Exercises for Partial Differential Equations II

1. Poisson-equation in 2D:
Gauss law (in dimensionless form) is given by \( \nabla \cdot \mathbf{E} = \rho \). With \( \mathbf{E} = -\nabla \phi \)
we derive a Poisson equation. Here we consider the problem in 2D:
\[
(\mathbf{E} = E_x(x, y)\mathbf{e}_x + E_y(x, y)\mathbf{e}_y)
\]
which leads to:
\[
-\frac{\partial^2 \phi(x, y)}{\partial x^2} - \frac{\partial^2 \phi(x, y)}{\partial y^2} = \rho(x, y)
\]

Solve this equation numerically with Dirichlet boundary conditions \( \phi = 0 \) on all boundaries. Compute the potential \( \phi(x, y) \) and the electric field \( \mathbf{E}(x, y) \).

(a) Jacobi method.
(b) Gauss-Seidel method.
(c) Successive Overrelaxation (SOR-method).
(d) Try to find the optimum relaxation factor \( w \) for the SOR-method.
(e) Investigate how the methods scale with the grid resolution. Say \( h = 0.4, 0.2, 0.1 \).

For an implementation in IDL you can start with the program
O : \wiegelmann\PDE_lecture\lecture_poisson2d_draft.pro
The program computes a distribution of electric charges \( \rho(x, y) \) on a
grid \( L_x = 12, L_y = 10 \) with a grid resolution \( \Delta x = \Delta y = h = 0.4 \)