Geomagnetic Dynamo Theory

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6. Literature
1600 Gilbert, De Magnete: "Magnus magnes ipse est globus terrestris." (The Earth’s globe itself is a great magnet.)

1838 Gauss: Mathematical description of geomagnetic field

\[ B = \sum_{l,m} B_l^m = -\sum \nabla \Phi_l^m = -R \sum \nabla \left( \frac{R}{r} \right)^{l+1} P_l^m(\cos \vartheta) \left( g_l^m \cos m\phi + h_l^m \sin m\phi \right) \]

sources inside Earth

- \( l \) number of nodal lines, \( m \) number of azimuthal nodal lines
- \( l = 1, 2, 3, \ldots \) dipole, quadrupole, octupole, ...
- \( m = 0 \) axisymmetry, \( m = 1, 2, \ldots \) non-axisymmetry

Earth: \( g_1^0 \approx -0.3 \) G, all other \( |g_l^m|, |h_l^m| < 0.05 \) G

mainly dipolar, dipole moment \( \mu = R^3 \left[ (g_1^0)^2 + (g_1^1)^2 + (h_1^1)^2 \right]^{1/2} \approx 8 \cdot 10^{25} \) G cm³

\( \tan \psi = \left[ (g_1^1)^2 + (h_1^1)^2 \right]^{1/2} / g_1^0 \), dipole tilt \( \psi \approx 11^\circ \)

dipole : quadrupole \( \approx 1 : 0.14 \) (at CMB)
Spatial structure of geomagnetic field

$B_r$ at surface 1990

$B_r$ at CMB 1990
Secular variation

$B_r$ at CMB 1890

$B_r$ at CMB 1990

Westward drift $0.18^\circ$/yr

$u \approx 0.3$ mm/sec
Secular variation continued

SINT-800 VADM (Guyodo and Valet 1999)

NGP (Ohno and Hamano 1992)
Polarity reversals

Introduction
Basic electrodynamics
Kinematic turbulent dynamos
MHD dynamos
Geodynamo simulations

Geomagnetic field
Dynamo hypothesis
Homopolar dynamo

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Geomagnetic Dynamo Theory
Dynamo hypothesis

- **Larmor (1919):** Magnetic field of Earth and Sun maintained by self-excited dynamo

- **Dynamo:** $u \times B \sim j \sim B \sim u$
  
  Faraday       Ampere       Lorentz

  motion of an electrical conductor in an ‘inducing’ magnetic field
  $\sim$ induction of electric current

- **Self-excited dynamo:** inducing magnetic field created by the electric current (Siemens 1867)

- **Example:** homopolar dynamo

- **Homogeneous dynamo** (no wires in Earth core or solar convection zone)
  $\sim$ complex motion necessary

- **Kinematic** ($u$ prescribed, linear)

- **Dynamic** ($u$ determined by forces, including Lorentz force, non-linear)
Homopolar dynamo

electromotive force $\mathbf{u} \times \mathbf{B}$ $\sim$ electric current through wire loop
$\sim$ induced magnetic field reinforces applied magnetic field

self-excitation if rotation $\Omega > 2\pi R/M$ is maintained
where $R$ resistance, $M$ inductance
Pre-Maxwell theory

Maxwell equations: cgs system, vacuum, $B = H, D = E$

\[ c \nabla \times B = 4\pi j + \frac{\partial E}{\partial t}, \quad c \nabla \times E = -\frac{\partial B}{\partial t}, \quad \nabla \cdot B = 0, \quad \nabla \cdot E = 4\pi \lambda \]

Basic assumptions of MHD:

- $u \ll c$: system stationary on light travel time, no em waves
- High electrical conductivity: $E$ determined by $\partial B/\partial t$, not by charges $\lambda$

\[ \frac{E}{L} \approx \frac{B}{T} \approx \frac{E}{B} \approx \frac{L}{c T} \approx \frac{u}{c} \ll 1, \quad E \text{ plays minor role: } \frac{e_{el}}{e_m} \approx \frac{E^2}{B^2} \ll 1 \]

\[ \frac{\partial E/\partial t}{c \nabla \times B} \approx \frac{E/T}{cB/L} \approx \frac{E}{B} \frac{u}{c} \approx \frac{u^2}{c^2} \ll 1, \quad \text{displacement current negligible} \]

