Cosmology / introductory remarks

- Very old science in human history
- Practical and speculative side
  - Calendar ephemeris
  - Magic/mythological ideas about origin world
    - Hindus: $T_{\text{universe}} = 1 \text{ Brahma day} = 4.32 \times 10^9 \text{ yr}$

- Very rapid evolution since ~1960
  - Speculative backyard → quantitative science

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Will have to skip many issues!
SPECIAL RELATIVITY

\[ t \quad \text{or} \quad x^0 = ct \]

Worldline

\[ \{ x^\alpha + \Delta x^\alpha \} \]

\[ \{ x^i \} \]

\[ \text{P} \cdot \{ x^\alpha \} \]

\[ \text{V} \]

\[ \text{Q} \]

\[ \Delta s^2 = (\Delta x^0)^2 - \Delta x^i \Delta x^i = \eta_{\alpha\beta} \Delta x^\alpha \Delta x^\beta \]

\[ (c \, dt)^2 \]

\[ \eta = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix} \]

\[ \text{Note sign convention} \]

- Notation: \( \Delta s^2 = (\Delta s)^2 \); \( dt^2 = (dt)^2 \)
- \( \Delta s^2 \) is invariant
\[ \Delta s^2 \text{ invariant} \rightarrow \text{all observers come to the same result} \]

"we can speak of "the" lightcone"
\[ P \{ x^a \} \quad \& \quad Q \{ x^a + \Delta x^a \} \]

\[ \Delta S^2 = \eta_{\alpha\beta} \Delta x^\alpha \Delta x^\beta \]

\[ \Delta S^2 > 0 \quad \text{TIMELIKE (MATTER)} \]
\[ = 0 \quad \text{NULL VECTOR (PHOTONS)} \]
\[ < 0 \quad \text{SPACELIKE (TACHYONS)} \]
- proper time
- Lorentz transformations
- tensors
GENERAL RELATIVITY

- Arbitrarily moving frames → force closely related to gravity

- Classical gravity
  \[ \nabla^2 \phi = 4 \pi G \rho ; \ \kappa = -m \phi \]
  holds only in one frame
  → \[ \nabla^2 \rightarrow \nabla^2 - \frac{1}{c^2} \frac{\phi^2}{2x^2} \]
  → SR-like theory, with one global frame seems dead end.

- Weak equivalence
  \[ m_i \ddot{r} = \text{applied force} \]
  \[ = -mg \nabla \phi \]
  \[ m_i/m_g \text{ same for all bodies, say } \equiv 1 \]
  (Eötvös 10^{-8}, now ~10^{-12})

∴ gravity can be transformed away locally, not globally.

Tidal force cannot be transformed away. Real gravity field is inhomogeneous.
$h = z_1 - z_2 = 22.5 \text{ m}$

(Pound, Rebka & Snider 1961, 1965)

- Experiment: $\Delta t_1 > \Delta t_0$ (redshift)

- Suggests that spacetime is curved due to gravity

$\Delta s^2 = \eta_{\alpha \beta} \Delta x^\alpha \Delta x^\beta \rightarrow ds^2 = g_{\alpha \beta} dx^\alpha dx^\beta$

(metric tensor determined by mass distribution)
Geometrical picture

Co-ordinate picture

\textit{Metric:}

\[ ds^2 = d\theta^2 + \sin^2 \theta \, d\phi^2 \]

\[ \theta \quad \theta \quad \phi \quad \phi \]
- Co-ordinate lines: do as you like
- Base vectors: tangent to co-ord. lines, in + direction

- Base vector span flat tangent space
  (presupposes existence flat embedding space)

- Metric in tangent space: arbitrary, but there is one preferred metric which is very handy

\[
\text{infinitesimal vector } ds = dx^\alpha e_\alpha \text{ has length } ds \text{ of Riemann space}
\]

\[
\Rightarrow \quad g_{\alpha \beta} = e_\alpha \cdot e_\beta
\]
Contra- & Covariant

- **Finite vectors in tangent space** \( A = A^\alpha e_\alpha \)

- *N.b. all vectors associated with a particle lie in the local tangent space (\( \nu, \alpha, \text{Spin} \))

