Solar convection and magnetic field

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Convection & magnetism: closely related
Outline

1) Basic physics of convection
2) Numerical simulation of convection
3) Overview of solar magnetism
4) Surface magneto-convection
5) Deep convection zone field & dynamo
The solar convection zone

- 200 Mm thick layer in turbulent motion
- Velocities range from 100 m/s (bottom) to 10 km/s (top)
- Energy flux nearly completely transported by convective motion
What is convection?

- Flow driven by thermal buoyancy
- Convective instability

➡ Viewgraphs...
**LABORATORY CONVECTION**  
(RAYLEIGH-BÉNARD CONV.)

\[ T_1 = T_2 \]

\[ \Delta T = T_2 - T_1 > 0 \]

"Fluid in a Box"

Conducted flux:  
\[ F = \alpha k \frac{dT}{dz} = \frac{k \Delta T}{d} \]

Hydrostatic equil. fluid static for \( \Delta T < \Delta T_c \)

Instability for \( \Delta T \geq \Delta T_c \)

Rayleigh number:  
\[ R = \frac{\alpha g \Delta T d^3}{\nu k} \geq R_c \approx 1700. \]

\( \alpha \): expansion coefficient  
\( [\text{Conv. Buoyancy-driven}] \)

\( \nu \): (kinematic) viscosity

\[ \rightarrow \Delta \text{ rolls:} \]

\( R \uparrow \): bifurcation \( \rightarrow \) chaos

\[ \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \kappa \nabla^2 T \]

Stars: \( \kappa = \kappa_{\text{rad}} \)

for \( R \geq R_c \) is convection "more efficient".

Nusselt No.  
\[ Nu = \frac{F_{\text{conv}}}{F_{\text{diff}(D=0)}} > 1 \]
3. CONVECTIVE INSTABILITY

HERE: "BLOB THEORY" → Fluctuations of an average state

\[ \tau + 5 \tau \]

\( \xi, \chi \)

\[ T \]

\( \xi, \chi \)

\[ \tau \]

\( \xi, \chi \)

\[ d \rightarrow \]

\[ d \ll H_p, \ \delta \xi/\xi \ll 1 \]

**Blob Timescale:** \( \tau_i = d/v \)

**Thermal:** \( \tau_{th} = d^2/\eta_k \)

**Dynamic:** \( \tau_{dy} = d/c_s \)

**Instability:** \( \delta_s^* < \delta^* \)

\[ \xi_i - \xi^* = \left[ \left( \frac{d \xi}{d \tau_{ad}} \right) - \frac{d \xi}{d \tau} \right] \cdot \delta \tau < 0 \]

**Eq. of State**

\( p = \rho R T / \mu \) & \( \xi_i^* = \xi_i \):

\[ \frac{dT}{d\tau} < \left( \frac{dT}{d\tau_{ad}} \right) + \frac{T}{\mu} \left[ \frac{d\mu}{d\tau} - \left( \frac{d\mu}{d\tau_{ad}} \right) \right] \]

\[ \frac{d\mu}{d\tau} + 0 \leftarrow \text{Composition Changes (Stellar Interiors)} \]

\[ \text{Ionization (Stellar Envelopes)} \]

→ CONSIDER SEPARATELY
**Schwarzschild Criterion**

\[
\frac{dT}{dr} < \left( \frac{dT}{dr} \right)_{ad}
\]

\[
\frac{1}{H_p} = - \frac{1}{P} \frac{dp}{dr}
\]

Rewrite:

\[
\frac{dT}{dr} = \frac{T}{P} \left( \frac{dl_{EU}}{dr} \right) \frac{dp}{dr} = - \frac{T}{H_p} \Delta
\]

\[ \Delta > \Delta_{ad} \]

- Instability for ideal gas, no ionization:

\[ \Delta_{ad} = \frac{y-1}{y} = 0.4 \quad (y=\frac{5}{3}) \]

**Relation to Entropy Gradient**

\[ TdS = pdV + dE \]

