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8 - 12 September 2008
Freiburg, Germany

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Monday 14:45-15:00

Meridional Circulation and Global Solar Oscillations

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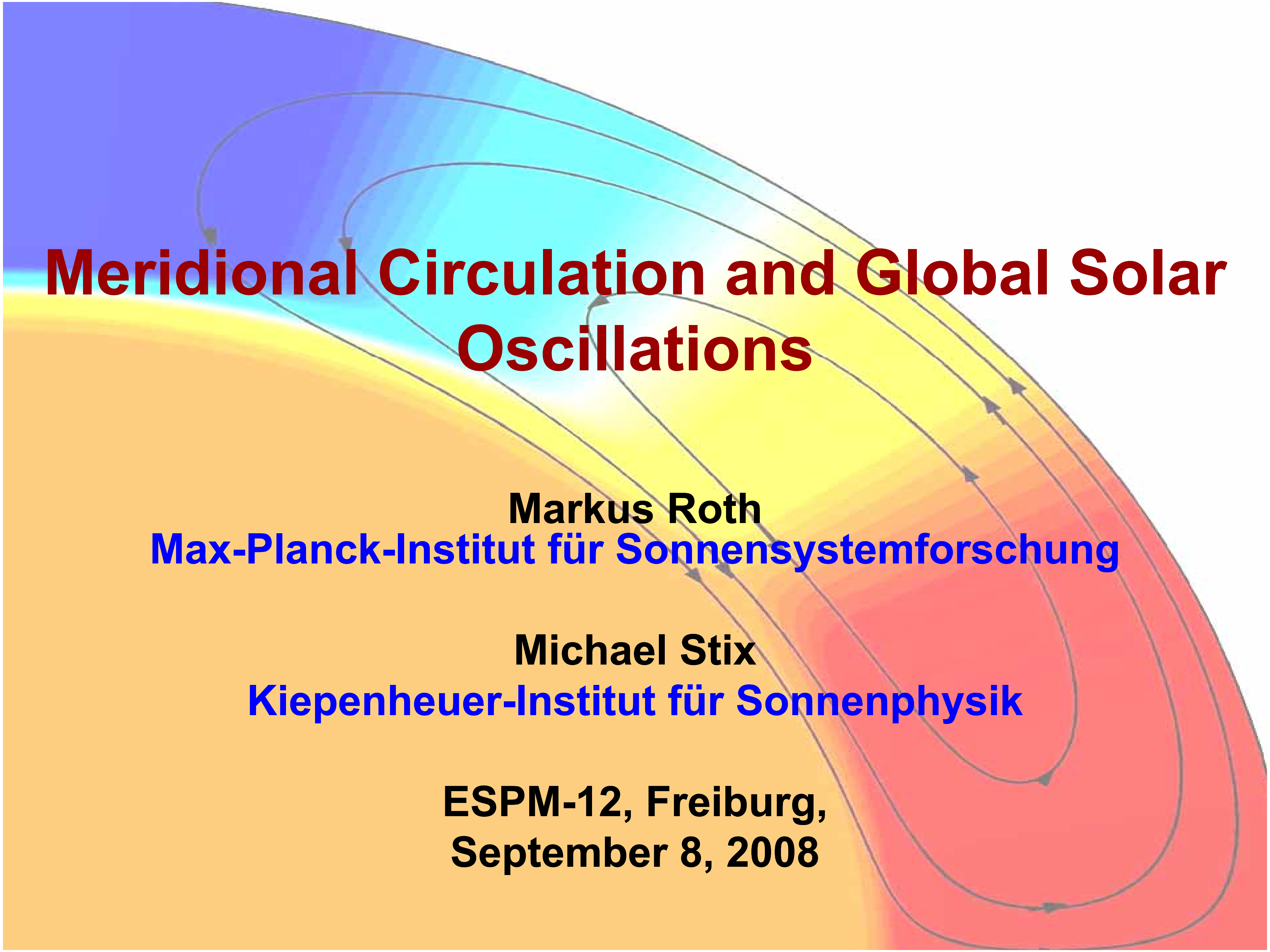
[1]*Max-Planck-Institut für Sonnensystemforschung;* [2]*Kiepenheuer-Institut für Sonnenphysik*

We investigate the influence of large-scale meridional circulation on solar p-modes by quasi-degenerate perturbation theory, as proposed by Lavely & Ritzwoller, 1992 (Roy. Soc. Lon. Phil. Trans. Ser. A, 339, 431). As an input flow we use various models of stationary meridional circulation obeying the continuity equation. This flow perturbs the eigenmodes of an equilibrium model of the Sun. We derive the signatures of the meridional circulation in the frequency multiplets of solar p modes. In most cases the meridional circulation leads to negative average frequency shifts of the multiplets. Further possibly observable effects are briefly discussed.



Meridional Circulation and Global Solar Oscillations

Markus Roth
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Michael Stix
Kiepenheuer-Institut für Sonnenphysik

ESPM-12, Freiburg,
September 8, 2008

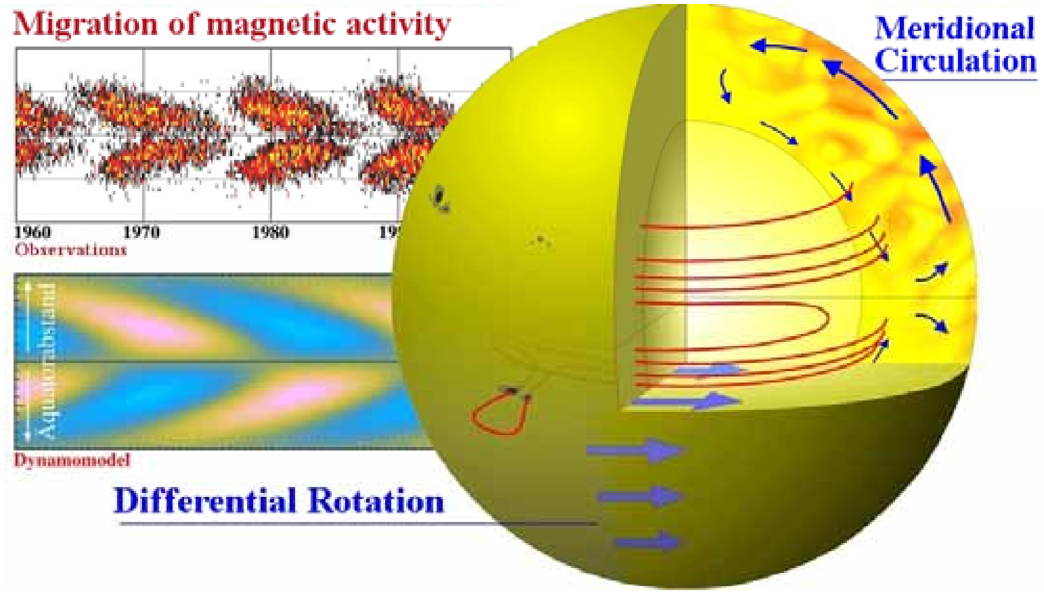
Meridional Circulation

Definition (Meridional Circulation):

A circulation in a vertical plane oriented along a meridian. It consists, therefore, of the vertical and the meridional (North or South) components of motion only.

Observations on solar surface:

- poleward flow
- v ~ 15m/s



(courtesy R. Arlt, AIP)

The Solar Case

Mass transported by the flow:

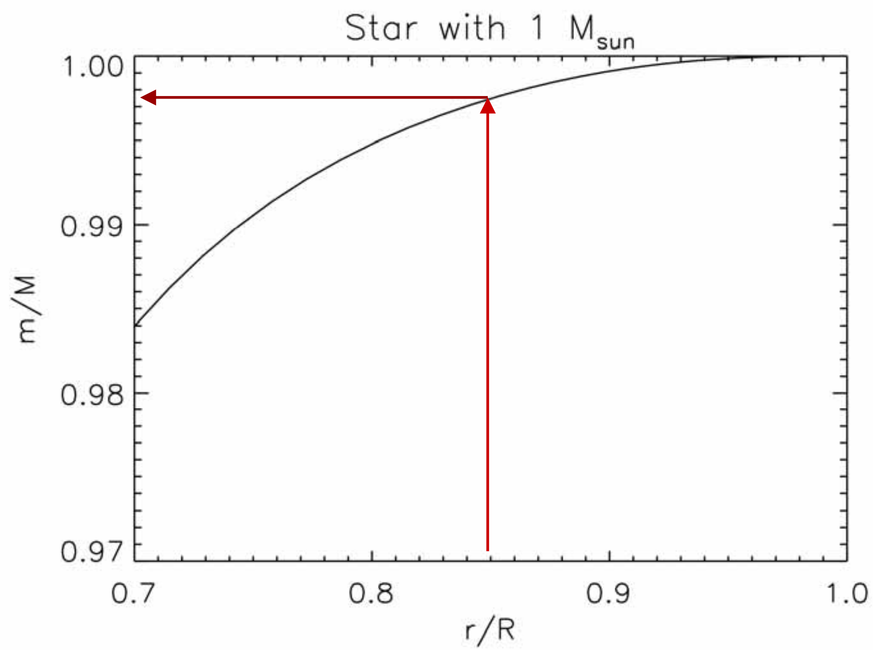
Top half of convection zone contains 0.25% of stellar mass

lower CZ contains ~ **5 times** as much mass

Mass Conservation:

poleward flow of 10 m/s in the outer half of the CZ requires equatorward return flow of 2 m/s in the lower convection zone

At the bottom of the CZ:
Magnetic flux transport time from middle latitudes to equator: **10 years**



Velocity of meridional circulation might determine length of the cycle.

But 2 m/s are hard to detect by seismology.

Role of the Meridional Circulation in the Sun

Important for solar convection zone dynamics and solar dynamo mechanism

Deep equatorward meridional flow needed in some solar dynamo models, esp. flux-transport dynamos

Other ingredients: **depth of penetration**

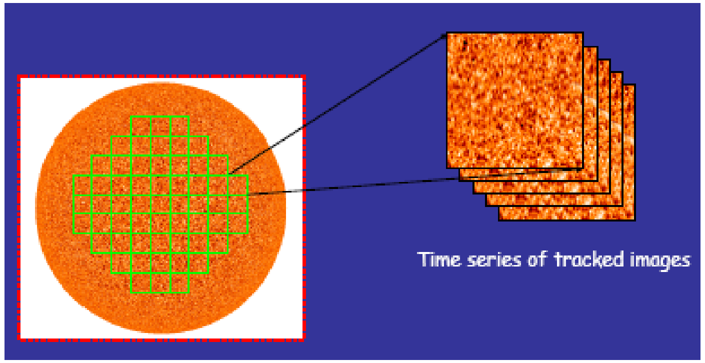
Mass transport from one latitude to another

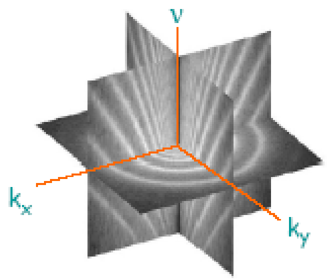
$ **transport of angular momentum and magnetic field**