- **Definition of** \( A_\alpha \):
  \[
  A \cdot A = A^\alpha A_\alpha \quad \text{(summation)}
  \]

  \[
  = g_{\alpha \beta} A^\alpha A^\beta \quad \text{(} A \cdot A = \text{length)}
  \]

  \[
  \therefore A_\alpha = g_{\alpha \beta} A^\beta \quad \text{Index lowering}
  \]

- **Index raising**:
  \[
  A^\alpha = g^{\alpha \gamma} A_\gamma
  \]
  \[
  = g^{\alpha \gamma} g_{\gamma \nu} A^\nu
  \]

  \[
  \therefore g^{\alpha \gamma} g_{\gamma \nu} = \delta^\alpha_\nu = \begin{cases} 1 & \alpha = \nu \\ 0 & \alpha \neq \nu \end{cases}
  \]

  or: \( \{ g^{\alpha \nu} \} \text{ is inverse of } \{ g_{\nu \nu} \} \)
Interpretation \( A^\alpha \) and \( A_\alpha \)

\[ A = A^\alpha e_\alpha \rightarrow A^\alpha \text{ components } A \text{ along basis (parallelogram construction)} \]

\[ A_\gamma = g_{\gamma\alpha} A^\alpha = e_\gamma \cdot e_\alpha A^\alpha = e_\gamma \cdot A \]

\[ \rightarrow A_\alpha \text{ is projection } A \text{ on base-vector } e_\alpha \]
- All this holds for any vector field $A(x^k) \rightarrow A^\alpha$.

- Tensors? Behaviour under coordinate transform:

$$\{ x^\alpha \} \rightarrow \{ \bar{x}^\beta \}$$

$$\delta \bar{x}^\beta = \frac{\partial \bar{x}^\beta}{\partial x^\alpha} \delta x^\alpha$$

DEF: Any set of numbers that transforms in this way is a (contravariant) tensor of 1st rank. i.e.

$$\bar{A}^\nu = \frac{\partial \bar{x}^\nu}{\partial x^\alpha} A^\alpha$$

Note: $\bar{L}^\alpha$

- Tensor of higher rank

$$T^{\nu \alpha} = A^\mu B^\nu$$

$$\Phi_{\beta}^\gamma = A^\alpha B^\beta C^\gamma$$

Each index transforms according to

$$\bar{T}^{\mu \nu} = \frac{\partial \bar{x}^\mu}{\partial x^\alpha} \frac{\partial \bar{x}^\nu}{\partial x^\beta} T^{\alpha \beta}$$

- Underlying geometries $T_{\mu \nu} = g_{\mu \chi} T^{\chi \beta}$

- Parallel displacement

- Geodesics (orbits of test particles)

- Curvature
FIELD EQUATION?

- $\nabla^2 \phi = 4\pi G \rho_0 \quad \Rightarrow \quad \text{what??}$

- CURVATURE OF \( (\cdot) \) TOTAL ENERGY
  SPACE TIME \( (\cdot) \) DENSITY

  \[
  G^{\mu\nu}(\{g_{\alpha\beta}\}) = -\frac{8\pi G}{c^2} \cdot T^{\mu\nu}
  \]

  By considering weak fields $\rightarrow$ classical mechanics
  Rest energy density pressure EM fields $\ldots$

- Simplest $T^{\mu\nu} = \rho_0 u^\mu u^\nu$ "Dust"

  4-velocity $\frac{1}{c} \frac{dx^\mu}{d\tau} = (\gamma, \nu_1/c)$

- Why relation between tensors of 2nd rank?

- \begin{align*}
  m_0 & \rightarrow m = \gamma m_0 \\
  m_0 & \rightarrow M = \gamma M_0 \\
  \therefore \quad \rho_0 = m_0 m_0 & \rightarrow \rho = m M = \gamma^2 \rho_0
  \end{align*}

  $\Rightarrow$ 0,0 component of 2nd rank tensor.

- Matter tells spacetime how to curve (\( g_{\alpha\beta} \))
  Spacetime tells matter how to move (along geodesics determined by \( g_{\alpha\beta} \))
GENERAL RELATIVITY / RECAPITULATION

- WEAK EQUIVALENCE
  Gravity is partly an apparent force

- ORBIT OF TEST PARTICLE IS GEODESIC
  "Straight" orbit in curved spacetime. Curvature due to energy densities

- WHY CURVATURE?