Assume ideal gas:

\[ dE = cvdT, \quad p = R \cdot \frac{T}{\gamma} \]

Use \((dS)_{ad} = 0\)

\[ R = \gamma - cv \]

⇒ ... Exercise ... 

\[
\frac{dS}{dr} = \frac{C_p}{T} \left[ \frac{dT}{dr} - \left( \frac{dT}{dr} \right)_{ad} \right] < 0 \text{ for inst.}
\]

⇒ Convective Instability

\[ \leftrightarrow \text{ Entropy decreases outward} \]

[ Entropy sink: stellar surface ]
- Composition Change, No Ionization

\[ \frac{dT}{dr} \mid_{\text{ad}} = 0 \quad \text{LEDoux Criterion} \]

He-enriched core \( \Rightarrow \frac{du}{dr} < 0 \Rightarrow \) stabilizing \( \mu \)-gradient

- Ionization, No Composition Changes

\[ \mu(T, p) \text{ given function (ext. & int.)} \]

\[ \left[ \frac{dT}{dr} - \left( \frac{dT}{dr} \right)_{\text{ad}} \right] \cdot \left[ 1 - \left( \frac{d \ln \mu}{d \ln T} \right)_p \right] < 0 \]

\( x_p > 0 \) always

- Schwarzchild Criterion \( \left[ \frac{dS}{dr} < 0 \right] \)

- Semi-convection [Early-type stars]

\[ \left( \frac{dT}{dr} \right)_{\text{ad}} + \frac{I}{\mu} \left( \frac{du}{dr} \right) < \frac{dT}{dr} < \left( \frac{dT}{dr} \right)_{\text{ad}} \]

Dynamically stable, but free energy available

\( T_i > T_e \Rightarrow \) Radiative Cooling

\( \Rightarrow \) \( \delta_i > \delta_{i,\text{ad}} \)

Overstable oscillations, growing amplitudes

\( T_i < T_e \Rightarrow \) Radiative heating

\( \Rightarrow \delta_i < \delta_{i,\text{ad}} \)

Th. relevant
\[ \frac{dT}{dr} = \frac{T}{P} \frac{dP}{d\ell_{up}} \frac{d\ell_{up}}{dr} = -\frac{T \gamma g}{P} \nabla = -\frac{T}{H_p} \nabla \]

**HYD. EQU.**  \[ H_p = \frac{\alpha T}{\gamma g} \]

**Ledoux:**  \[ \nabla > \nabla_{ad} + \nabla_k \]

**Schwarzschild:**  \[ \nabla > \nabla_{ad} \]

- **Criteria are sufficient and necessary**
  
  *Legolita (1966) Full Linear Stability Analysis*

- **4 Gradients**
  
  **Radiative Energy Transport:**  \[ \nabla = \nabla_{rad} = \frac{3 k_b T^4}{4 \pi c^2} \frac{L(v)}{4 \pi v^2} \]
  
  **Adiabatic Stratification:**  \[ \nabla = \nabla_{ad} = \frac{(y - 1)}{y} \]

  **"Blob Gradient" With Radiative Exchange:**  \[ \nabla_L > \nabla_{ad} \]

  **Relation in a Convective Region:**

  \[ \nabla_{rad} > \nabla_L > \nabla_{dr} > \nabla_{ad} \]

  **Convective Driver**

  **Energy Exchange**

  **MIXES**

  **DAMPING**

- **Formal Stability Analysis:**

  \[ \rightarrow \text{Review by P. Ledoux} \]

  E.A. SIEGBEL & J.P. ZAHN,

  "PROBLEMS OF STELLAR CONVECTION",

  LECTURE NOTES IN PHYSICS 71, p. 87

  SPRINGER (1977)
"Exploding Granules"

& Granule Sizes

\[ P' > 0 \]

\[ \rightarrow \text{ACCELERATION OF HORIZONTAL FLOW} \]
\[ \rightarrow \text{BUOYANCY BRAKING} \]
\[ \rightarrow \text{UPFLOW CHOKED} \]
\[ \rightarrow \text{COOLING BY RADIATION} \]