$ **#feedback on differential rotation**

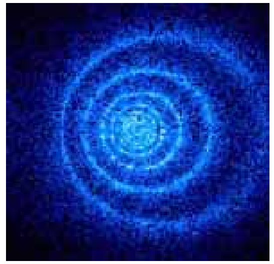
Local Helioseismology Techniques

- Ring Diagrams



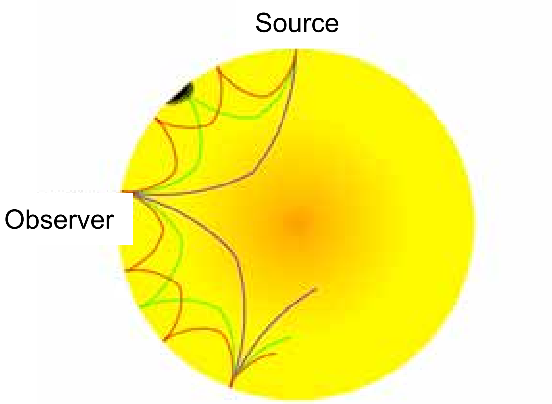


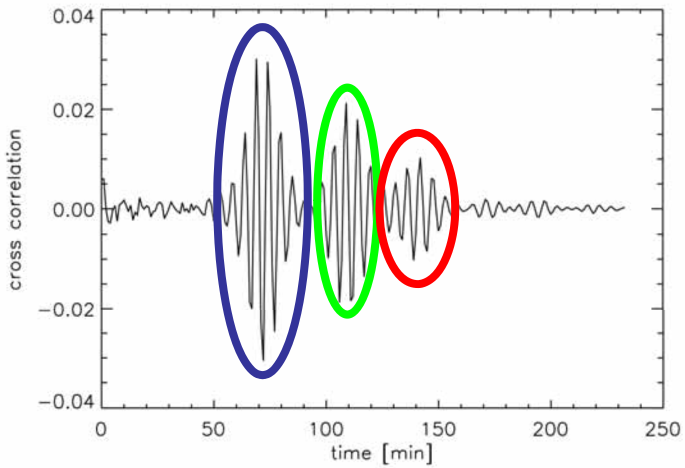
Acoustic Power



v_x (r) → Zonal flow
v_y (r) → Meridional flow

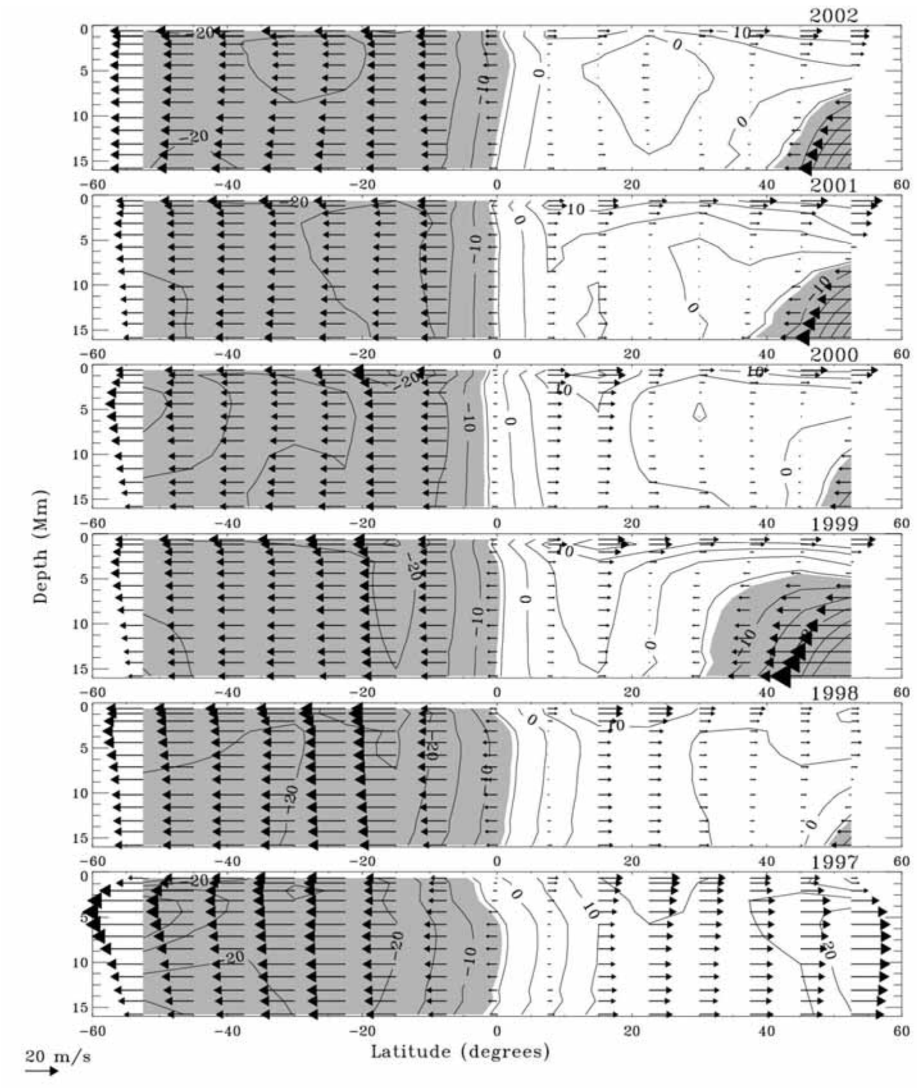
- Time-Distance Helioseimology





\$ Limitation in depth due to smaller areas

Meridional Circulation as seen by Ring-Diagram Analysis

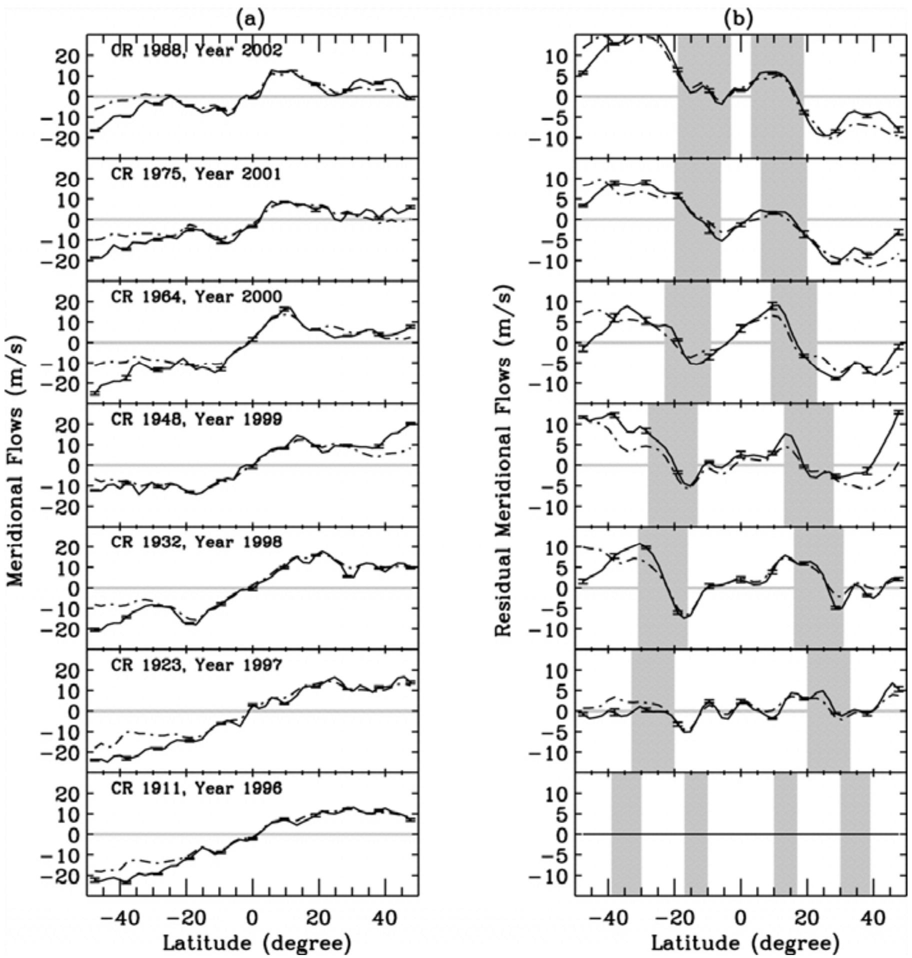


(Haber et al. 2002)

General findings:

- Mainly poleward flow
- Flow strength ***down to 15Mm*** very variable: 0-60 m/s
- Weak horizontal flow at the equator
- In 2002 a "*counter cell*" was reported on the Northern hemisphere, that started evolving from 1998.
- Results may be contaminated in part by errors in the orientation of the MDI / GONG dopplergrams (Zaatri et al. 2006, Beckers 2007).

Meridional Circulation as seen by Time-Distance Helioseismology



(Zhao & Kosovichev 2004)

Meridional flow properties:

• Poleward flow at all latitudes and depths (***down to 12 Mm***) in the order of 15 m/s

• Flow residuals show convergence towards active regions.

$ Extra-meridional circulation rolls on each side of the mean latitude of activity.

Only top layers are probed,
no information about return flow

Effect of Meridional Circulation on Global Solar Oscillations

Forward Problem:

- Use a 1D solar model → `Model S´
  (spherically symmetric, no magnetic field, no rotation, no flows) → Eigenoscillations
- Apply perturbations to this model, e.g. flows
  → model for meridional circulation
- Determine the *oscillation frequency shifts* theoretically

Flow Models

Forward problem (effect of flow on oscillations) well studied:

Decomposition of meridional flow into spherical harmonics:

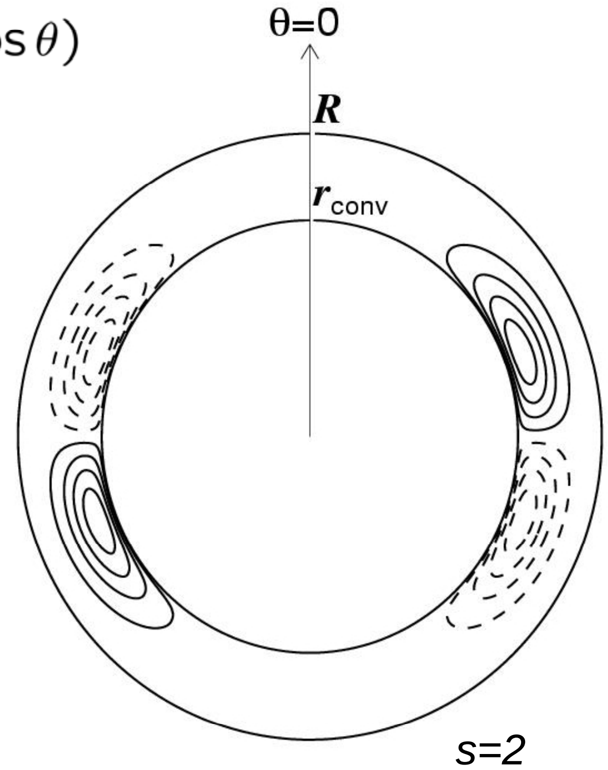
$$\boldsymbol{v}_{\text{mer}}(\boldsymbol{r}) = \sum_s u_s(r) P_s(\cos\theta)\mathbf{e}_r + v_s(r)\nabla_h P_s(\cos\theta)$$

Mass conservation (anelastic condition):

$$\nabla \cdot (\rho_0 \boldsymbol{v}_{\text{mer}}) = 0$$

$$\Rightarrow v_s = \frac{\partial_r (r^2 \rho_0 u_s)}{\rho_0 r s(s+1)}$$

only $u_s(r)$ is needed to describe the flow



Flow Models

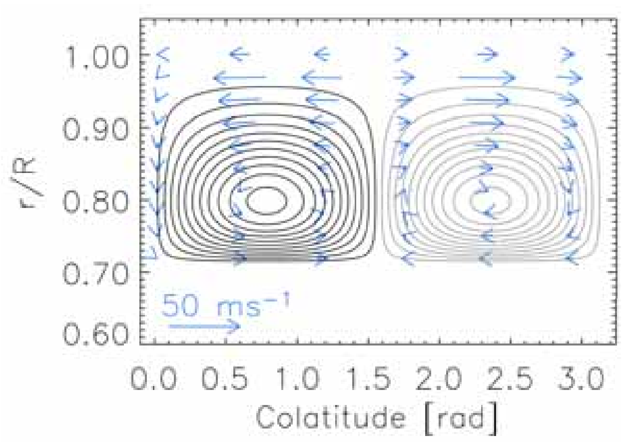
Study the effect of 15 different flow models: s=2,...,6 latitudinal dependence

Model for radial dependence $u_s(r)$:

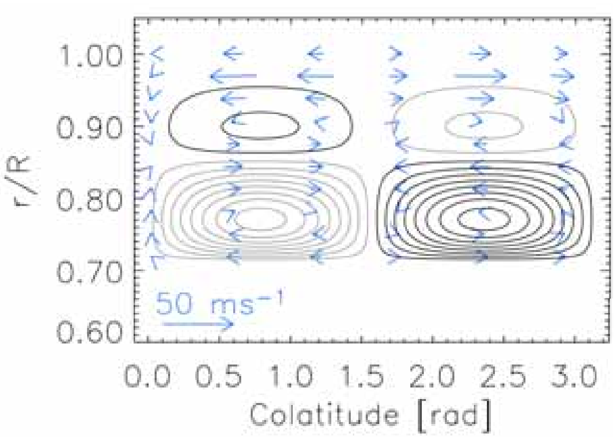
$$u_s(r) = A \sin\left(n_c \pi \frac{r - r_b}{R_\odot - r_b}\right) \quad \text{for } r_b \le r \le R_\odot \,,$$
$$u_s(r) = 0 \quad \text{otherwise} \,.$$

Amplitude A selected such that $v_s(R_\odot)$ has a maximum of 15 m/s.

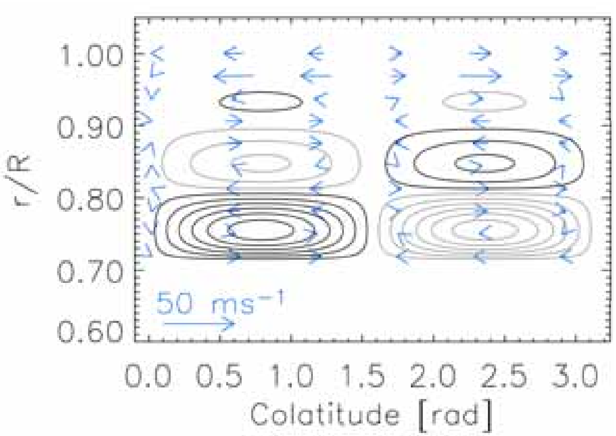
s=2, n_c=1



s=2, n_c=2



s=2, n_c=3



Flow Models

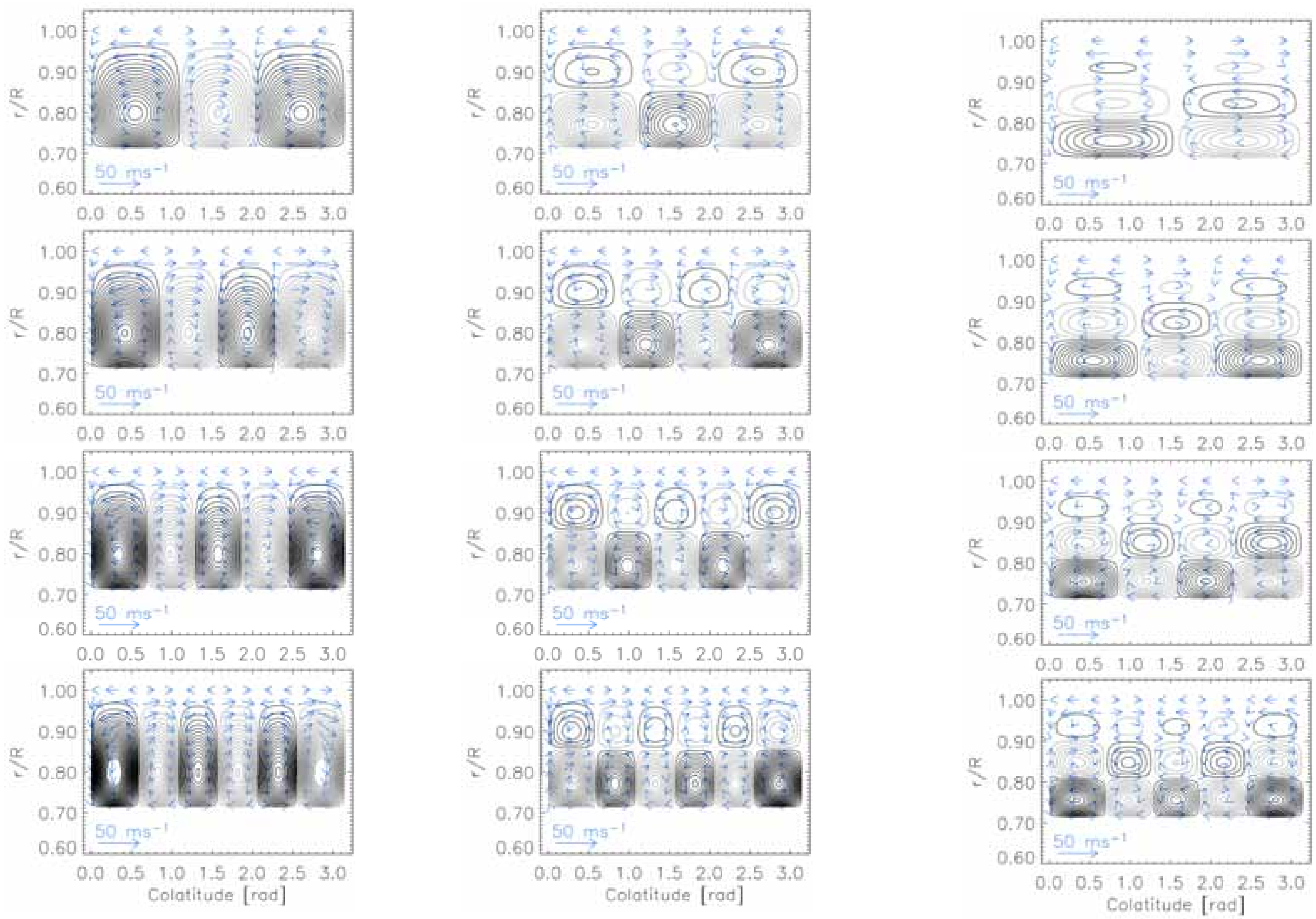
$n_c=1$ $n_c=2$ $n_c=3$

s=3

s=4

s=5

s=6



Mode Coupling

The equation governing the eigenmodes is

$$\mathcal{L}_0 \boldsymbol{\xi}_k = -\rho_0 \omega_k^2 \boldsymbol{\xi}_k \ .$$

Describing the disturbing effect of a stationary flow we replace

$$\begin{aligned} \mathcal{L}_0 &\rightarrow \mathcal{L}_0 + \mathcal{L}_1 \ , \\ \omega_k^2 &\rightarrow \tilde{\omega}_j^2 \ , \\ \boldsymbol{\xi}_k &\rightarrow \tilde{\boldsymbol{\xi}}_j \ . \end{aligned}$$