- RIEMANN SPACES
  Tangent space, finite vectors, contra- & covariant components. Tensor: transformation.

\[ D^2 \phi = 4 \pi G \rho \rightarrow ??? \]

\[ G^{\alpha \beta} \{g_{\mu \nu}\} = - \frac{8 \pi G}{c^2} T^{\mu \nu} \]

\[ c \text{ nonlinear in } g^{\mu \nu} \]
THE EVOLUTION
OF OUR UNIVERSE
**COSMOLOGY - WHY GR??**

- Universe is compact object

\[ R = \frac{2GM}{c^2} = \frac{2G}{c^2} \frac{4\pi}{3} \rho R^3 \]

\( \text{Schwarzschild radius} \)

\[ \begin{align*}
\nu &= H_0 d \\
C &= H_0 R \rightarrow R = c/H_0
\end{align*} \]

\[ \rho \approx \frac{3H_0^2}{8\pi G} = \rho_c \quad \text{critical density} \]

- \( H_0 = 100 \ h \ \text{km s}^{-1} \ \text{Mpc}^{-1} \)

\( h = 0.71 \pm 0.04 \quad \text{(WMAP)} \)

\( H_0 = (2.3 \pm 0.1) \times 10^{-18} \ \text{s}^{-1} \)

- \( \rho_c \approx 10^{-29} \ \text{g cm}^{-3} \)
\[
\begin{array}{ll}
\text{TYPE} & \Omega = \rho / \rho_c \\
\text{MATTER (\(\Omega_m\))} & 0.27 \\
\text{Luminous baryons} & 0.006 \\
\text{dark matter} & 0.038 \quad \Omega_b \approx 0.04 \\
\text{WIMPS} & 0.23 \quad \text{unknown} \\
\text{DARK ENERGY (\(\Omega_{\Lambda}\))} & 0.73 \quad \text{unknown} \\
\text{TOTAL \(\Omega_m + \Omega_{\Lambda}\)} & 1.02 \pm 0.02 \quad \text{flat geometry}
\end{array}
\]

Matter distribution isotropic
also within redshift classes
WMAP IMAGE CMB, $\lambda = 3.2$ mm.

- $T = 2.725$ K
- Monopole & dipole subtracted, foreground emission not yet black: $-200 \mu K$ red: $+200 \mu K$

---

Energy densities

$\varepsilon_{\text{matter}} = \frac{\rho_m c^2}{\rho_c} = 2.4 \times 10^{-9}$ erg cm$^{-3}$

$\varepsilon_{\text{CMB}} = \frac{4\pi}{3} \frac{\hbar^2}{c^3} T^4 = 4.2 \times 10^{-13}$ erg cm$^{-3}$

$\uparrow 2.725$ K

$\varepsilon_{\nu\bar{\nu}} = 2.8 \times 10^{-13}$

$\varepsilon_{\text{radiation}} = 7 \times 10^{-13}$ erg cm$^{-3}$
- past lightcone: "nested shells"
- isotropy → shells $\Sigma$ are homogeneous ($\Sigma_1 \neq \Sigma_2$)
- cosmological principle → spaces $t$ = constant are homogeneous
- Rest $\equiv$ not moving w.r.t. Hubble flow
- Spatial coordinate, galaxies are constant (we ignore their small peculiar velocities)
- worldlines vertical
- coordinate distance $B \& C$ is constant, geometrical distance $B \& C$ grows (Expansion!)
- cosmological principle needed, seems OK, but may prove incorrect in the future!
\[ ds^2 = (dx_0)^2 - \delta^2(t) \left[ dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \]

\( r, \theta, \phi \) spatial coordinates

\[ ds^2 = c^2 dt^2 - \delta^2(t) \left( \text{physical distance} \right)^2 \]

- actually \( k=0, \pm 1 \) types, but \( \Omega_m + \Omega_k = 1 \rightarrow \text{flat} \) (without proof)

\[ (\delta c^2 S^3) + p(S^3) = 0 \quad \text{d}U + p dV = 0 \]

1. \[ (\frac{\delta^2}{S^2}) = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} \]

classical gravity

\[ \text{cosmological constant} \rightarrow S \propto \exp(t) \]