Subsonic \[ 0 \approx \nabla \cdot (\rho u) = \frac{\partial (\rho u^2)}{\partial z} + \frac{1}{\sqrt{\gamma}} \frac{\partial}{\partial r} (\rho u v r) \]

\[ u_r \sim k r \]
\[ u_z \sim \text{const.} \]
\[ \frac{\rho u^2}{H_p} = \frac{2 \rho u_r}{r} \Rightarrow R = 2H_p \left( \frac{u_r}{u_z} \right) \]

- \[ \frac{u_r}{u_z} \perp \text{WITH SIZE OF THE STRUCTURE} \]
- \[ P' \perp \text{TO DRIVE INCREASING U_R} \]
  \[ \rightarrow \text{BUOYANCY BRAKING CHOKES UPFLOW} \]
- Granules cannot exceed a critical size
  \[ (u_z = 2 \text{ km/s}, \ u_r = c_s = 10 \text{ km/s} \]
  \[ \text{sun: } R \leq 10H_p \approx 2000 \text{ km} \]
- Stellar Granules scale with \( H_p \)
  \[ (\text{DRAVINS \& NORMUND, 1990}) \]
4. MIXING LENGTH DESCRIPTION

Convection as mixture of blobs which move vertically over a distance \( l \) (the mixing length) and dissolve [Ferndt (1925), Biermann (1948), Spik (1950), Böhm-Vitense (1953)]. Typically

\[
l = \alpha \cdot H_p, \quad \dot{l} = O(1)
\]

Aim: calculate energy flux and mean quantities \((\nabla \cdot S; \Delta T; \Delta t, \mu \) of blobs, ...)

Temperature difference: [ignore factors of \( O(1) \)]

\[
\Delta T = \left( \frac{dT}{dr} \right)_i - \left( \frac{dT}{dr} \right)_o \quad \epsilon V = (\nabla - \nabla_i) T \frac{\delta r}{H_p} = (\nabla - \nabla_i) T \alpha
\]

Convective flux: [ergs cm\(^{-3}\) s\(^{-1}\)]

\[
F_c = \Delta T \cdot \epsilon C_p \cdot \epsilon = \epsilon C_p \epsilon T (\nabla - \nabla_i) \alpha
\]

Velocity: [acceleration by buoyancy]

\[
\ddot{r} = -g \frac{\Delta \rho}{\rho} = g \chi_p \Delta T / T = g \chi_p (\nabla - \nabla_i) \frac{\delta r}{H_p}
\]

\[
\Rightarrow \quad \text{(blob homogeneous)} \quad \dot{\epsilon}^2 = \frac{g \chi_p}{H_p} (\nabla - \nabla_i) \epsilon^2
\]

\[
\Rightarrow \quad \epsilon = \left[ \frac{g \chi_p}{H_p} (\nabla - \nabla_i) \right]^{1/2} l
\]

Convective flux:

\[
F_c = \epsilon C_p T (g \chi_p H_p)^{1/2} \chi^2 (\nabla - \nabla_i)^{3/2} \sim \epsilon \cdot 10^{4} \chi^2 \sim 10^{12}
\]

Energy flux:

\[
F_{rad} + F_c = L_0 / 4 \pi r^2
\]
• \( V_1 = V_a \rightarrow \text{READY}, \quad \gamma(x) \text{ DETERMINED} \)

• \( V_1 > V_a \text{ DUE TO RADIATION} \rightarrow \ldots \rightarrow \text{CUBIC EQUATION} \)

• **TYPICAL VALUES IN DEEP CONVECTION ZONE:**
  
  \[
  \frac{V - V_a}{V_a} \approx 10^{-5} \ll 1 \\
  \Delta T = 2 \, K \ll T \\
  u = 100 \, m/s \ll C_s
  \]

  \( \rightarrow \text{CONVECTION IS VERY "EFFICIENT"} \)