The perturbation operator $\mathcal{L}_1$ is defined in terms of the meridional circulation $\boldsymbol{v}_{\mathrm{mer}}$

$$\mathcal{L}_1(\boldsymbol{\xi}_k) = -2\mathrm{i}\omega_{\mathrm{ref}}\rho_0(\boldsymbol{v}_{\mathrm{mer}} \cdot \nabla)\boldsymbol{\xi}_k \ .$$

(Ritzwoller & Lavely, 1992)

Mode Coupling

The perturbed eigenfunctions are expressed as normal mode expansion

$$\tilde{\boldsymbol{\xi}}_j = \sum_{k \in \mathcal{K}} a_k^j \boldsymbol{\xi}_k \ ,$$

The problem is turned into an algebraic eigenvalue problem

$$\sum_{k \in \mathcal{K}} a_k^j Z_{k'k} = \sum_{k \in \mathcal{K}} a_k^j \lambda_j \delta_{k'k} \qquad \text{for } k' \in \mathcal{K} \ .$$

The elements of the matrix $\mathbf{Z}$ are given by

$$Z_{k'k} = \frac{1}{N_{k'}} \left\{ H_{n'n,l'l}^{m'm} - (\omega_{\mathrm{ref}}^2 - \omega_k^2) N_k \delta_{k'k} \right\} \ ,$$

with

$$H_{n'n,l'l}^{m'm} = - \int \boldsymbol{\xi}_{k'}^* \cdot \mathcal{L}_1 \boldsymbol{\xi}_k \, d^3 r \ .$$

1st order perturbation theory

Coupling of Angular Momenta

- Some matrix elements vanish, i.e. $H_{nn',ll'}^{m\,m'}=0$

- Only certain combinations of l, l', s give a non-vanishing entry in the coupling matrix
  → coupling of angular momenta
  → selection rules:
  - ***l,l´,s* must form a triangle**
  - ***l+l´+s* must be even**
  - ***m´=m***

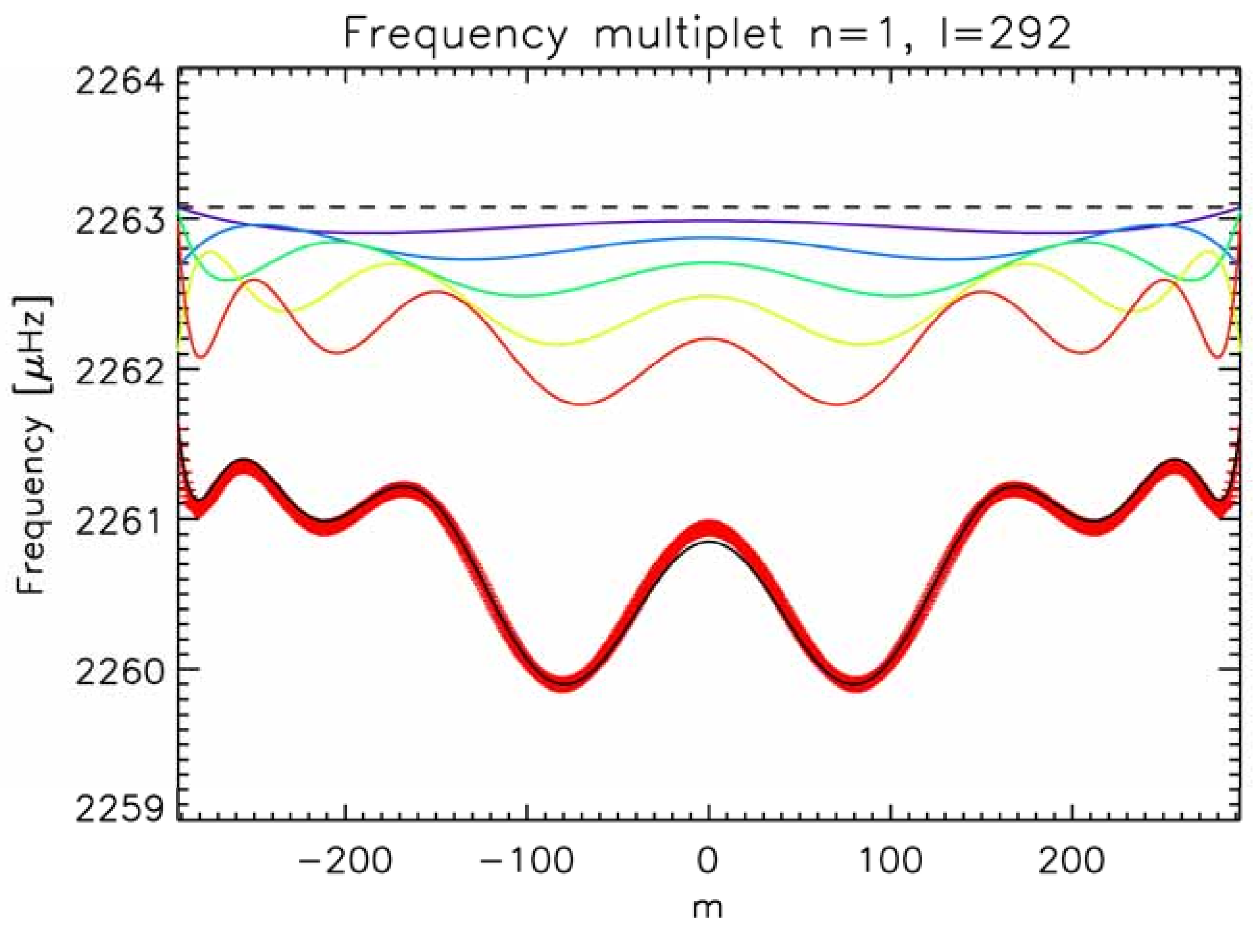
Procedure:

→ Calculate the full coupling matrix ***Z*.**

→ Perturbed frequencies are eigenvalues of this matrix.

Lifting of Degeneracies

Mode coupling due to the meridional circulation leads to splitting of multiplets



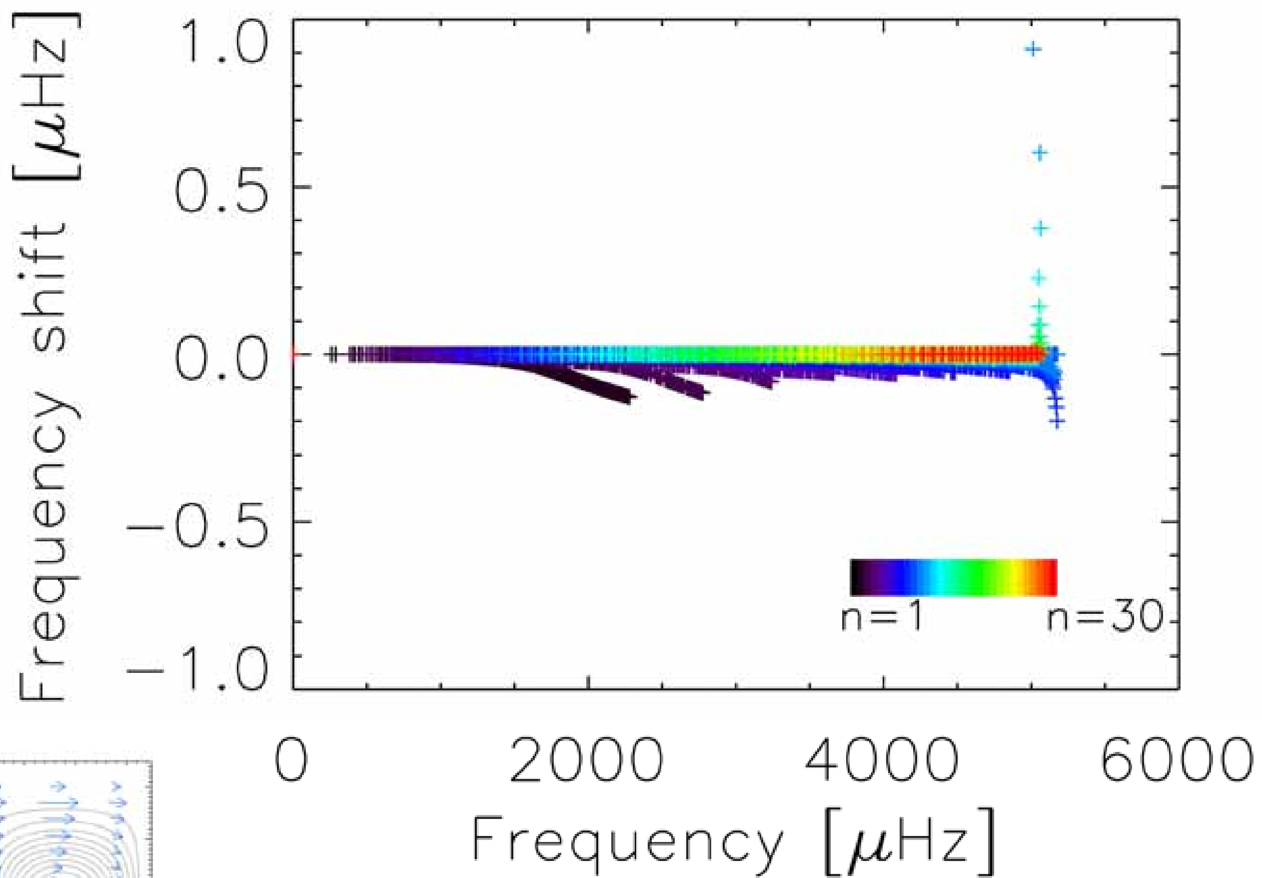
s=2, n_c=1
s=3, n_c=1
s=4, n_c=1
s=5, n_c=1
s=6, n_c=1