- Three unknowns: \( S, p, S' \)

need e.g. \( p(S) \), or rather \( p(S') \)
CLASICAL GRAVITY

NEWTON:

\[ m \ddot{S} = -G m \left( \frac{4\pi}{3} S^3 \rho \right) \cdot \frac{1}{S^2} \]

\[ \ddot{S} = - \frac{4\pi G}{3} \rho S \]

\[ \rho S^3 = \rho_o S_o^3 \]

\[ \ddot{S} = - \frac{4\pi G \rho_o S_o^3}{3} \frac{1}{S^2} \]

* Integrate

\[ S^2 = \frac{8\pi G \rho_o S_o^3}{3} \frac{1}{S} + \text{const} \]

\[ = 0 \]

\[ \rho_o S_o^3 / S = \rho S^2 \]
COSMOLOGICAL CONSTANT

- $T^{\mu \nu}$ of classical fluid in rest-frame:

$$T^{\mu \nu} = \frac{1}{c^2} \left( \begin{array}{cc} \phi^2 & \phi p \\ \phi & pp \end{array} \right)$$

- Note: pressure $p$ is form of energy and if $p \sim \rho c^2$ it generates gravity
  This causes collapse $NS \rightarrow BH$ !

- Note: $\frac{dp}{dr}$ supports star; $p \rightarrow gravity$

- Accept this $T^{\mu \nu}$ as $T^{\mu \nu}$ of vacuum

  $\phi - \rho V$, $p = \rho V$

- Vacuum identical in all inertial frames,

  $T^{\mu \nu} \propto \eta^{\mu \nu} = \left( \begin{array}{ccc} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right)$

  $T^{\mu \nu} = \mathcal{S}_V \left( \begin{array}{ccc} 1 & -1 & \phi \\ -1 & 1 & \phi \\ 0 & 0 & 0 \end{array} \right)$ and $p_V = -\mathcal{S}_V c^2 < 0$ !

- $\mathcal{S}_V$: universal attraction: space tries to contract $p_V < 0$: space blows itself up

- Ultimate explanation: Quantum Gravity
\[
\rho S^3 = \text{constant} \quad \Rightarrow
\]

\[
\dot{u} = H_0 \left( -\Omega_m u^{-1} + \Omega_\Lambda u^2 \right)^{1/2} \quad u = s/s_0.
\]

\[
\left[
\begin{array}{l}
\Omega_m = \frac{s_0}{s_c} \approx 0.27 \\
\Omega_\Lambda = \frac{s_\Lambda}{s_c} \approx 0.73
\end{array}
\right]
\]

- \( u \ll 1 \rightarrow \dot{u} \propto u^{-1/2} \rightarrow u(\cdot) t^{2/3} \)
- \( u \gg 1 \rightarrow \dot{u} \propto u \rightarrow u(\cdot) \exp(-t) \)
- Singularity \( u = 0 \) must occur if \( \Omega_\Lambda < 1 \)
- Age \( \simeq H_0^{-1} \approx 14 \text{ Gyr} \).
$1/a \approx 3300$

$(1/a)(S/S_0)$

- Radiation Era
- Matter Era
- $\tau^{2/3}$
- Recombination

$\tau = t/t_m$

$\tau^{1/2}$

$\approx 9 \times 10^4$ yr

$T_{rad} = T_{mat} \cdot S^{-1}$

$\epsilon_{rad} > \epsilon_{mat}$

$\epsilon_{rad} \propto S^{-4} \propto T^4$

$T_{rad} \approx 3000K$

$S_0/S \approx 1100$

Universe very homogeneous

No galaxies
THE BIG BANG

Elementary Particles

Quarks  6 types, 3 colour charge, + antiquarks \rightarrow 36 in total

Leptons  e^-, \nu_e, \mu^-, \nu_\mu, \tau^-, \nu_\tau + antiparticles, 12 in total