• **... AND NEAR THE SURFACE**
  
  \[
  \frac{V - V_a}{V_a} \approx 0.6 \\
  \Delta T = 2000 \, K \\
  u = 2 \, km/s
  \]

  \( \rightarrow \text{ASSUMPTIONS BECOME INVALID} \)

• **MIXING LENGTH DESCRIPTION**

  \( \approx \text{TURBULENT DIFFUSION OF ENTROPY} \) \( \text{LITH} \quad \gamma \approx \nu \cdot l \)

• **IONSATION (H, He) REDUCES V_a (LATENT HEAT)**

  \( \rightarrow \text{DESTABILIZING} \)

• **DRAWBACKS OF M.L.D.:**

  • **LOCAL** (NO OVERSHEAT)
  
  • **ADJUSTABLE PARAMETERS** (NO PREDICTIONS)
  
  • **EFFECTIVELY INCOMPRESSIBLE, NEGLIGEKS**

  PRESSURE FLUCTUATIONS, STATIFICATION
  
  (BOUSSINESQ - APPROX.)

  (INAPPLICABLE TO OBSERVABLE SURFACE FLOWS)
$\nabla_{rad}$, $\nabla_{ad}$, in a standard mixing length model of the solar convection zone.

\[ a) \]

\[ M = 1 M_\odot \]
(SPECULATIVE) Picture of Solar Convection: Inverse Cascade
(Spruit et al., 1990)

Filamentary downdrafts merge (due to horiz. flows on larger scales,)
driven by the entropy deficit of the downdrafts themselves
Onset of convection: 2D simulation

Colours: temperature
Granulation: Solar surface convection
Solar granulation
Granulation und laboratory convection
Granulation as a convective phenomenon
Supergranulation
Supergranulation and magnetic field: the Ca$^+$ network
Granulation, sunspots, & small-scale magnetic field
‘Realistic’ solar simulations

- elaborate physics: partial ionization, radiation, compressible, open box, transmitting boundaries, spectral line diagnostics (Stokes profiles)

+ : approximation to solar conditions

+ : direct comparison with observations

– : computational restrictions (box size, resolution)

– : Reynolds numbers much below solar values
Approach: Local simulation box including photosphere

- Radiative energy transport
- Convective energy transport
The MURaM code: equations

**Continuity equation**
\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0
\]

**Momentum equation**
\[
\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \left( \rho \mathbf{u} \mathbf{u} + \left( p + \frac{|\mathbf{B}|^2}{8\pi} \right) \mathbf{1} - \frac{\mathbf{BB}}{4\pi} \right) = \rho \mathbf{g} + \nabla \cdot \mathbf{T}
\]

**Energy equation**
\[
\frac{\partial e}{\partial t} + \nabla \cdot \left( \mathbf{u} \left( e + p + \frac{|\mathbf{B}|^2}{8\pi} \right) - \frac{1}{4\pi} \mathbf{B} (\mathbf{u} \cdot \mathbf{B}) \right)
\]
\[
= \frac{1}{4\pi} \nabla \cdot (\mathbf{B} \times \eta \nabla \times \mathbf{B}) + \nabla \cdot (\mathbf{u} \cdot \mathbf{T}) + \nabla \cdot (\chi \rho \nabla e)
\]
\[
+ \rho (\mathbf{g} \cdot \mathbf{u}) - Q_{\text{rad}},
\]

**Radiative Transfer Equation**
\[
Q_{\text{rad}} = -\nabla \cdot \mathbf{F} = 4\pi \rho \int \kappa_\nu (I_\nu - S_\nu) d\nu
\]

**Induction equation**
\[
\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{uB} - \mathbf{Bu}) = -\nabla \times (\eta \nabla \times \mathbf{B}).
\]

**Radiative Transfer Equation**
\[
\frac{dI_\nu}{ds} = -\kappa_\nu \rho (I_\nu - S_\nu)
\]
Computer-simulated convection

- realistic simulation
- ionization, rad. transfer
- 3D, 288×288×100 mesh
- 6 Mm × 6 Mm × 1.4 Mm
- granulation

Vögler et al. (2005)