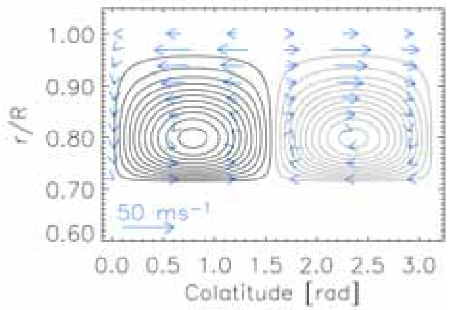
overall effect

overall effect, but n_c=3

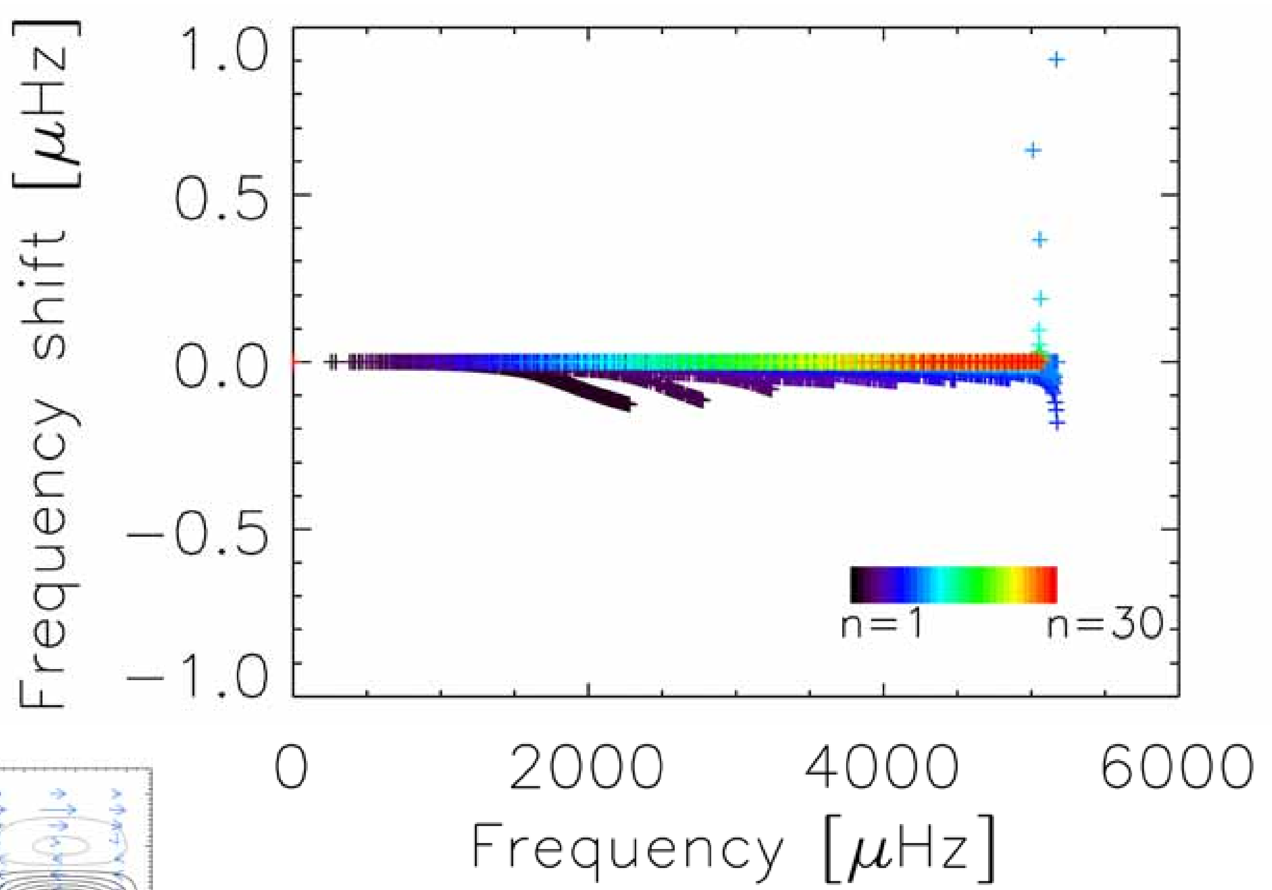
s=2, n_c=1

Mean frequency shifts of the multiplets





s=2, n_c=2


Frequency shift [μHz]
1.0
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0.0
−0.5
−1.0
n=1
n=30
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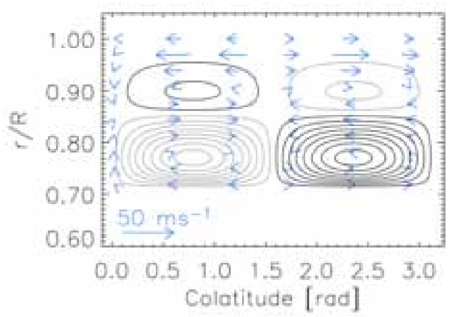
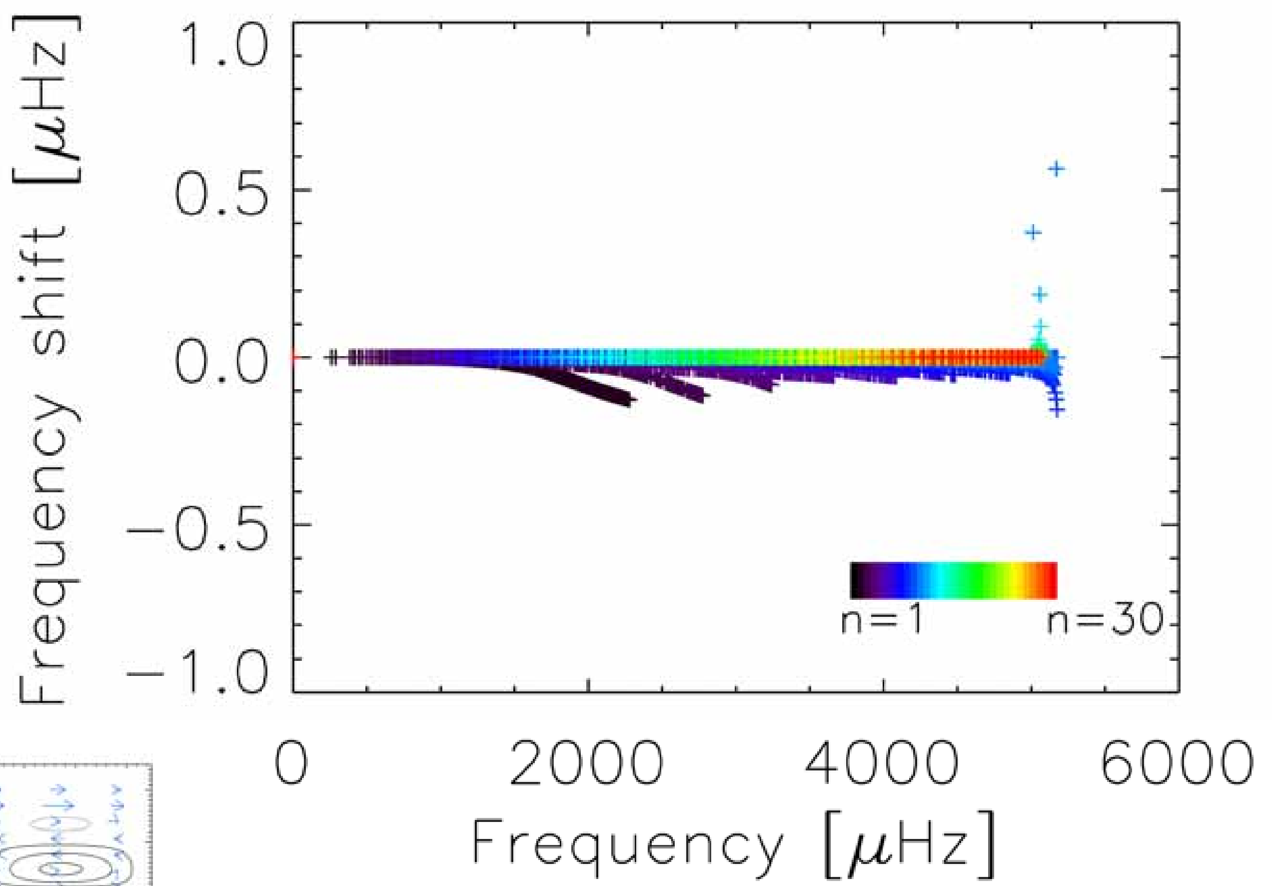
2000
4000
6000
Frequency [μHz]


1.00
0.90
0.80
0.70
0.60
r/R
50 ms⁻¹
0.0 0.5 1.0 1.5 2.0 2.5 3.0
Colatitude [rad]

$s=2, n_c=3$


Frequency shift [μHz]
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n=1
n=30
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4000
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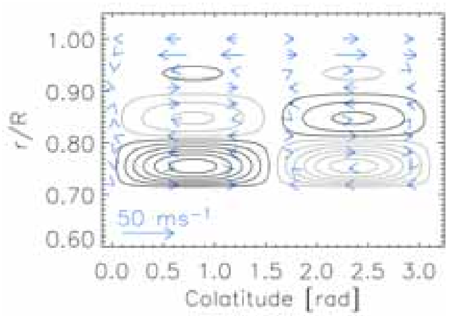
Frequency [μHz]


r/R
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0.90
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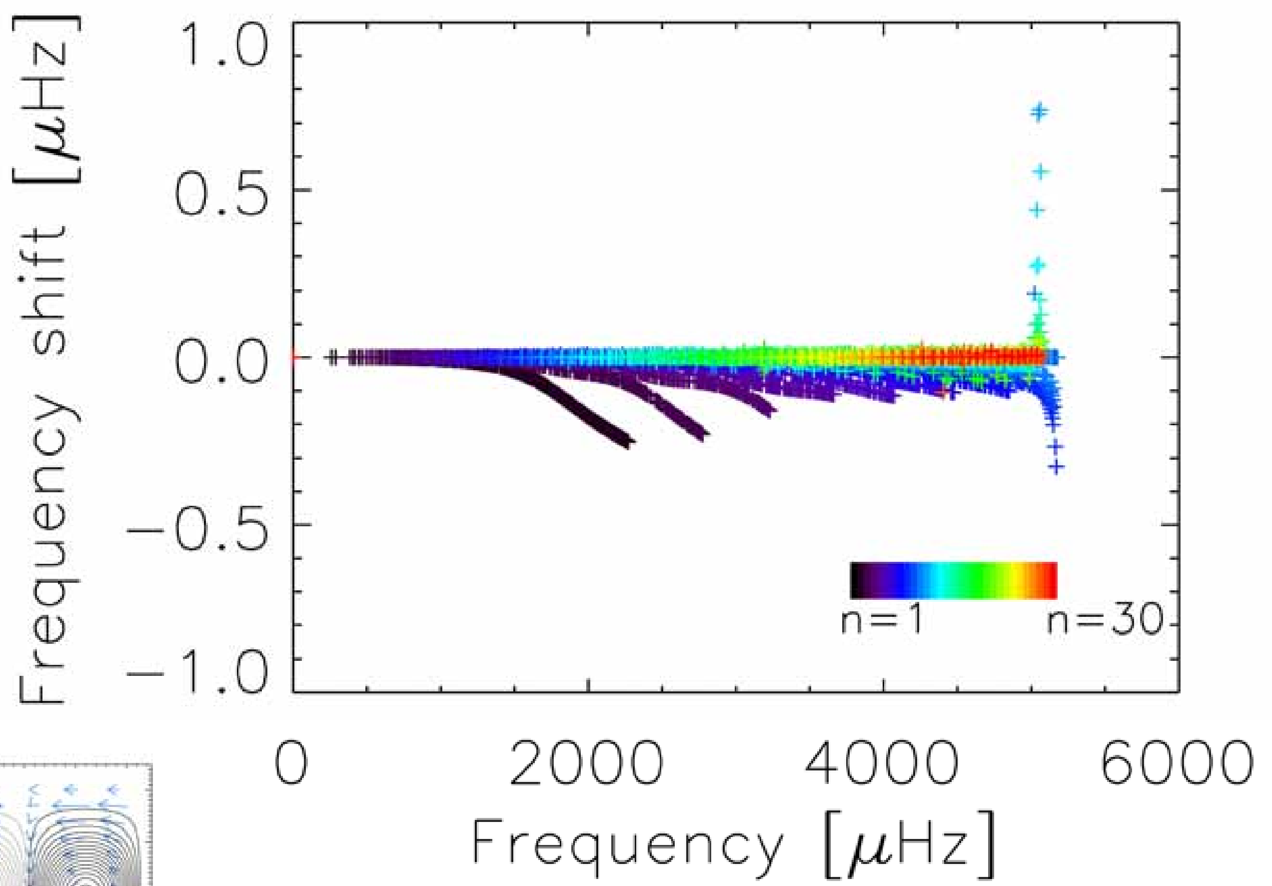
0.70
0.60
50 ms⁻¹
0.0 0.5 1.0 1.5 2.0 2.5 3.0
Colatitude [rad]

s=3, n_c=1


Frequency shift [μHz]
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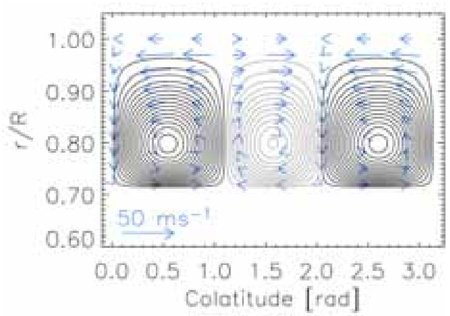
Frequency [μHz]
n=1
n=30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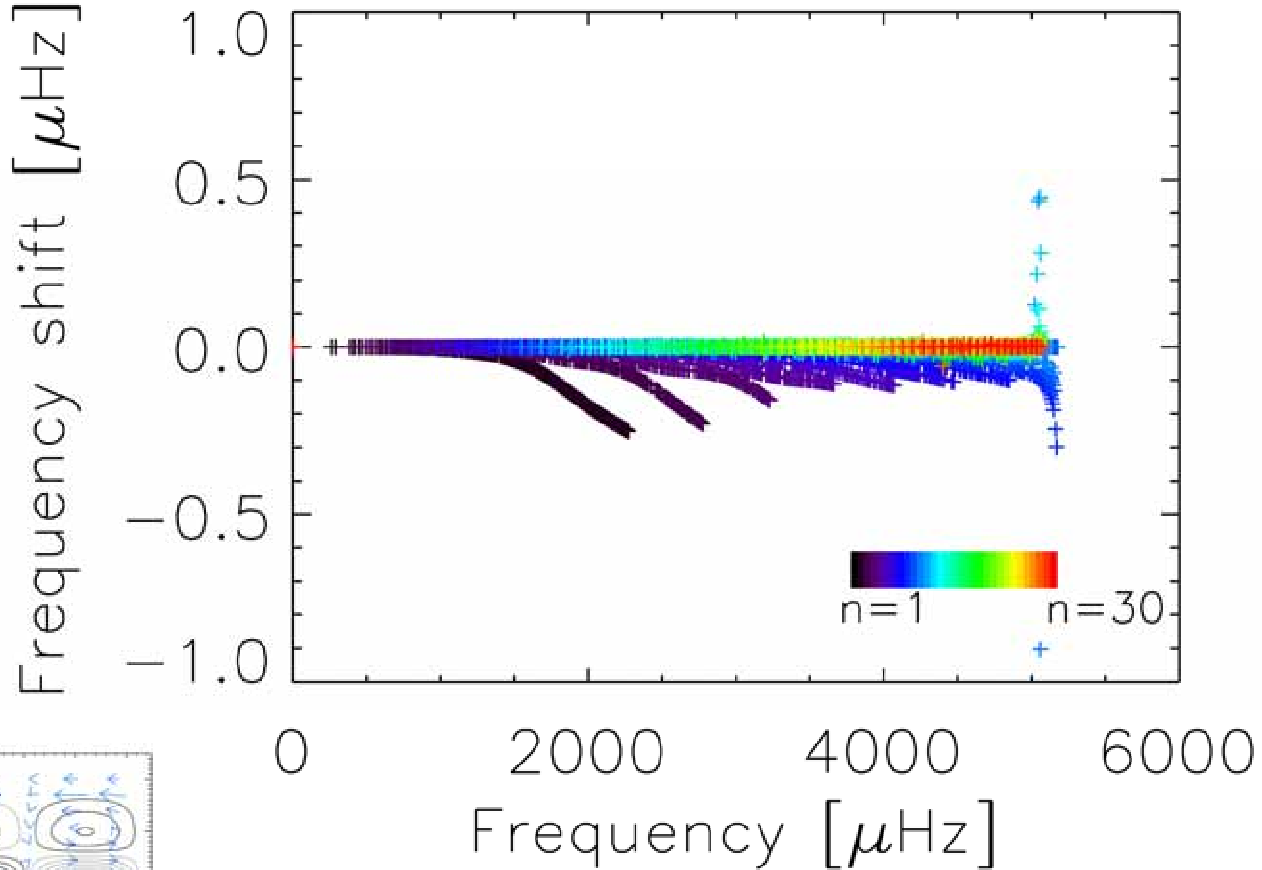
0.70
0.60
r/R
50 ms⁻¹
0.0 0.5 1.0 1.5 2.0 2.5 3.0
Colatitude [rad]