Gauge bosons  gluons  vector bosons  photon  graviton

WIMPS (WIMP)
Table 2. Overview of the evolution of the universe

<table>
<thead>
<tr>
<th>age (s)</th>
<th>temperature (K)</th>
<th>size (S/S₀)</th>
<th>composition a</th>
<th>baryons</th>
<th>leptons</th>
<th>gauge bosons</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 10⁻⁷</td>
<td>&gt; 10¹³</td>
<td>&lt; 2 x 10⁻¹³</td>
<td>q q̄</td>
<td>l l̄</td>
<td>γ, g, W⁺⁻, Z⁰⁻</td>
<td></td>
</tr>
<tr>
<td>10⁻⁶</td>
<td>5 x 10¹²</td>
<td>5 x 10⁻¹³</td>
<td>p p̄, n n̄, l l̄</td>
<td>γ, g</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10⁻⁴</td>
<td>10¹²</td>
<td>3 x 10⁻¹²</td>
<td>p, n</td>
<td>e⁻⁺ e⁺, ν ν̄</td>
<td>γ, g, x</td>
<td></td>
</tr>
<tr>
<td>10²</td>
<td>10⁹</td>
<td>3 x 10⁻⁹</td>
<td>p, n</td>
<td>e⁻, ν ν̄</td>
<td>γ, g, x</td>
<td></td>
</tr>
<tr>
<td>10³</td>
<td>3 x 10⁸</td>
<td>10⁻⁸</td>
<td>¹H, ⁴He</td>
<td>e⁻, ν ν̄</td>
<td>γ, g</td>
<td></td>
</tr>
<tr>
<td>&gt; 10¹³</td>
<td>&lt; 3000</td>
<td>&gt; 10⁻³</td>
<td>H, He atoms</td>
<td>ν ν̄</td>
<td>γ, g</td>
<td></td>
</tr>
<tr>
<td>4 x 10¹⁷</td>
<td>3</td>
<td>1</td>
<td>galaxies</td>
<td>neutrino, microwave and graviton background</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a Boldface printed particles have comparable number densities, and these are about 10⁹ times larger than those of the other particles on the same line.

**REACTIONS**

\[ A + B \leftrightarrow C + D \]

\[ \rightarrow \text{p + q} \]

Whole network

- Available time: \( \sim (S/S₀)^{-1} \sim \text{age universe} \)
- Thermal equilibrium \( \rightarrow \) freeze out
- M̅M̅ asymmetry \( \sim 10⁻⁸ \) unknown origin
- \( \frac{M̅}{P} = \exp (\Delta E/kt) = 1 \rightarrow \text{freeze out at } \frac{M̅}{M_0} = 0.16 \)
- Decrease (M̅ decay) to \( \sim 0.13 \) at onset synthesis of elements.
- All n's end up in $^4\text{He} \rightarrow \gamma = 2 \times 0.13 = 0.26$
- no time for synthesis heavier element!
Observational Issues

1. how do we observe the universe
2. the horizon (problem)
3. a common misconception
4. the angular correlation spectrum of the CMB
- Horizon delimits sphere of influence around each observer
  
  Light emitted by matter outside horizon has not (yet) been able to reach the observer

- Visible Universe = sphere inside horizon of the observer

- Horizon Problem only solved with advent of inflation theory
**Distance to Horizon**

\[ ds^2 = c^2 dt^2 - s^2 \left( dr^2 + r^2 d\theta^2 \right) \]

\[ d\theta = d\phi = 0 \]

\[ ds = 0 \text{ (photon)} \quad \rightarrow \quad dr = c dt / s \]

\[ r = c \int_0^t \frac{dt}{s} \quad \rightarrow \quad \text{distance} = Sr = cs \int_0^t \frac{dt}{s} \]

- take \( S(\cdot) t^\alpha \)
- distance = \( \frac{ct}{1 - \alpha} \)
  \[ \begin{align*}
  &\alpha = \frac{2}{3} \\
  &\alpha = \frac{1}{2}
  \end{align*} \]
- photons superluminal??
A common misconception: The Big Bang as a point explosion

- $\bar{\rho} + \text{AGE} \Rightarrow T$ to boundary of explosion is $\ll 1$

- Universe isotropic $\Rightarrow$ we are in the center of a spherically symmetric explosion

Universe visible universes
POWER SPECTRUM CMB FROM WMAP DATA

\[ \overline{I} = \text{PREVIOUS RESULTS} \]

1. \[ \Delta T = T(\vartheta, \phi) - \langle T \rangle \quad \quad \langle . \rangle = \frac{1}{4\pi} \int d\Omega \]

2. \[ C_\ell = 2\pi \int_0^\pi \langle \Delta T(\vartheta) \Delta T(\vartheta) \rangle |_{\vartheta = \vartheta_1, \vartheta} P_\ell (\cos \vartheta) \sin \vartheta d\vartheta \]