Pre-Maxwell equations:

\[ c \nabla \times B = 4\pi j, \quad c \nabla \times E = -\frac{\partial B}{\partial t}, \quad \nabla \cdot B = 0 \]
Pre-Maxwell theory continued

Pre-Maxwell equations Galilei-covariant:

\[ E' = E + \frac{1}{c} u \times B, \quad B' = B, \quad j' = j \]

Relation between \( j \) and \( E \) by Galilei-covariant Ohm’s law:

\[ j' = \sigma E' \]

in resting frame of reference, \( \sigma \) electrical conductivity

\[ j = \sigma \left( E + \frac{1}{c} u \times B \right) \]

Magnetohydrokinematics:

\[ c \nabla \times B = 4\pi j \]
\[ c \nabla \times E = -\frac{\partial B}{\partial t} \]
\[ \nabla \cdot B = 0 \]
\[ j = \sigma \left( E + \frac{1}{c} u \times B \right) \]

Magnetohydrodynamics:

additionally

Equation of motion
Equation of continuity
Equation of state
Energy equation
Induction equation

Evolution of magnetic field

\[
\frac{\partial B}{\partial t} = -c \nabla \times E = -c \nabla \times \left( \frac{j}{\sigma} - \frac{1}{c} \mathbf{u} \times \mathbf{B} \right) = -c \nabla \times \left( \frac{c}{4\pi\sigma} \nabla \times \mathbf{B} - \frac{1}{c} \mathbf{u} \times \mathbf{B} \right)
\]

\[
= \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times \left( \frac{c^2}{4\pi\sigma} \nabla \times \mathbf{B} \right) = \nabla \times (\mathbf{u} \times \mathbf{B}) - \eta \nabla \times \nabla \times \mathbf{B}
\]

with \( \eta = \frac{c^2}{4\pi\sigma} = \text{const magnetic diffusivity} \)

induction, diffusion

\[
\nabla \times (\mathbf{u} \times \mathbf{B}) = -\mathbf{B} \nabla \cdot \mathbf{u} + (\mathbf{B} \cdot \nabla) \mathbf{u} - (\mathbf{u} \cdot \nabla) \mathbf{B} \quad \text{[} + \mathbf{u} \nabla \cdot \mathbf{B} = 0 \text{]}
\]

expansion/contraction, shear/stretching, advection

\( \nabla \cdot \mathbf{B} = 0 \) as initial condition, conserved
Alfven’s theorem

Ideal conductor $\eta = 0$:

$$\frac{\partial B}{\partial t} = \nabla \times (u \times B)$$

Magnetic flux through floating surface is constant:

$$\frac{d}{dt} \int_F B \cdot dF = 0$$

(Alfvén 1942)
Alfven’s theorem continued

Frozen-in field lines: impression that magnetic field follows flow, but \( cE = -u \times B \) and \( c \nabla \times E = -\partial B / \partial t \)

\[
\frac{\partial B}{\partial t} = \nabla \times (u \times B) = -B \nabla \cdot u + (B \cdot \nabla)u - (u \cdot \nabla)B
\]

(i) star contraction: \( \overline{B} \sim R^{-2}, \overline{\rho} \sim R^{-3} \sim \overline{B} \sim \overline{\rho}^{2/3} \)

Sun \( \sim \) white dwarf \( \sim \) neutron star: \( \rho [\text{g cm}^{-3}] \): \( 1 \sim 10^6 \sim 10^{15} \)

(ii) stretching of flux tube: \( \overline{B}d^2 = \text{const}, \overline{ld}^2 = \text{const} \sim B \sim l \)

(iii) shear, differential rotation
Differential rotation

\[ \frac{\partial B_\phi}{\partial t} = r \sin \theta \nabla \Omega \cdot B_p \]
Magnetic Reynolds number