s=3, n_c=2


Frequency shift [μHz]
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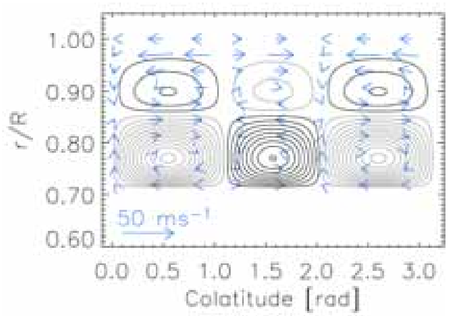
Frequency [μHz]
n=1
n=30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50 ms⁻¹
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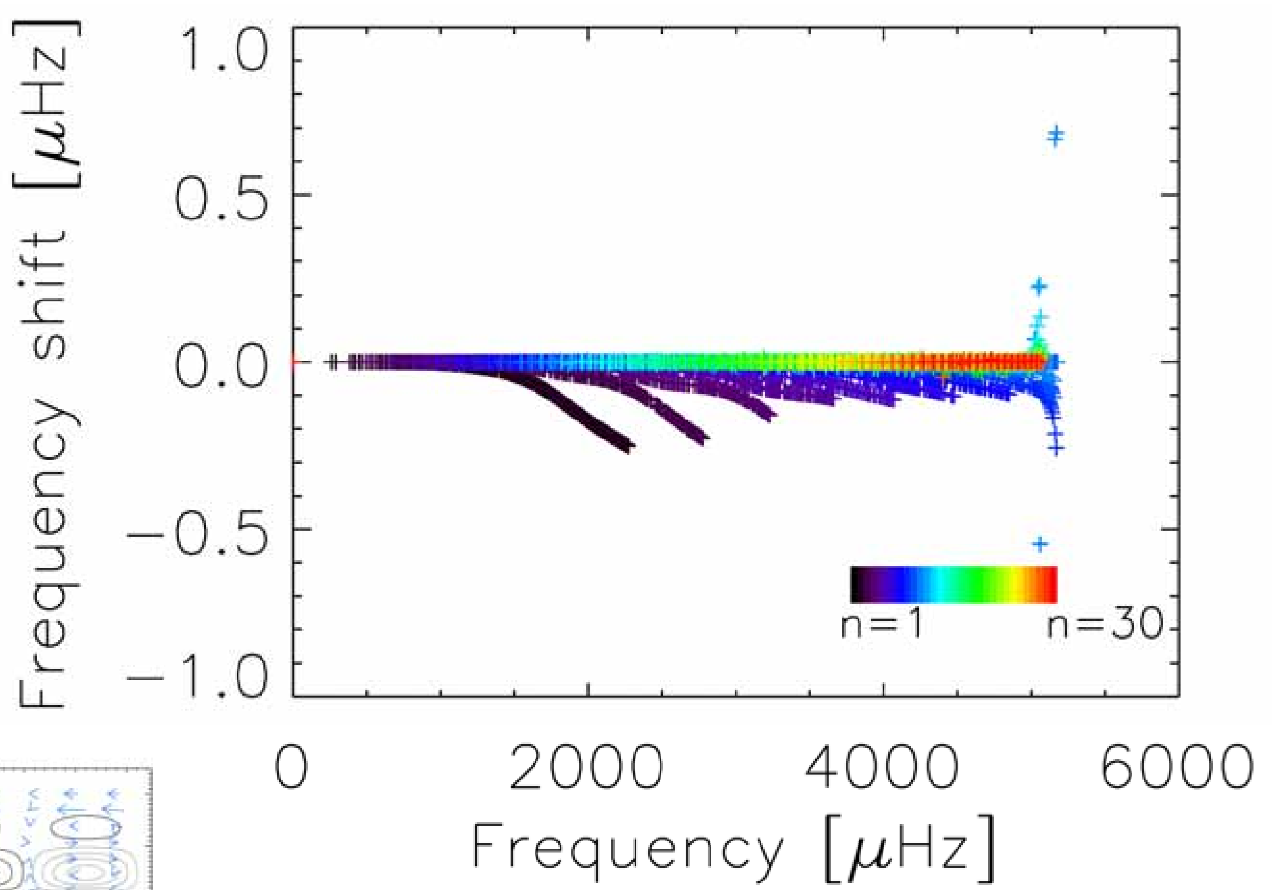
Colatitude [rad]

s=3, n_c=3


Frequency shift [μHz]
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n=1
n=30
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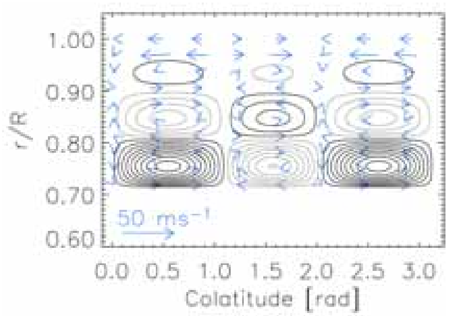
Frequency [μHz]


r/R
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50 ms⁻¹
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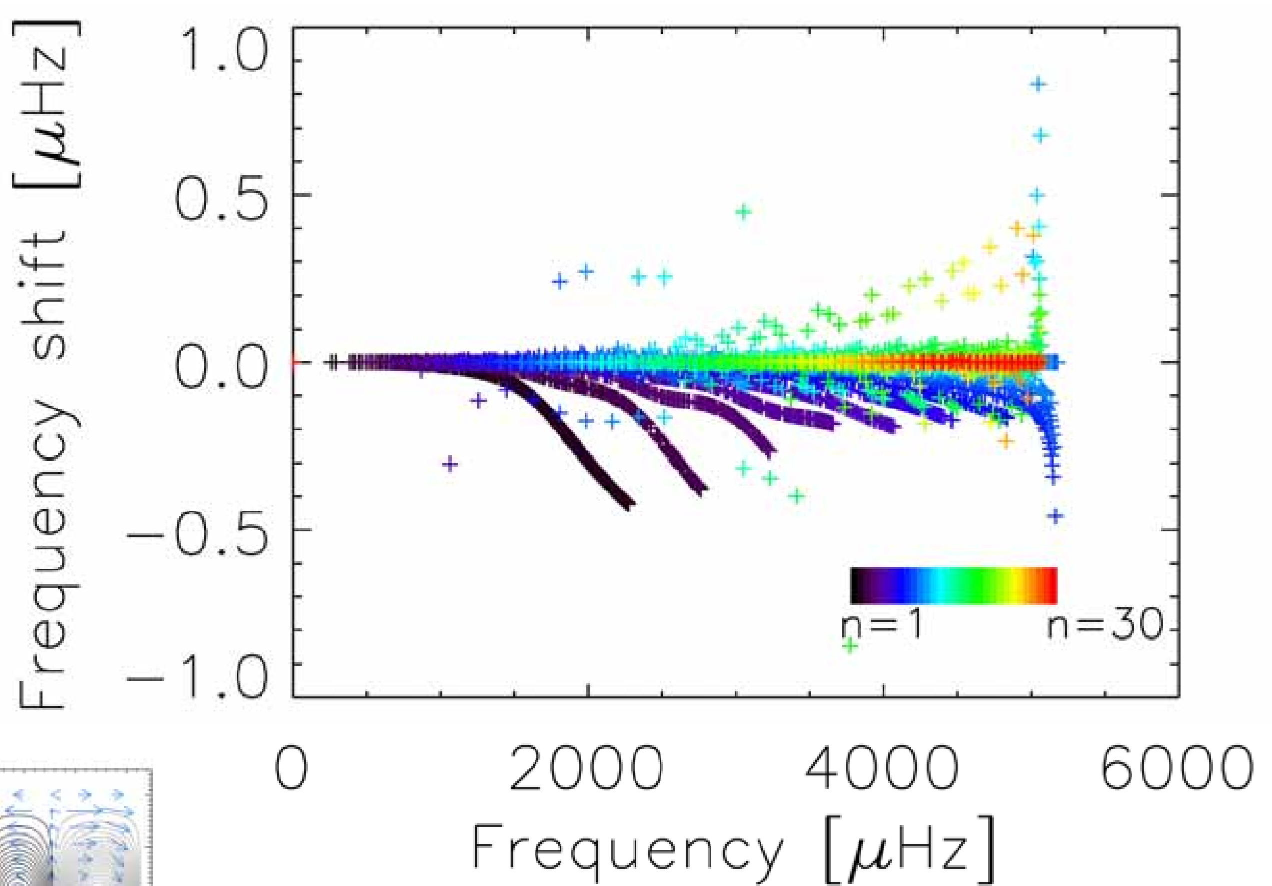
Colatitude [rad]

s=4, n_c=1


Frequency shift [μHz]
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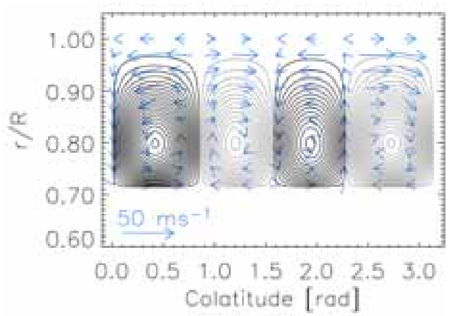
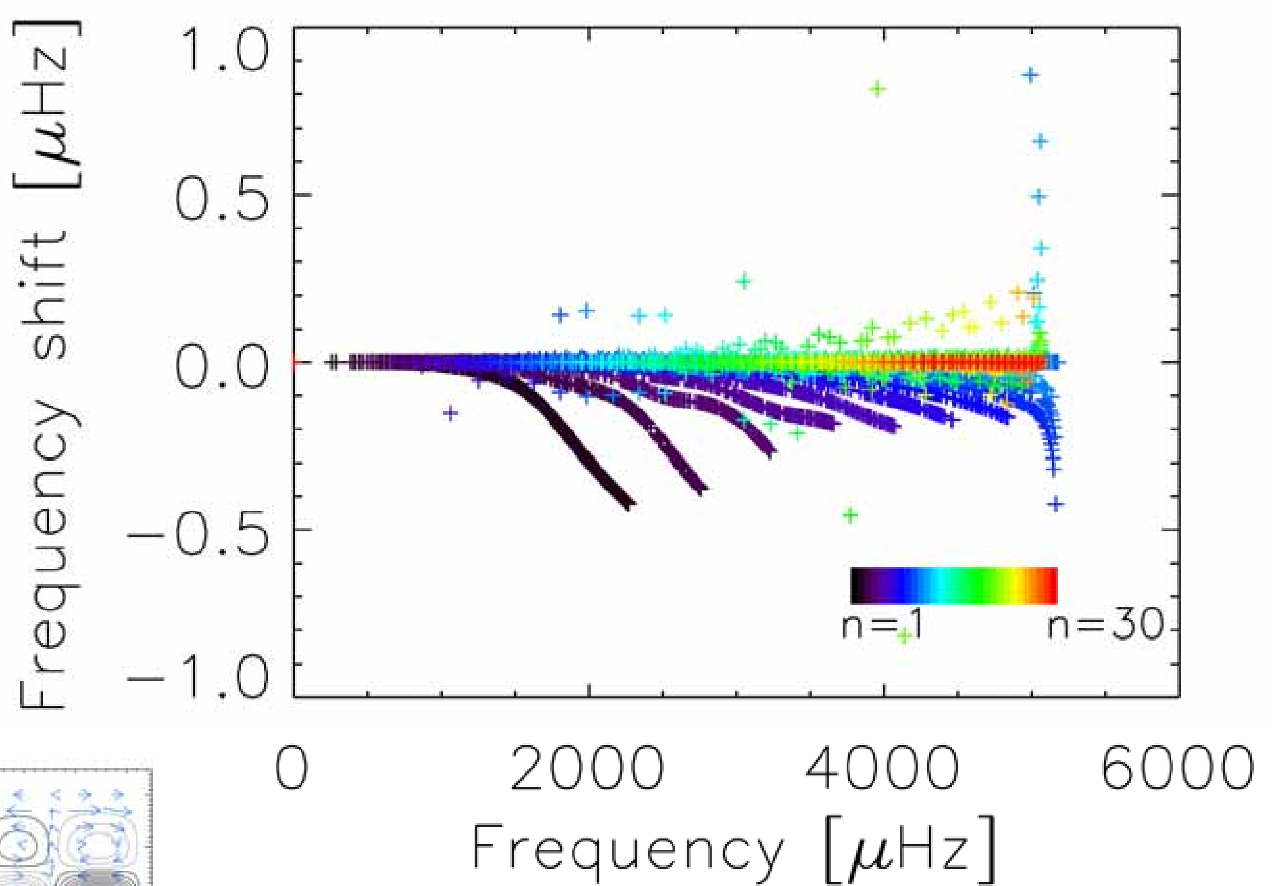
Frequency [μHz]
n=1
n=30


r/R
1.00
0.90
0.80
0.70
0.60
50 ms⁻¹
0.0 0.5 1.0 1.5 2.0 2.5 3.0
Colatitude [rad]