3. Figure shows "power spectrum" \( \ell (\ell + 1) C_\ell / 4\pi \)
THE PLAYERS

(1). SCALAR FIELD $\phi$

   At end of inflation $\phi \rightarrow \rho \& \delta \rho$
   (matter)

(2). "BARYONIC FLUID"

   - $\rho$, $^4\text{He}$, $e^-$, $\gamma$
   - $\Omega_B = 0.04$
   - A tightly coupled system until $t_{rec}$
   - High 'sound' speed $c/\sqrt{3}$ ($10^9$ $\text{m}$ per baryon)

(3). NONBARYONIC DARK MATTER "DM"

   - No EM interactions
   - $\Omega_{\text{dark}} = 0.23 \rightarrow \text{DM fixes } \phi, \delta \phi$
   - Cold $\rightarrow$ Low 'sound' speed ("CDM model")
THE PLOT

(1) DM and bary communicate only through gravity

(2) $\epsilon_m = \epsilon_r$

$t_{\text{mat}}$ (60 kyr) $t_{\text{rec}}$ (380 kyr)

$\delta \rho / \rho$ evolves

bary fluid deintegrates; imprint of characteristic $\delta T$

- L line of sight
- Comoving with Hubble flow
- in L.S.S. at $t_{\text{rec}}$
\[ \frac{\delta p}{\rho} = \sum \text{fourier modes} \quad \text{(only in LSS)} \]

- \( (\delta p/\rho)_{\text{DM}} \) grows, but modes do not move
- \( (\delta p/\rho)_{\text{be}} \) damps, but modes do move

- All be modes travel same distance \( D \), shown is that mode for which \( D = \lambda/2 \) at \( t_{\text{rec}} \)

- \[ \frac{\delta T}{T} = \begin{cases} \frac{\delta\phi/c^2}{\rho} > 0 \text{ in } A & < 0 \text{ in } B \\ \frac{1}{3} (\frac{\delta p}{\rho})_{\text{be}} > 0 \text{ in } A & < 0 \text{ in } B \end{cases} \]

\[ \theta = \frac{\lambda/2}{d} = \frac{D}{d} \rightarrow \frac{D}{(2n+1)d} = \frac{0.6}{2n+1} \]
ANALYSIS WMAP DATA

- Complex modelling required

\[ A \]
\[ B \]
\[ C \]
\[ \Omega_0 \]

WMAP + HST Key \( H_0 \) + SNIa \( \rightarrow \) \( \Omega_m + \Omega_\Lambda = 1 \)

\[ B/C \] \( \rightarrow \) \( \Omega_b \ h^2 \)

\[ B/A \] \( \rightarrow \) \( \Omega_m \ h^2 \)

\[ \Omega_0 \] \( \rightarrow \) \( h \)
**INFLATION**

- Most important **theoretical** development in cosmology of last 25 yr.

- Successes of F.R.W. Universe
  - expansion velocities distant galaxies
  - the relics: CMB + chemical composition 
    \( (H, D, ^4He, ^3He, ^7Li) \)

- **BUT**

  - why is universe spatially flat?
  - horizon problem?
  - why expansion?

\[
\begin{align*}
&\cdot \text{ m \in asymmetry} \\
&\cdot \text{ vacuum energy}
\end{align*}
\]
THE ESSENCE OF INFLATION

\[ S(t) \propto t^{1/2} \]

1. **Distance to horizon**
   \[ c S \int_0^t \frac{dt}{S(t)} \propto t \]
   for \( S(\tau) \propto \tau^x \)

   Change \( S(t) \) near \( t=0 \) \( \Rightarrow \) much larger horizon distance!

   \((S(t) \text{ at later time, virtually fixed})\)

2. \( S(\tau) \propto \tau^x \rightarrow S \uparrow \propto a, \ t \downarrow 0 \)

   Expansion speed infinite \( \rightarrow \) universe desintegrates into separate parts

3. Take universe initially extremely small
   wait size/c sec \( \rightarrow \) causal contact
   blow up to huge proportion \( \rightarrow \) horizon distance also huge
NORMAL FRW EXPANSION ~ $10^{25}$

INFLATION $10^{50} - 10^{(108)}$

PREEXISTING SPACETIME

SIZE ~ PLANCK LENGTH ($\sim 10^{-33}$ cm)

(QUANTUM FLUCTUATION IN METRIC)

$10^{-33}$ S

$10^{-43}$ S (PLANCK TIME)

$10^{18}$ S
- Guth (1981): Scalar fields may cause inflation

- Scalar fields correspond to hypothetical, heavy, zero-spin bosons that may occur in GUTs.