Emerging intensity
Computer-simulated convection

Emerging intensity

Vertical velocity (red: down, blue: up)
Computer-simulated convection

Upper photosphere
“Mesogranulation” ?
Simulated long-lived convective downflows

Virtual “corks” are carried by the horizontal flow. They accumulate in downflow regions.
Averaged energy fluxes in a simulation of solar convection

Stein & Nordlund, 2000
Simulation and observation

Simulation (original)

Simulation (smoothed)

Observation
Change of downflow topology

Stein & Nordlund, 1998
Simulated convection in a solar-like spherical shell

Miesch 1998
Magnetic fields on the Sun

Sunspots

A large sunspot group
What is the nature of sunspots?

Smoke clouds?
Holes?
Tornadoes?
The magnetic nature of sunspots

Sunspot with spectrograph slit

Magnetically split spectral line
Magnetic variability

Full-disk magnetogram

Magnetic patterns on the rotating Sun
Hot plasma draws magnetic field lines...
Hot plasma draws magnetic field lines...
The solar magnetic field...

... continues into interplanetary space.

Its variability in the course of the 11-year cycle and its long-term modulation...

- affects cosmic rays,
- perturbs the terrestrial magnetic field.
G-band observations

KIS/VTT, Obs. del Teide, Tenerife
G-band observations

Dutch Open Telescope, Obs. del Roque de los Muchachos, courtesy P. Sütterlin
What is magneto-convection?

- Interaction between convective flows and magnetic field in an electrically well-conducting fluid
- High Reynolds numbers: nonlinear dynamics, structure and pattern formation
- Interference with convective energy transport
Regimes of solar magneto-convection

- $<B>$ increases:
  - quiet Sun $\rightarrow$ plage $\rightarrow$
  - umbra

- horizontal scale of convection decreases

- convective energy transport decreases

T. Berger, SVST 12 May 1998, Obs. del Roque de los Muchachos
Adapted from a figure by Thierry Emonet, Univ. Chicago
Good electrical conductors: “frozen field”

Initially field-free volumes remain field-free

Magnetic flux through a given volume remains constant
“Frozen field” in the Sun

Magnetic flux is transported to the downflow regions of the convective flow patterns.

“magnetic network”
Simulation of flux expulsion

(Weiss, 1966)

- the magnetic flux is expelled from the area of closed streamlines and concentrated in narrow sheets

b: final state for $\text{Re}_m = 40$

a: streamlines of the fixed velocity field

c-j: time evolution for $\text{Re}_m = 1000$

- evolution of an initially vertical magnetic field under the influence of a fixed flow field

- kinematic, 2D

- the magnetic flux is expelled from the area of closed streamlines and concentrated in narrow sheets
Flux expulsion and intermittency

- N.O. Weiss (1964): *first simulations*

(Hupfer, KIS Freiburg, 2001)
Flux expulsion and intermittency

- N.O. Weiss (1964): *first simulations*

(Hupfer, KIS Freiburg, 2001)
$B_0 = 200 \text{ G (plage): time evolution}$

horizontal cuts near $\tau=1$

$6000 \text{ km} \times 6000 \text{ km} \times 1400 \text{ km}$

$288 \times 288 \times 100$ grid points

vertical magnetic field

brightness

vertical velocity

$B_0 = 200 \text{ G (plage): time evolution}$
$B_0 = 200 \, \text{G (plage)}$

Vertical magnetic field component

+2 kG

-2 kG

6 Mm
$B_0 = 200 \text{ G (plage)}$
Convective intensification

- Flux advection by horizontal flow (flux expulsion)
- Suppression of convection, cooling and downflow
- Evacuation, field intensification
The magnetically variable Sun

11-year cycle of magnetic activity and surface flux
The 11-year solar cycle

Solar magnetic activity varies with a period of roughly 11 years. Long-term variations are superposed upon this cycle.
$^{14}$C: Solar activity back to AD 1000
Butterfly diagram

DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS

SUNSPOT AREA IN EQUAL AREA LATITUDE STRIPS (% OF STRIP AREA)

> 0.0%  > 0.1%  > 1.0%

DATE

AVGARAGE DAILY SUNSPOT AREA (% OF VISIBLE HEMISPHERE)

DATE

http://wwwsl.msfc.nasa.gov/solpad/solar_images/bfly.gif

NASA MSFC HATHAWAY 91-97
Hale’s Polarity Law:

The polarity of the leading spots in one hemisphere is opposite that of the leading spots in the other hemisphere and the polarities reverse from one cycle to the next.