s=4, n_c=2


Frequency shift [μHz]
1.0
0.5
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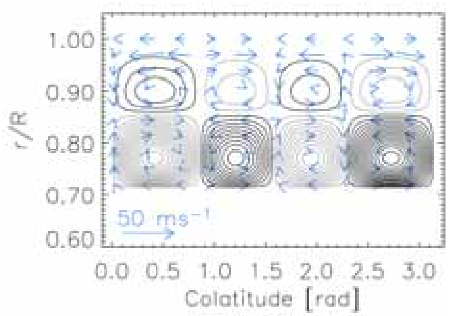
−1.0
n=1
n=30
0
2000
4000
6000
Frequency [μHz]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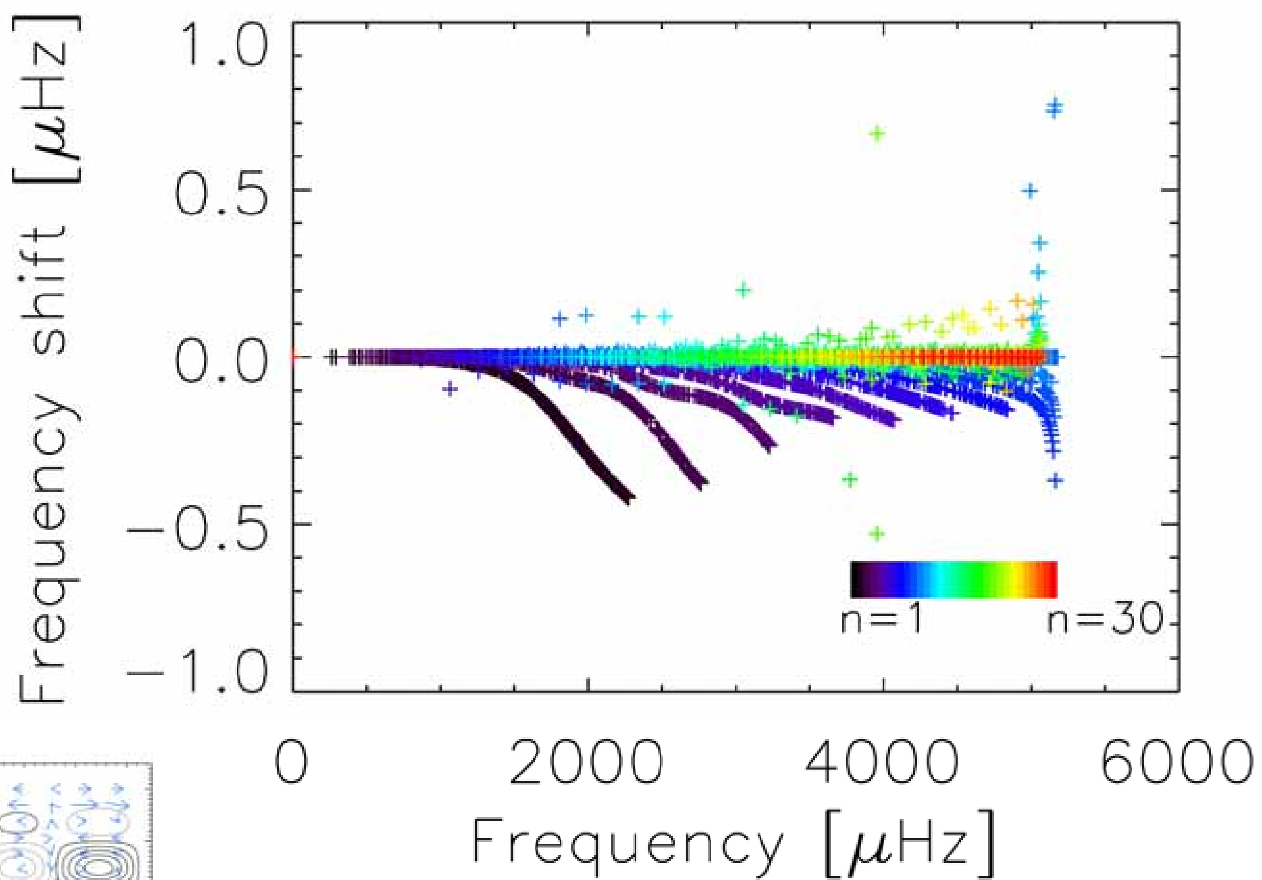
50 ms⁻¹
0.0 0.5 1.0 1.5 2.0 2.5 3.0
Colatitude [rad]

$s=4,\ n_c=3$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Frequency shift [μHz]
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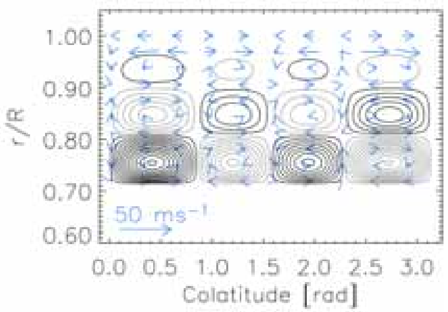
Frequency [μHz]
n=1
n=30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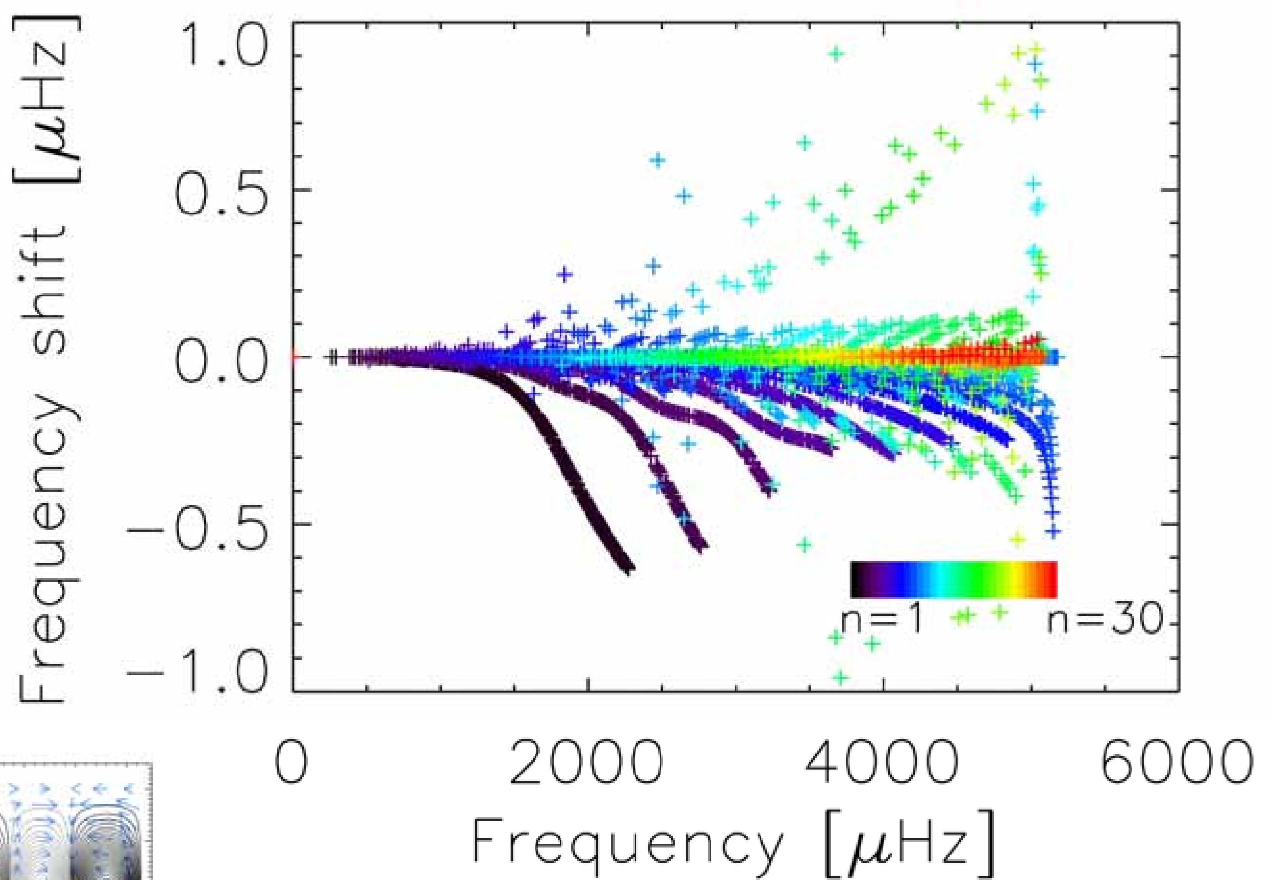
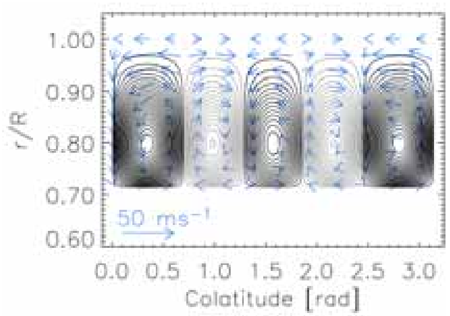
50 ms⁻¹
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Colatitude [rad]

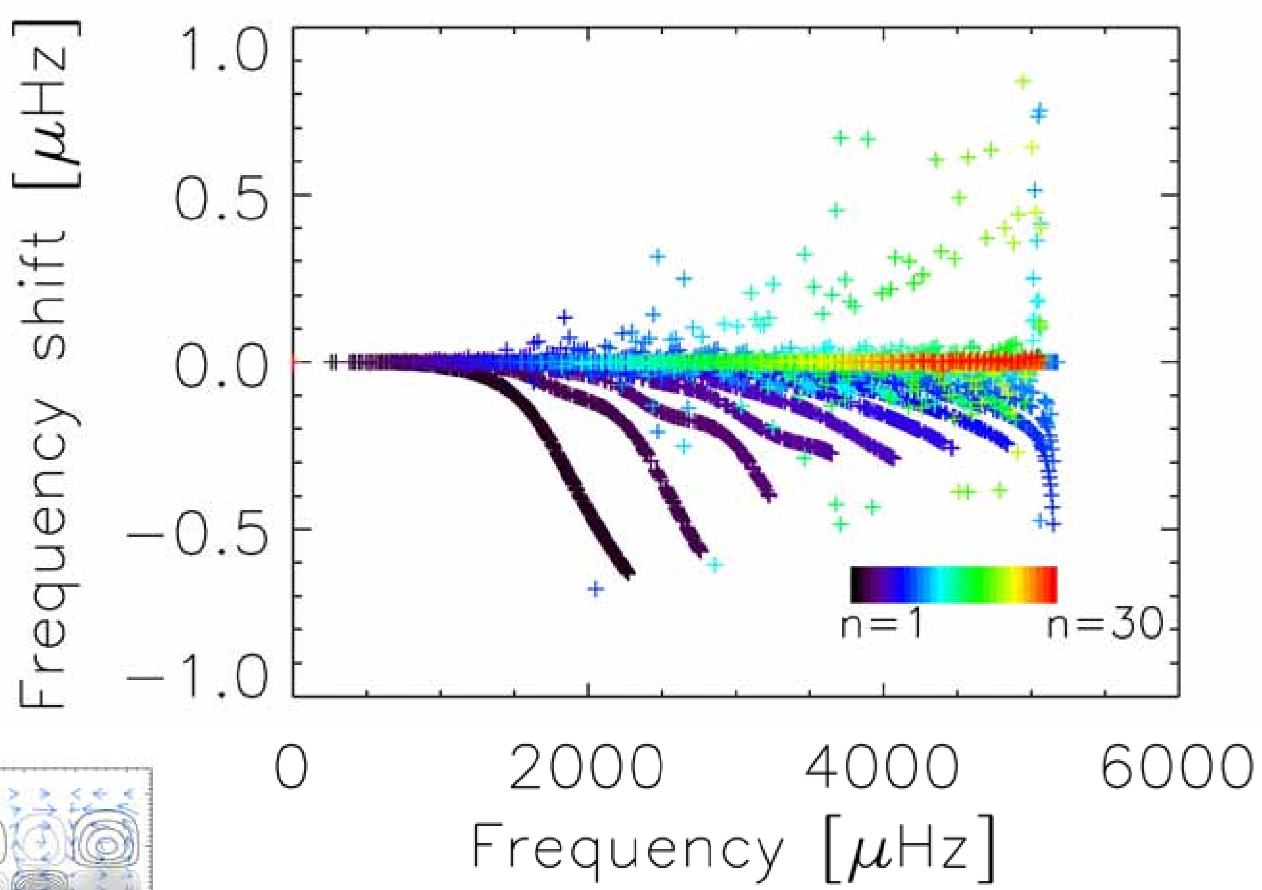
Meridional Circulation and
Global Solar Oscillations

s=5, n_c=1





s=5, n_c=2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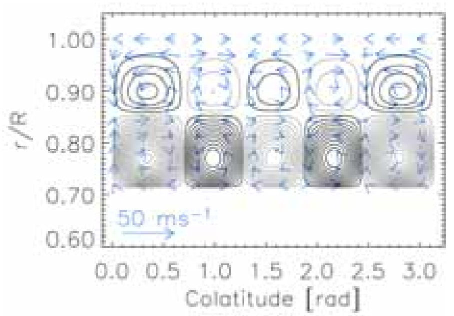
Frequency [μHz]
n=1
n=30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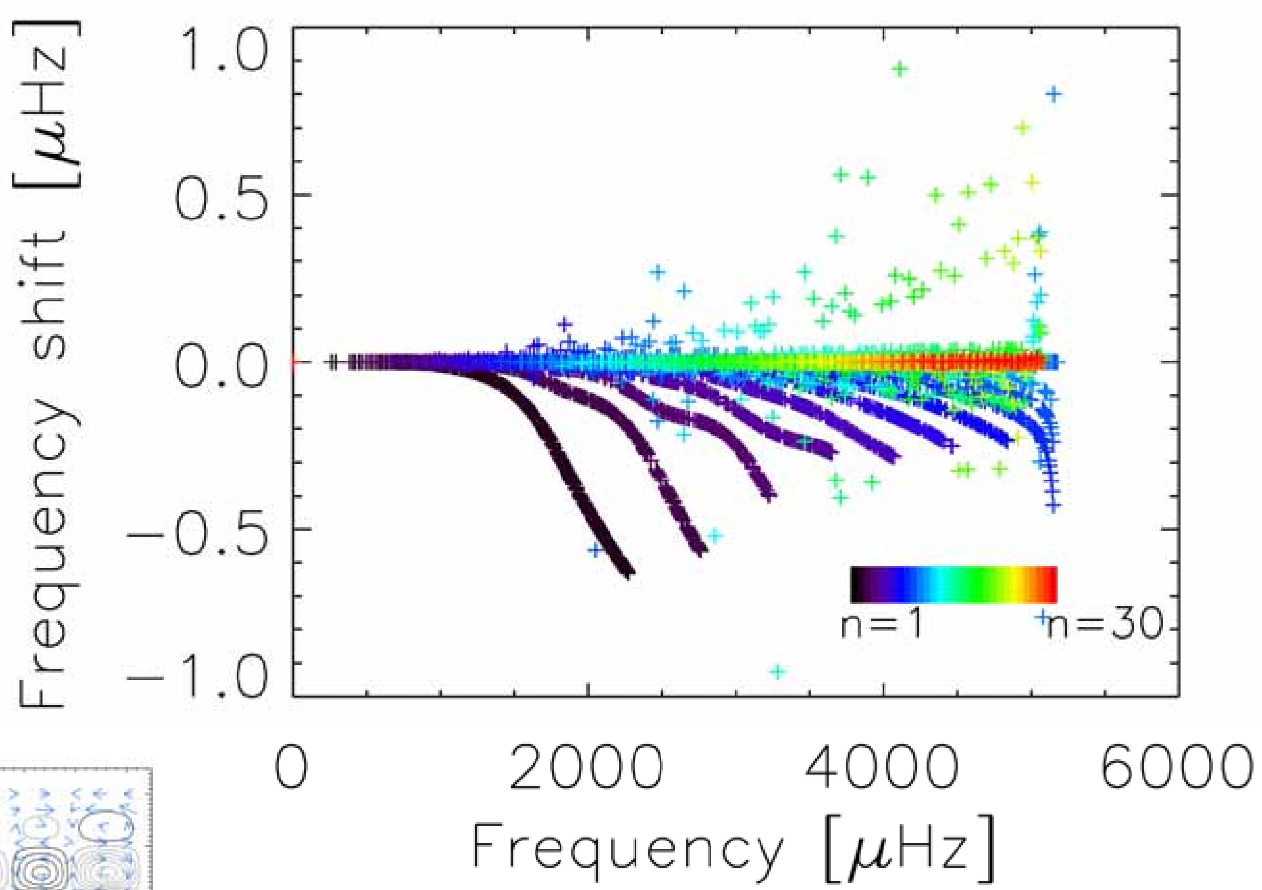
r/R
50 ms⁻¹
0.0 0.5 1.0 1.5 2.0 2.5 3.0
Colatitude [rad]