Dimensionless variables: length $L$, velocity $u_0$, time $L/u_0$

$$\frac{\partial B}{\partial t} = \nabla \times (u \times B) - R_m^{-1} \nabla \times \nabla \times B \quad \text{with} \quad R_m = \frac{u_0 L}{\eta}$$

as combined parameter

laboratorium: $R_m \ll 1$, cosmos: $R_m \gg 1$

induction for $R_m \gg 1$, diffusion for $R_m \ll 1$, e.g. for small $L$

example: flux expulsion from closed velocity fields
Flux expulsion

(Weiss 1966)
Poloidal and toroidal magnetic fields

Spherical coordinates \((r, \vartheta, \varphi)\)

**Axisymmetric fields:** \(\partial / \partial \varphi = 0\)

\[
\mathbf{B}(r, \vartheta) = (B_r, B_\vartheta, B_\varphi)
\]

\[
\nabla \cdot \mathbf{B} = 0 \Leftrightarrow \frac{1}{r^2} \frac{\partial r^2 B_r}{\partial r} + \frac{1}{r \sin \vartheta} \frac{\partial \sin \vartheta B_\vartheta}{\partial \vartheta} + \frac{1}{r \sin \vartheta} \frac{\partial B_\varphi}{\partial \varphi} = 0
\]

\[
\mathbf{B} = \mathbf{B}_p + \mathbf{B}_t \text{ poloidal and toroidal magnetic field}
\]

\[
\mathbf{B}_t = (0, 0, B_\varphi) \text{ satisfies } \nabla \cdot \mathbf{B}_t = 0
\]

\[
\mathbf{B}_p = (B_r, B_\vartheta, 0) = \nabla \times \mathbf{A} \text{ with } \mathbf{A} = (0, 0, A_\varphi) \text{ satisfies } \nabla \cdot \mathbf{B}_p = 0
\]

\[
\mathbf{B}_p = \frac{1}{r \sin \vartheta} \left( \frac{\partial r \sin \vartheta A_\varphi}{\partial \vartheta}, -\frac{\partial r \sin \vartheta A_\varphi}{\partial r}, 0 \right)
\]

axisymmetric magnetic field determined by the two scalars: \(r \sin \vartheta A_\varphi\) and \(B_\varphi\)
Poloidal and toroidal magnetic fields continued

**Axisymmetric fields:**

\[
j_t = \frac{c}{4\pi} \nabla \times B_p, \quad j_p = \frac{c}{4\pi} \nabla \times B_t
\]

\[r \sin \vartheta A_\varphi = \text{const} : \quad \text{field lines of poloidal field in meridional plane}
\]

\[\text{field lines of } B_t \text{ are circles around symmetry axis}
\]

**Non-axisymmetric fields:**

\[
B = B_p + B_t = \nabla \times \nabla \times (Pr) + \nabla \times (Tr) = -\nabla \times (r \times \nabla P) - r \times \nabla T
\]

\[r = (r, 0, 0), \quad P(r, \vartheta, \varphi) \quad \text{and} \quad T(r, \vartheta, \varphi) \quad \text{defining scalars}
\]

\[\nabla \cdot B = 0, \quad j_t = \frac{c}{4\pi} \nabla \times B_p, \quad j_p = \frac{c}{4\pi} \nabla \times B_t
\]

\[r \cdot B_t = 0 \quad \text{field lines of the toroidal field lie on spheres, no } r \text{ component}
\]

\[B_p \text{ has in general all three components}
\]
Antidynamo theorems

**Cowling’s theorem** (Cowling 1934)

Axisymmetric magnetic fields can not be maintained by a dynamo.

**Toroidal velocity theorem** (Elsasser 1947, Bullard & Gellman 1954)

A toroidal motion in a spherical conductor can not maintain a magnetic field by dynamo action.

