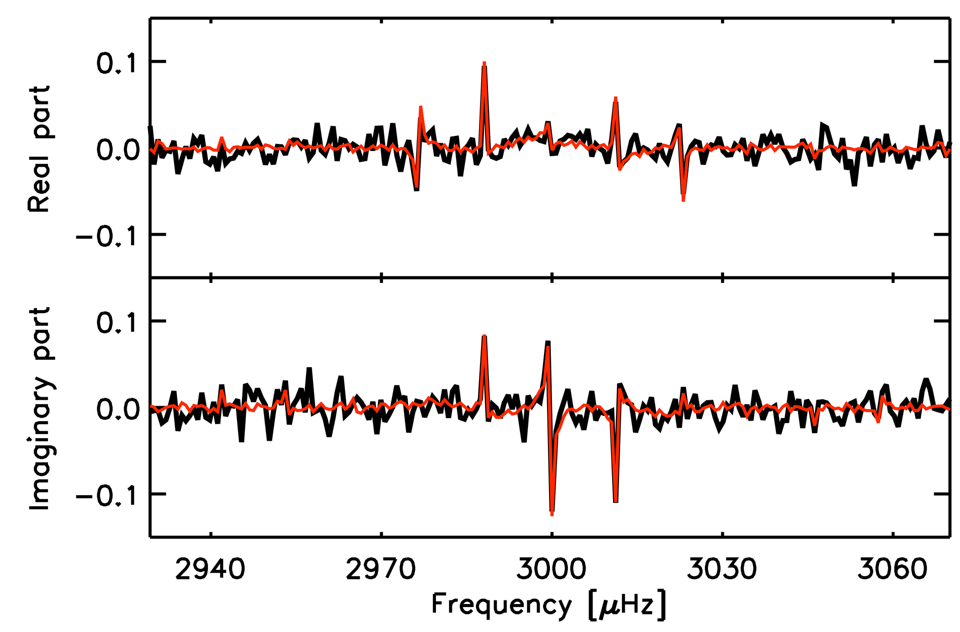

Figure 6: *Example fit (red) to one particular realization (black) in complex Fourier space. (window function: $T = 16$ days, 30% duty cycle).*

7. Conclusions

For solar-like oscillations, our improved fitting method in Fourier space, which takes the correlations of the data into account, provides less biased and more accurate estimates, especially when the signal-to-noise ratio is low and the observation window has large gaps (here: $T = 16$ days, 30% duty cycle). In that particular case, the frequency uncertainty is decreased by a factor of two with respect to the old fitting method and parameters like the mode amplitude and the mode linewidth are significantly less biased.

For the case of a deterministic sinusoidal oscillation on top of a white noise background, the old and new fitting are equivalent. We confirm that the frequency can be retrieved with a precision which is much better than 1/T ('super-resolution').

8. Future Applications

Our new fitting method could be applied to stars exhibiting solar-like oscillations. α Cen A might be a suitable target for which excellent data already exists (e.g. Butler et al. 2004). A revised table of oscillation parameters with more precise mode frequencies will enable us to verify and to extend the mode identification of the detected modes.

We intend to apply the new fitting method to BiSON data. The duty cycle of the BiSON observations is in the range of 60-80%. Therefore, an analysis of the data taking into account the effects of the gaps properly will lead to more precise measurements of the global modes of solar oscillations.

References

Anderson, E. R., Duvall, Jr., T. L., & Jefferies, S. M. 1990, *Astrophys. J.*, 364, 699

Butler, R.P., Bedding, T.R., Kjeldsen, H., McCarthy, C., O'Toole, S.J., Tinney, C.G., Marcy, G.W., and Wright, J.T. 2004, *Astrophys. J.*, 600, L75.

Cuypers, J. 1987, *Medelingen van de Koninklijke Academie voor Wetenschappen, Letteren en Schone Kunsten van België*, Jg. 49 Nr. 3 (Brussel: Palais der Academieën), 21.

Gabriel, M. 1994, *Astron. Astrophys.*, 287, 685