s=5, n_c=3


Frequency shift [μHz]
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n=1
n=30
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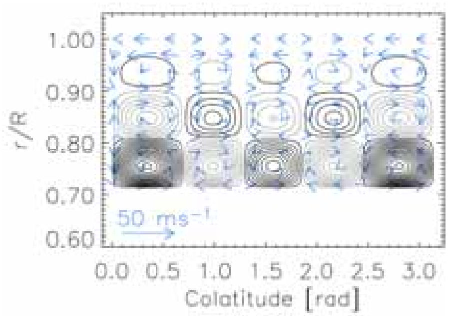
Frequency [μHz]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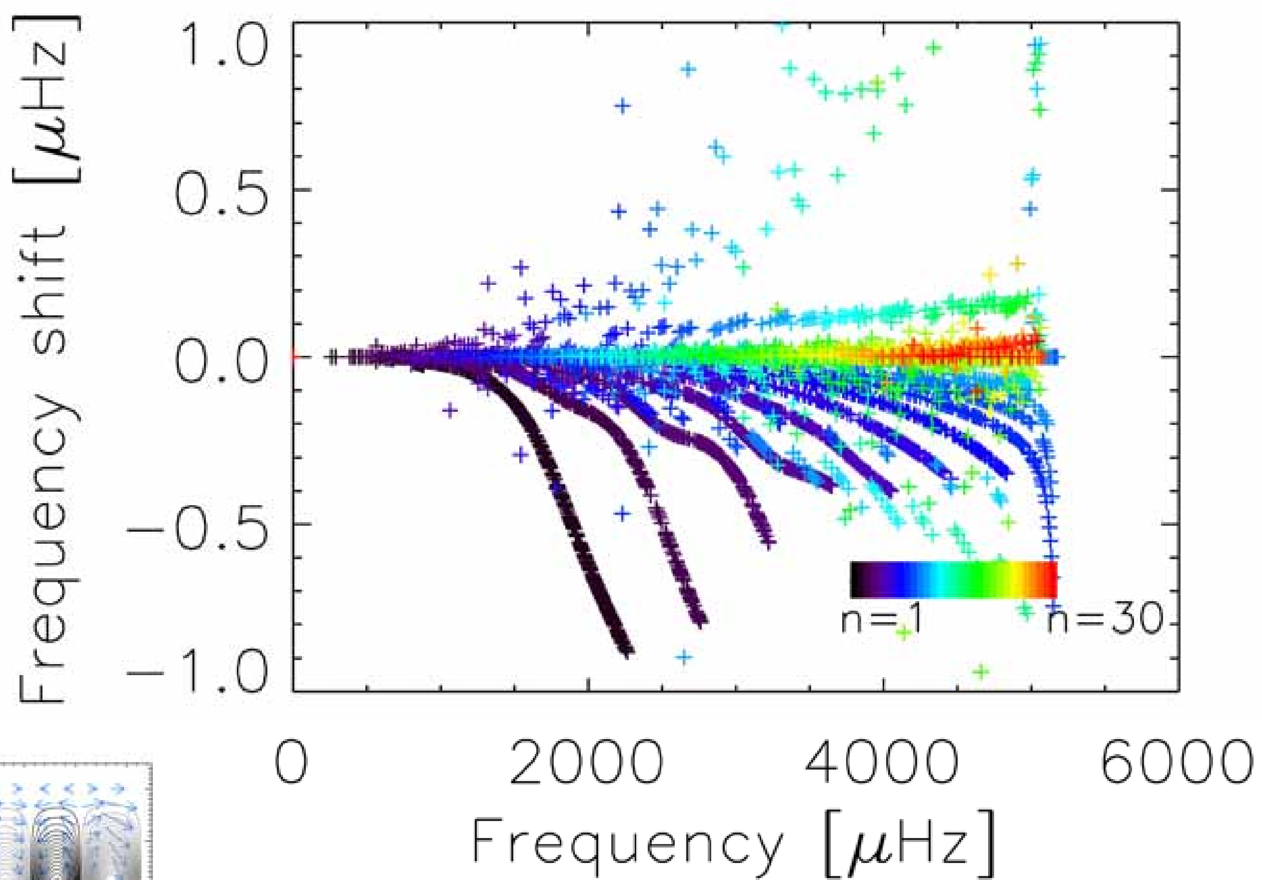
r/R
50 ms⁻¹
0.0 0.5 1.0 1.5 2.0 2.5 3.0
Colatitude [rad]

s=6, n_c=1


Frequency shift [μHz]
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n=30
0
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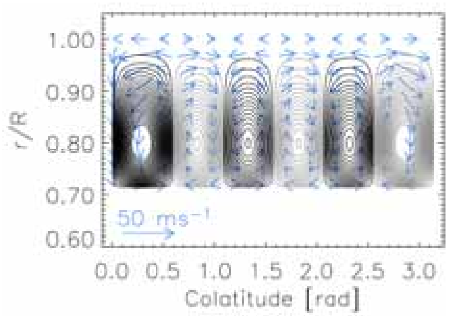
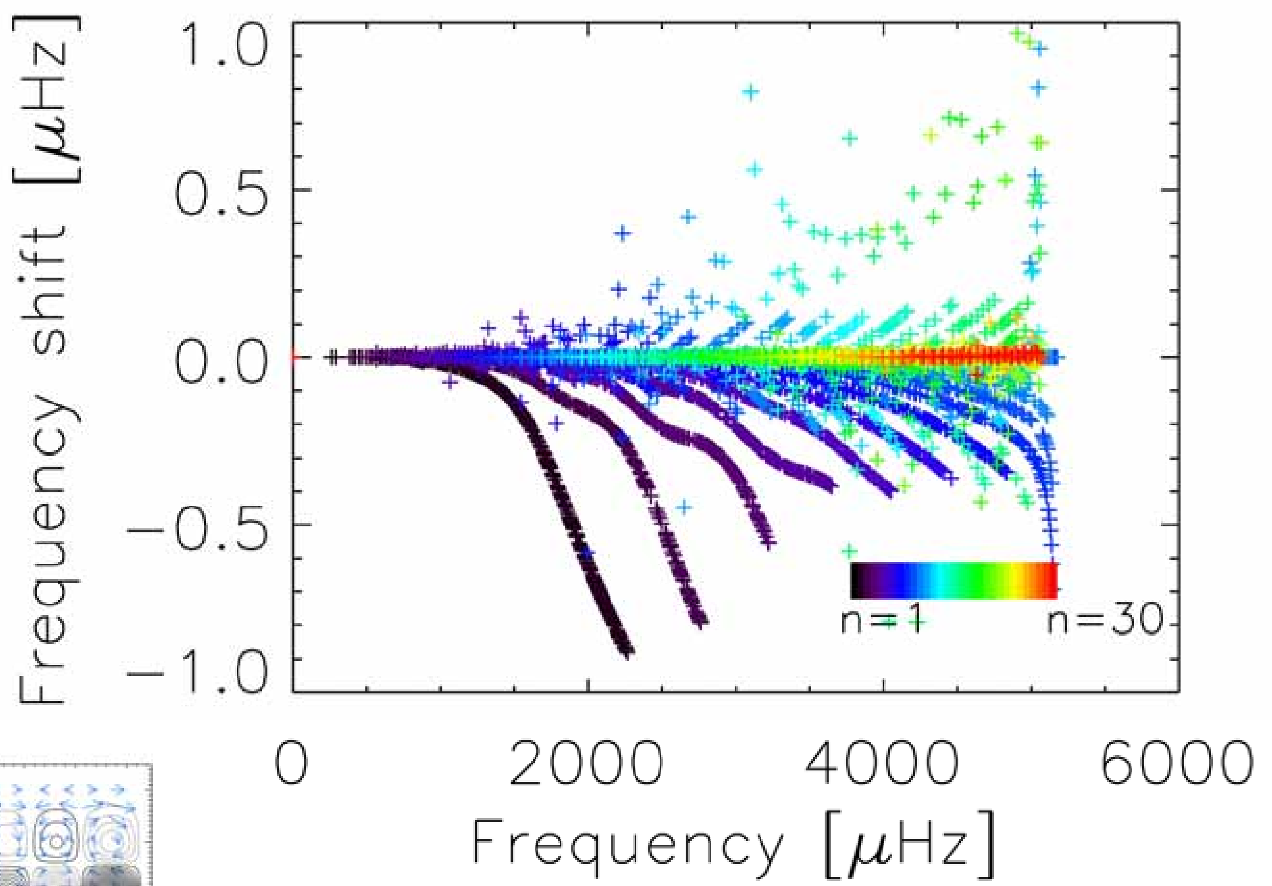
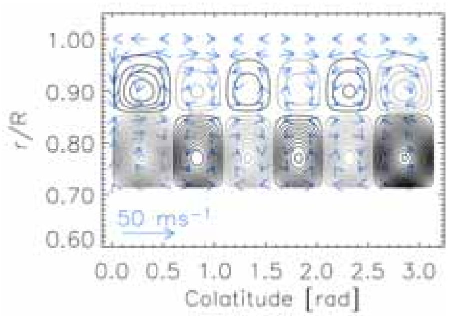
6000
Frequency [μHz]

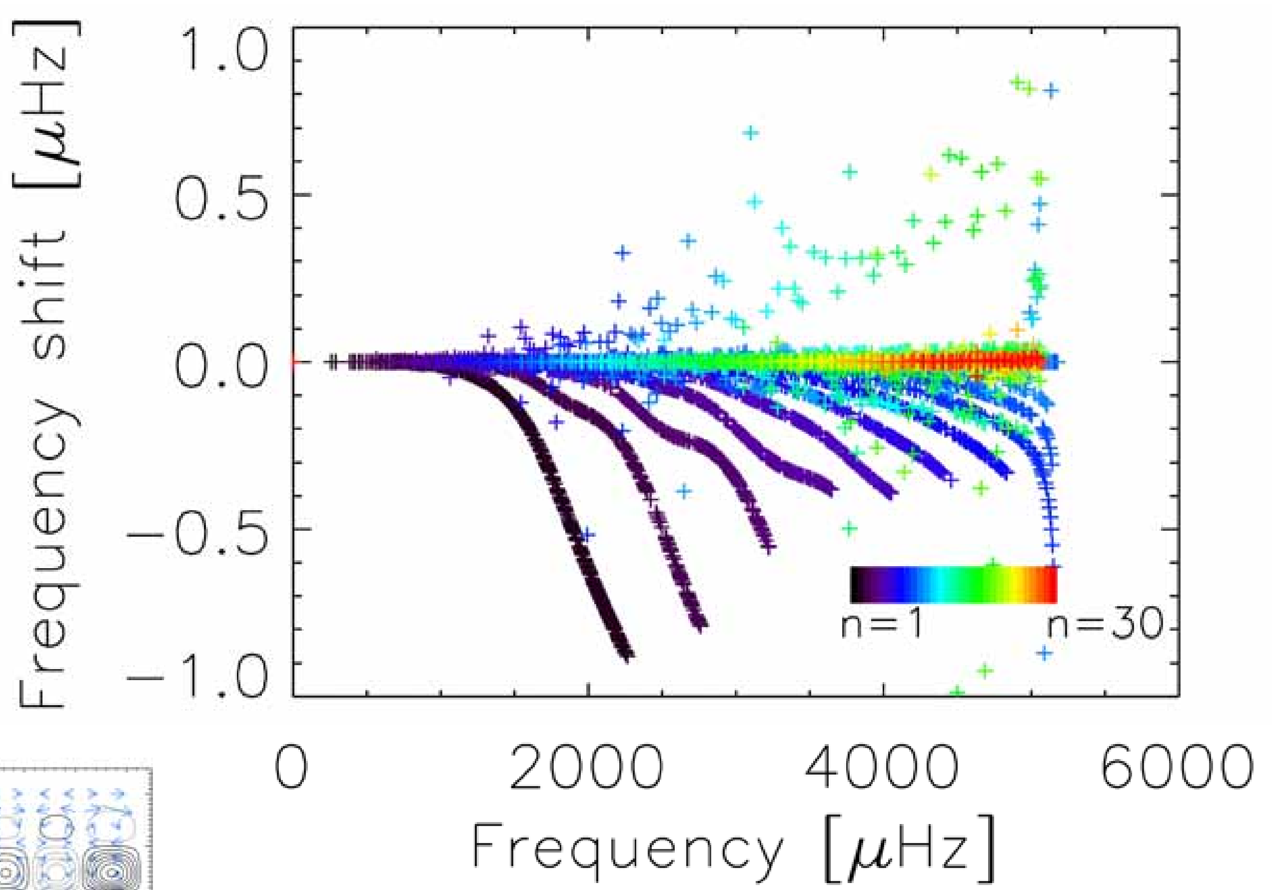

r/R
1.00
0.90
0.80
0.70
0.60
50 ms⁻¹
0.0 0.5 1.0 1.5 2.0 2.5 3.0
Colatitude [rad]

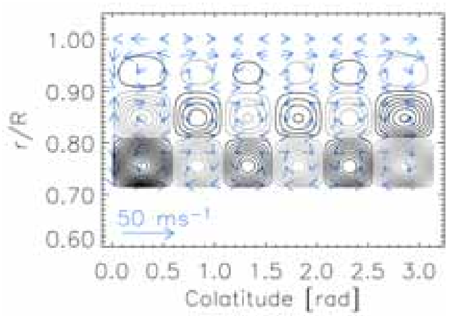
s=6, n_c=2


Frequency shift [μHz]
1.0
0.5
0.0
−0.5
−1.0
n=1
n=30
0
2000
4000
6000
Frequency [μHz]


1.00
0.90
0.80
0.70
0.60
r/R
50 ms⁻¹
0.0 0.5 1.0 1.5 2.0 2.5 3.0
Colatitude [rad]

s=6, n_c=3


Frequency shift [μHz]
1.0
0.5
0.0
−0.5
−1.0
0
2000
4000
6000
Frequency [μHz]
n=1
n=30


r/R
1.00
0.90
0.80
0.70
0.60
50 ms⁻¹
0.0 0.5 1.0 1.5 2.0 2.5 3.0
Colatitude [rad]