Cycle 21 Maximum

Cycle 22 Maximum
Where do the surface fields come from?
Origin of sunspots

- Convection zone
Origin of sunspots

• Overshoot layer
Origin of sunspots

- Magnetic flux tube
Origin of sunspots

- Parker instability
Origin of sunspots

- Magnetic buoyancy
Origin of sunspots

- Magnetic buoyancy
Origin of sunspots

- Tube expansion and decreasing field strength
Origin of sunspots

- Eruption at the solar surface
Origin of sunspots

- Formation of a bipolar sunspot pair/group
Magnetic buoyancy of a flux tube

Pressure equilibrium
\[ P_a = P_i + \frac{B^2}{8\pi} \]

\( B \neq 0 \Rightarrow P_i < P_a \)
\( \Rightarrow \rho_i < \rho_a \)
\( \Rightarrow \text{buoyancy} \)

\( P_a, \rho_a \text{ external pressure, density} \)
\( P_i, \rho_i \text{ internal pressure, density} \)

Parker instability
Magnetic buoyancy of a flux tube

Pressure equilibrium

\[ P_a = P_i + \frac{B^2}{8\pi} \]

\[ B \neq 0 \Rightarrow P_i < P_a \]
\[ \Rightarrow \rho_i < \rho_a \]
\[ \Rightarrow \text{buoyancy} \]

\( P_a, \rho_a \) external pressure, density
\( P_i, \rho_i \) internal pressure, density

Parker instability
Magnetic buoyancy of a flux tube
Magnetic buoyancy of a flux tube
Magnetic buoyancy of a flux tube
Generation of magnetic flux...

- ... requires an electrically conducting medium
  - plasma (ionized gas)

- ... requires fluid motion for induction
  - convective flows
  - (differential) rotation

- ... how is the field maintained against dissipation?
  - (self-excited) dynamo process
The induction principle

- Conductor moving in a magnetic field
- perpendicular electrical field and force
- electrical current
- new magnetic field
- Lenz’s rule!

(no perpetuum mobile)
A simple dynamo

Initially weak “seed field”

- Rotation induces electrical field between axis and edge
- Current closed by wire
- Current generates a magnetic field which amplifies the seed field
- Sun: no isolated wires
- “homogeneous dynamo”
Local dynamo

Vögler & Sch. 2007
Differential rotation generates azimuthal (toroidal) magnetic field

Azimuthal flow of differential rotation

The longer the arrow the faster the flow

Meridional magnetic field is transformed into azimuthal magnetic field
Internal rotation of the Sun as determined by helioseismology

- Convection zone rotates similar to surface
- Core rotates nearly rigidly
- Steep transition at the bottom of the convection zone; width \( \sim 2\% R_{\text{sun}} \)
- Region of strongest shear \( \Rightarrow \) Dynamo!
Internal rotation of the Sun as determined by helioseismology
The solar dynamo (1)

Dipol field in the convection zone

Winding up of the field by differential rotation
⇒ strong toroidal field
The solar dynamo (2)

Twist of the erupting field by Coriolis force
→ reversed dipole field

Rise and eruption of magnetic flux tubes
→ sunspots
Twisting of a field line in a rising & expanding convective flow by the action of the Coriolis force (Parker, 1955)
Reversal of the meridional field
Reversal of the meridional field
Flux transport dynamo scheme

M. Dikpati, HAO/NCAR, Boulder/CO
$B = 1 \text{ Tesla}$

$B = 10 \text{ Tesla}$
The end...