- QM in expanding space-time

\[ E^2 = (pc)^2 + (mc^2)^2 \]
\[ E \rightarrow i\hbar \frac{\partial}{\partial \phi} \]
\[ p \rightarrow -i\hbar \nabla \]
\[ \mu = mc/\hbar \]

Klein-Gordon equation for wave function

free relativistic s=0 boson

\[ \eta^{\alpha \beta} \psi_{\alpha \beta} + \mu^2 \psi = 0 \rightarrow \eta^{\alpha \beta} \psi_{\alpha \beta} + \mu^2 \psi = 0 \]

ordinary covariant derivative

- Assume \( \Box \psi = 0 \)

\[ \psi,_{0,0} + \frac{3}{c^2} \frac{s}{S} \psi,_{0} + \mu^2 \psi = 0 \]

\( s = \hbar c, d \)

Damped harmonic oscillator
\[ \left( \frac{\dot{S}}{S} \right)^2 = \frac{8\pi G S}{3} + \frac{\Lambda c^2}{3} \]

- **Important in early Univ.**
- replace by energy density \( \Psi \) field
- add curvature term \( k c^2 / S^2 \)

\[
\left( \frac{\dot{S}}{S} \right)^2 + \frac{k c^2}{S^2} = \frac{4\pi G}{3} \left\{ \Psi_0^2 + \mu^2 \Psi^2 + \left| \dot{\phi} \right|^2 \right\}
\]

- think of energy \( \dot{\phi}^2 + \mu^2 \phi^2 \)
- of harmonic oscillator!

---

- **Make dimensionless**

\[
M = \left( \frac{\hbar c}{G} \right)^{\frac{1}{2}} = \text{Planck Mass} \left( 2.2 \times 10^{-5} \text{ g} \right)
\]

\( (\text{Planck length} = \lambda_{\text{compt}}) \)

\[
l_p = \frac{\hbar}{MC} = 1.6 \times 10^{-33} \text{ cm}
\]

\[
t_p = l_p / c = 5.4 \times 10^{-44} \text{ s}
\]

Substitute \( G = \frac{\hbar c}{M^2} \), then set \( \hbar = c = 1 \)
\[ \ddot{\psi} + 3H \dot{\psi} + m^2 \psi = 0 \quad \text{(1)} \]

\[ H^2 + \frac{k}{S^2} = \frac{4\pi}{3M^2} (\psi^2 + m^2 \psi^2) \quad \text{(2)} \]

p.m. \( \dot{\psi} = 0 \) and \( H = \dot{S}/S \)

closed set equations for \( \psi \) and \( S \)

- **Initial conditions** \( \Rightarrow \)

- **Solution of equations**

  Assume \( H \gg m \)  
  (strong damping) \[ \{ \Rightarrow \dot{\psi} = 0 ; \dot{\psi} \ll m \psi \]

\[ H = \dot{S}/S \sim \text{constant} \Rightarrow S(t) \text{ exponential and } k/S^2 \text{ rapidly ignorable} \]

\[ 3H \dot{\psi} + m^2 \psi = 0 \quad \text{(1)} \]

\[ H^2 = \frac{4\pi m^2}{3M^2} \psi^2 \quad \text{(2)} \]

- \( \psi \rightarrow m^2 \psi^2 = -3H \dot{\psi} \rightarrow \text{in (2)} \)

\[ H^2 = \frac{4\pi}{3M^2} (-3H \psi \dot{\psi}) \rightarrow H = -\frac{4\pi}{M^2} \psi \dot{\psi} \quad \text{(3)} \]

\[ \therefore \frac{\dot{S}}{S} + \frac{4\pi}{M^2} \psi \dot{\psi} = 0 \rightarrow \text{integrate}! \]
Initial conditions at $t = t_p$

- All energy "quantum bubble" $\sim L_p$ may contain resides in one scalar field $\phi$