Frequency Shifts due to Meridional Circulation

- m-averaged frequency shifts are *mostly negative*, only a few positive shifts

  m-averaged shift magnitude O(1µHz)

- As more cells in latitude as stronger the effect:

  difference between models O(0.1µHz)

- Changing number of cells with depth makes a weaker effect:

  difference between models O(0.03µHz)

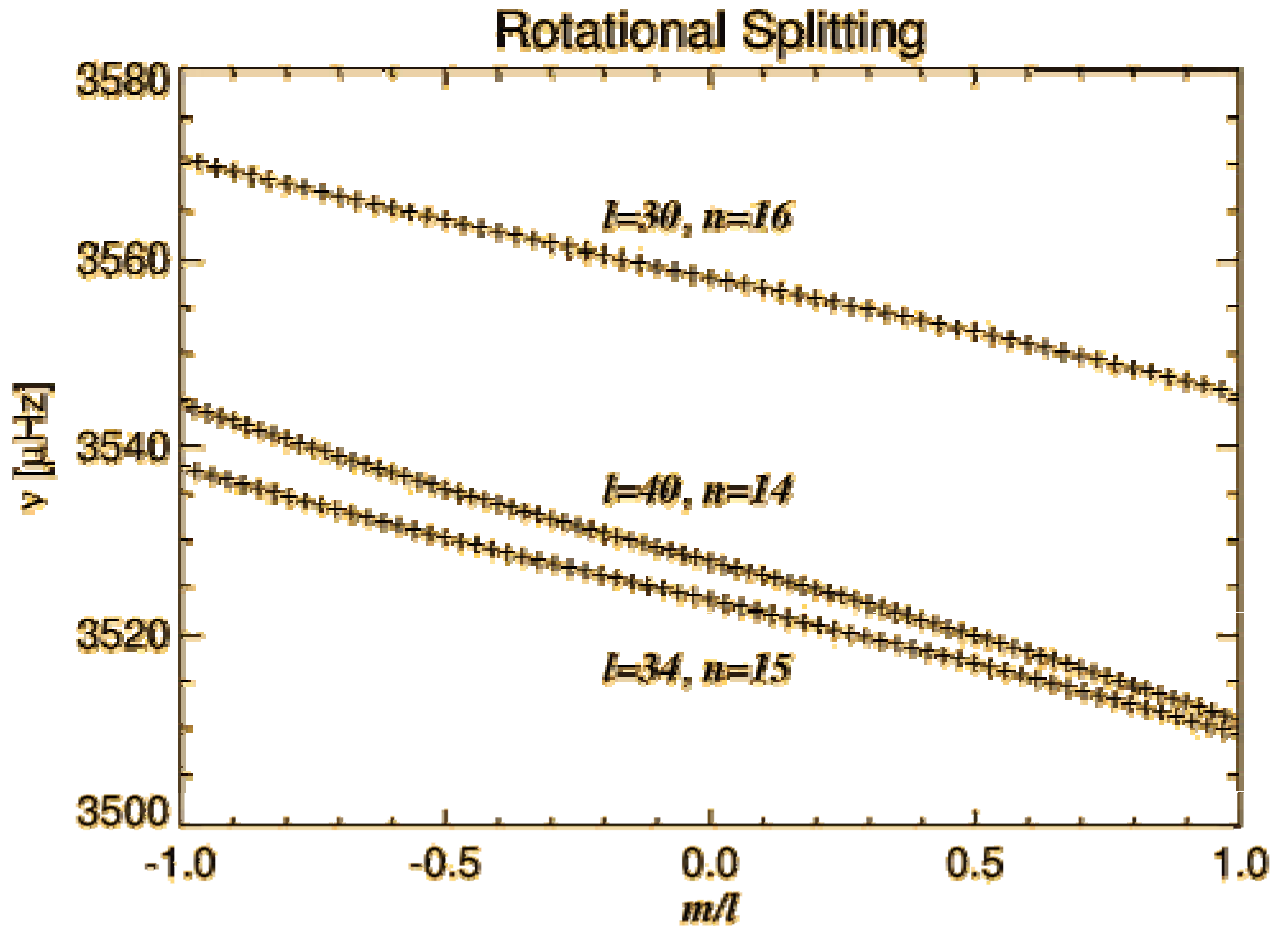
Frequency Shifts due to Meridional Circulation

- Strongest effect for low n, high l modes
- Strongest single mode shift can be up to 70 μHz (for the high frequency modes)
- Splitting of multiplets even with respect to m=0
- Frequency shifts due to meridional circulation is an observable effect in long time series (T_{obs}~ 1 year).

Differential Rotation

Lifting of degeneracies by differential rotation is odd to *m*=0

$ #Multiplets much stronger affected



Conclusions

- Various models of meridional circulation were used to compare effect on global solar oscillation frequencies
  - large-scale flows lead to a reduction of the mode frequencies
  - with respect to m=0 there is an even lifting of degeneracies
- Effect is in the observable range, however long time series are needed to obtain frequency resolution
  - latitudinal structure possibly easier to assess than depth dependence
- Usage of 3D simulations of large-scale flows is under preparation
- Other effects (asphericities, B) need to be modelled, too.
- **In future:**
  use mode frequencies to constrain the models

  →Global seismology of the meridional circulation?
    - **long time averages**
    - **ϕ-independent**

(Roth & Stix, Sol. Phys. 2008)

Negative Frequency Shifts

Shifted frequencies are given by

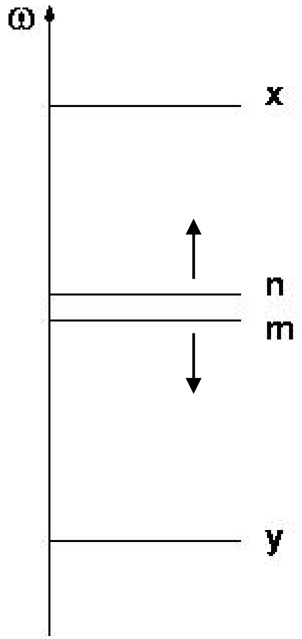
$$\begin{aligned} \tilde{\omega}^2_{nlm} &= \omega_{nlm} + \frac{|H_{n'n,l'l}(m)|^2}{\omega^2_{nl} - \omega^2_{n'l'}} \\ \tilde{\omega}^2_{n'l'm} &= \omega_{n'l'm} - \frac{|H_{n'n,l'l}(m)|^2}{\omega^2_{nl} - \omega^2_{n'l'}} \ , \end{aligned}$$

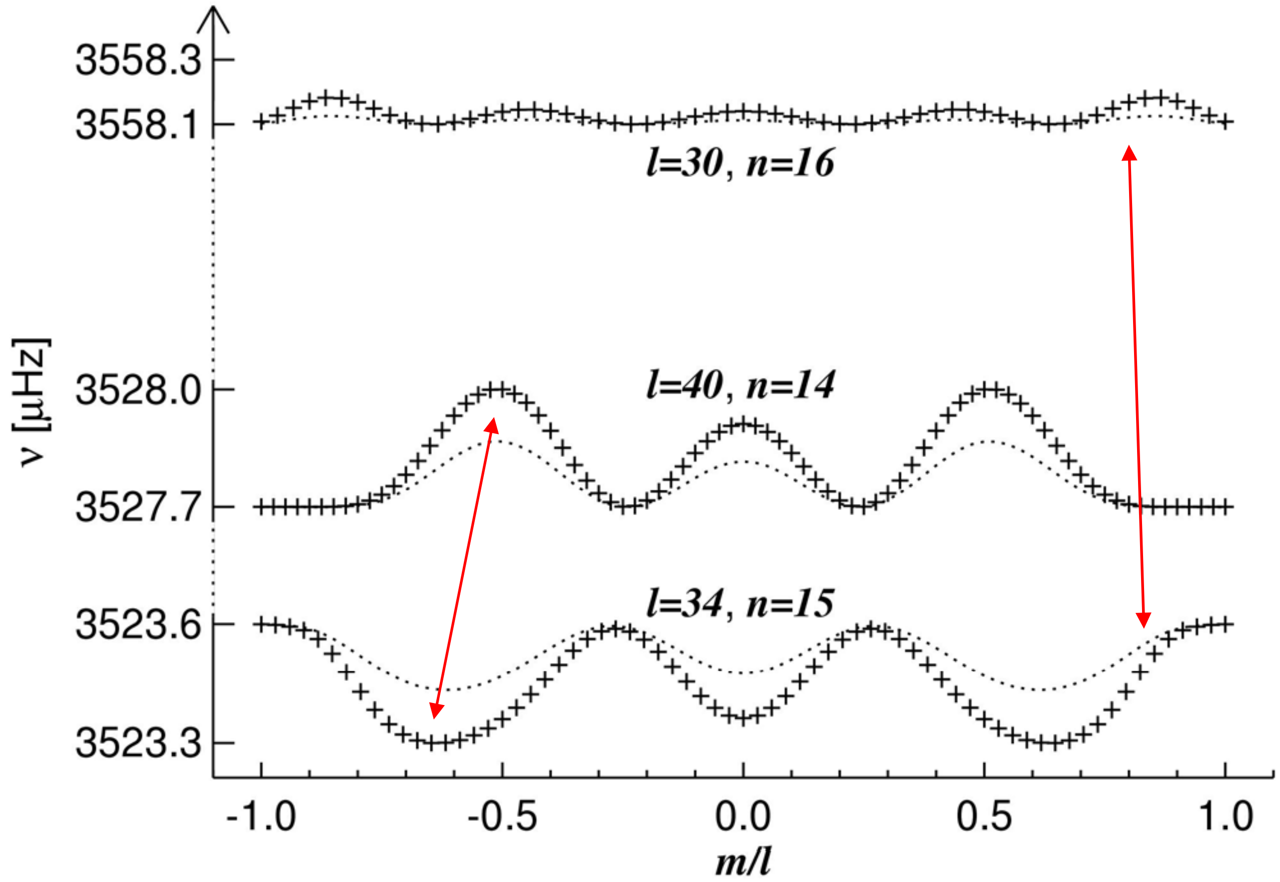
The absolute magnitudes of the frequency shifts are equal for both couplers, but the sign is negative for the mode with the lower frequency

$$\delta\omega(m) = \frac{|H_{n'n,l'l}(m)|^2}{2\omega(\omega^2_{nl} - \omega^2_{n'l'})}$$

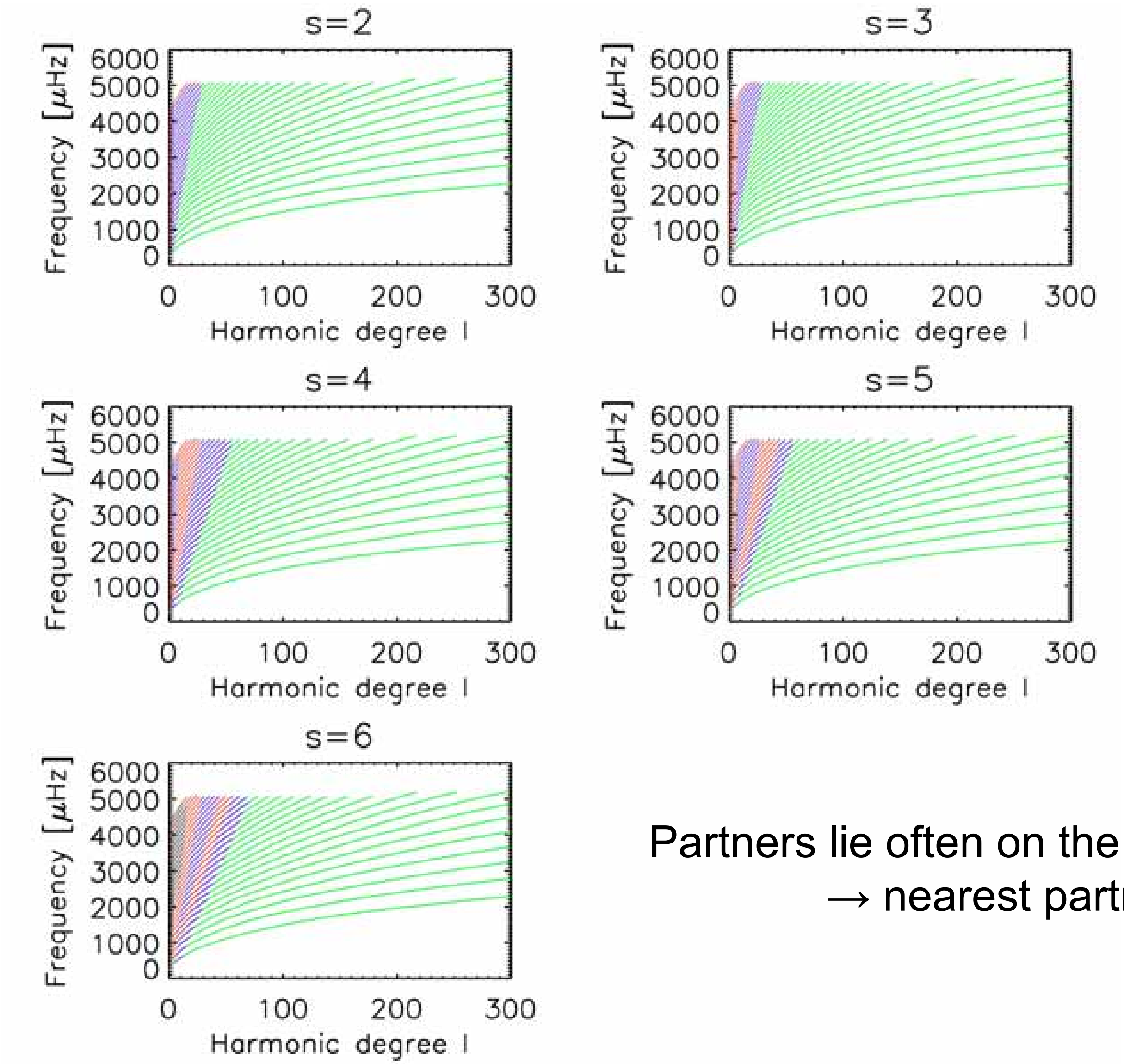
Nearest neighbor has strongest effect.

Negative Frequency Shifts





Position of Partners in the l-ν Diagram



Partners lie often on the same ridge at higher l

→ nearest partner has a higher frequency