**Toroidal field theorem / Invisible dynamo theorem** (Kaiser et al. 1994)

A purely toroidal magnetic field can not be maintained by a dynamo.
Parker’s helical convection

velocity \( u \)

vorticity \( \omega = \nabla \times u \)

helicity \( H = u \cdot \omega \)

\[(Parker \ 1955)\]
Mean-field theory

Statistical consideration of turbulent helical convection on mean magnetic field (Steenbeck, Krause and Rädler 1966)

\[
\frac{\partial B}{\partial t} = \nabla \times (u \times B) - \eta \nabla \times \nabla \times B
\]

\[u = \bar{u} + u', \quad B = \bar{B} + B'\] Reynolds rules for averages

\[
\frac{\partial \bar{B}}{\partial t} = \nabla \times (\bar{u} \times \bar{B} + E) - \eta \nabla \times \nabla \times \bar{B}
\]

\[E = u' \times B'\] mean electromotive force

\[
\frac{\partial B'}{\partial t} = \nabla \times (\bar{u} \times B' + u' \times \bar{B} + G) - \eta \nabla \times \nabla \times B'
\]

\[G = u' \times B' - \bar{u}' \times \bar{B}'\] usually neglected, FOSA = SOCA

\[B'\] linear, homogeneous functional of \(\bar{B}\)

approximation of scale separation: \(B'\) depends on \(\bar{B}\) only in small surrounding

Taylor expansion: \((u' \times B')_i = \alpha_{ij} \bar{B}_j + \beta_{ijk} \partial \bar{B}_k / \partial x_j + \ldots\)
(\mathbf{u}' \times \mathbf{B}')_j = \alpha_{ij} \mathbf{B}_j + \beta_{ijk} \partial \mathbf{B}_k / \partial x_j + \ldots

\alpha_{ij} \text{ and } \beta_{ijk} \text{ depend on } \mathbf{u}' \text{ and are, in general, tensors}

\text{homogeneous, isotropic } \mathbf{u}' : \alpha_{ij} = \alpha \delta_{ij}, \beta_{ijk} = -\beta \varepsilon_{ijk} \text{ then}

\mathbf{u}' \times \mathbf{B}' = \alpha \mathbf{B} - \beta \nabla \times \mathbf{B}

\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} + \alpha \mathbf{B}) - (\eta + \beta) \nabla \times \nabla \times \mathbf{B}

Two effects :

(1) $\alpha$ – effect: mean current parallel mean magnetic field

$$\alpha = -\frac{1}{3} \mathbf{u}' \cdot \nabla \times \mathbf{u}' \tau^* = -\frac{1}{3} \mathbf{H} \tau^* \quad \text{where } \mathbf{H} \text{ helicity}, \tau^* \text{ correlation time}$$

(2) turbulent diffusivity: $\beta = \frac{1}{3} \mathbf{u}'^2 \tau^* \gg \eta, \quad \eta + \beta = \beta = \eta \tau$
Mean-field dynamos

Dynamo equation:

\[
\frac{\partial \overline{B}}{\partial t} = \nabla \times (\overline{u} \times \overline{B} + \alpha \overline{B} - \eta_T \nabla \times \overline{B})
\]

- spherical coordinates, axisymmetry
- \( \overline{u} = (0, 0, \Omega(r, \vartheta) r \sin \vartheta) \)
- \( \overline{B} = (0, 0, B(r, \vartheta, t)) + \nabla \times (0, 0, A(r, \vartheta, t)) \)

\[
\frac{\partial B}{\partial t} = r \sin \vartheta (\nabla \times A) \cdot \nabla \Omega - \alpha \nabla^2 A + \eta_T \nabla^2 B
\]

\[
\frac{\partial A}{\partial t} = \alpha B + \eta_T \nabla^2 A \quad \text{with} \quad \nabla^2 = \nabla^2 - (r \sin \vartheta)^{-2}
\]

rigid rotation has no effect

no dynamo if \( \alpha = 0 \)

\[
\begin{align*}
\frac{\alpha-\text{term}}{\nabla \Omega-\text{term}} & \approx \frac{\alpha_0}{|\nabla \Omega| L^2} \\
& \begin{cases} 
\gg 1 & \alpha^2-\text{dynamo with dynamo number} \ R_\alpha^2 \\
\sim 1 & \alpha^2 \Omega-\text{dynamo} \\
\ll 1 & \alpha \Omega-\text{dynamo with dynamo number} \ R_\alpha R_\Omega
\end{cases}
\end{align*}
\]
Sketch of an $\alpha\Omega$ dynamo

\[ \partial \Omega / \partial r < 0 \]

\[ \alpha \sim \cos \theta \]