[Nb: Usually energy divided over many different fields, but these regions do not inflate]

\[ M^2 \cdot t_p \sim L_p \]  (Heisenberg)

\[ \therefore \frac{\dot{\phi}^2 + m^2 \phi^2}{\rho} \approx \frac{M^2}{L_p^3} = M^4 \]

energy density

strong damping $\Rightarrow \phi$ very small

\[ \varphi_p \approx \psi(t_p) = \frac{M^2}{m} \]

\[ \varphi_p \text{ not needed} \]

further:

\[ s_p = s(t_p) = L_p \]

- Why $\nabla \phi \approx 0$?

\[ |\nabla \phi|^2 \leq M^4 \Rightarrow \delta \psi \approx |\nabla \phi| L_p \leq M^2 M^{-1} M \]

\[ \therefore \delta \psi / \psi \approx M / (M^2 / m) = m / M \ll 1 \]

scalar boson mass $\ll$ Planck mass

$\therefore$ all energy in $\phi$ field implies homogeneity of $\phi$ over $L_p$
\[
S = S_p \exp \left[ \frac{2\pi}{M^2} \left( \Psi^2 - \Psi_t^2 \right) \right]
\]

(4)

3. \( \dot{\psi} = -\frac{mM}{\sqrt{12\pi}} \)

**Evolution of \( \psi \)**

- Since \( H(\psi) \), see 2, eventually weak damping limit is reached \( \rightarrow \) oscillations
- Coupling with other (weak) quantum fields becomes important \( \rightarrow \) creation of matter \( \rightarrow \) begin of hot Big Bang
- Inflation creates homogeneous, expanding, hot, flat FRW Universe

**Evolution of \( S \)**

\[
\text{small } t : \quad S = S_p \exp \left[ \frac{2\pi}{M^2} \left\{ \Psi_t^2 - (\Psi - \dot{\Psi}t)^2 \right\} \right]
= S_p \exp \left[ \frac{4\pi \Psi \dot{\Psi}}{M^2} + \frac{\dot{\Psi}^2}{M^2} \right]
\]

\( \therefore \text{Exponential Expansion} \)
Large $t$

$$S = S_0 \exp \left[ \frac{2\pi}{M^2} (4\rho^2 - \psi^2) \right]$$

$\approx 0$ at end of inflation

$$\therefore \frac{S_e}{S_0} \sim \exp \left[ 2\pi \frac{M^2}{m^2} \right] = \text{HUGE}$$
- Many inflation scenarios, none wholly acceptable

- this one called chaotic inflation, see Linde, Phys. Today, Sept ’82, p. 61.

- conceptual picture ⇒

- pro & cons

++ all energy in \( \phi \)-field ⇒ dynamics \( \phi \) probably reasonably described by equation for free particle.
No speculative particle physics needed!

-- classical reasoning right at \( t = t_p \) !
does Quantumgrav. permit instability of the vacuum?

- What drives inflation?

  \( \phi \)-field does not scale as \( s^{-3} \) like matter

  Evolution is driven by equivalent \( \rho, \rho' \):

  \[
  \rho = \frac{1}{2} \dot{\psi}^2 + \frac{1}{2} m^2 \psi^2 = \frac{1}{2} m^2 \psi^2
  \]

  \[
  \rho = -\frac{1}{2} \dot{\psi}^2 - \frac{1}{2} m^2 \psi^2 \approx -\frac{1}{2} m^2 \psi^2
  \]

  vacuum with huge cosm. constant \( \Lambda \) \( m^2 \psi^2 \)
  huge antigravity from \( \rho < 0 \)

  Global energy conservation does not exist in GR!!
Rand quantum-bel

Quantumfluctuaties in de metrik
$t = t_0 \sim 10^{-43}$ s

Onze positie

Horizon

Spontane fluctuatie in onze ruimte-tijd

$x^0$

$ct_4$

$ct_3$

$ct_2$

$ct_1$

$\{x^i\}$

Horizon

$t_4$

$t_3$

$t_2$

$t_1$

Ruinthe
THE BIG QUESTIONS

**Observational**
- Where are the optically dark baryons?
- Structure formation
- Relic $\nu\bar{\nu}$ ($T = 1.95k$)

**Particle Physics**
- Origin $\bar{\mu}\mu$ asymmetry
- Nature nonbaryonic dark matter
- Properties quark-gluon plasma

**Quantum Gravity**
- Origin $\Lambda$?
- Acceptable inflation theory

Cosmologists have a great phantasy:

Universe ($\gg$ our visible universe) originates from tiny quantum fluctuation!? True or not? Only future will tell