Poloidal field by differential rotation;
Electric currents by $\alpha$-effect

Periodically alternating field, here antisymmetric with respect to equator
Sketch of an $\alpha^2$ dynamo

stationary field, here antisymmetric with respect to equator
**MHD equations of rotating fluids in non-dimensional form**

**Navier-Stokes equation including Coriolis and Lorentz forces**

\[
E \left( \frac{\partial u}{\partial t} + u \cdot \nabla u - \nabla^2 u \right) + 2 \hat{z} \times u + \nabla \Pi = \frac{Ra \ E \ r}{Pr \ r_0} T + \frac{1}{Pm} (\nabla \times B) \times B
\]

- Inertia
- Viscosity
- Coriolis
- Buoyancy
- Lorentz

**Induction equation**

\[
\frac{\partial B}{\partial t} = \nabla \times (u \times B) - \frac{1}{Pm} \nabla \times \nabla \times B
\]

- Induction
- Diffusion

**Energy equation**

\[
\frac{\partial T}{\partial t} + u \cdot \nabla T = \frac{1}{Pr} \nabla^2 T + Q
\]

**Incompressibility and divergence-free magnetic field**

\[
\nabla \cdot u = 0, \quad \nabla \cdot B = 0
\]
### Control parameters (Input)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Force balance</th>
<th>Model value</th>
<th>Earth value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rayleigh number</td>
<td>$Ra = \alpha g_0 \Delta T d / \nu k$</td>
<td>buoyancy/diffusivity</td>
<td>$1 - 50 Ra_{\text{crit}}$</td>
<td>$\gg Ra_{\text{crit}}$</td>
</tr>
<tr>
<td>Ekman number</td>
<td>$E = \nu / \Omega d^2$</td>
<td>viscosity/Coriolis</td>
<td>$10^{-6} - 10^{-4}$</td>
<td>$10^{-14}$</td>
</tr>
<tr>
<td>Prandtl number</td>
<td>$Pr = \nu / \kappa$</td>
<td>viscosity/thermal diff.</td>
<td>$2 \cdot 10^{-2} - 10^3$</td>
<td>$0.1 - 1$</td>
</tr>
<tr>
<td>Magnetic Prandtl</td>
<td>$Pm = \nu / \eta$</td>
<td>viscosity/magn. diff.</td>
<td>$10^{-1} - 10^3$</td>
<td>$10^{-6} - 10^{-5}$</td>
</tr>
</tbody>
</table>

### Diagnostic parameters (Output)

<table>
<thead>
<tr>
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<th>Definition</th>
<th>Force balance</th>
<th>Model value</th>
<th>Earth value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elsasser number</td>
<td>$\Lambda = B^2 / \mu \rho \eta \Omega$</td>
<td>Lorentz/Coriolis</td>
<td>$0.1 - 100$</td>
<td>$0.1 - 10$</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>$Re = u d / \nu$</td>
<td>inertia/viscosity</td>
<td>$&lt; 500$</td>
<td>$10^8 - 10^9$</td>
</tr>
<tr>
<td>Magnetic Reynolds</td>
<td>$Rm = u d / \eta$</td>
<td>induction/magn. diff.</td>
<td>$50 - 10^3$</td>
<td>$10^2 - 10^3$</td>
</tr>
<tr>
<td>Rossby number</td>
<td>$Ro = u / \Omega d$</td>
<td>inertia/Coriolis</td>
<td>$3 \cdot 10^{-4} - 10^{-2}$</td>
<td>$10^{-7} - 10^{-6}$</td>
</tr>
</tbody>
</table>

Earth core values: $d \approx 2 \cdot 10^5$ m, $u \approx 2 \cdot 10^{-4}$ m s$^{-1}$, $\nu \approx 10^{-6}$ m$^2$s$^{-1}$
**Proudman-Taylor theorem**

Non-magnetic hydrodynamics in rapidly rotating system

\[ E \ll 1, \quad Ro \ll 1: \quad \text{viscosity and inertia small} \]

balance between Coriolis force and pressure gradient

\[ \nabla p = 2\rho \Omega \times u, \quad \nabla \times (\Omega \cdot \nabla)u = 0 \]

\[ \frac{\partial u}{\partial z} = 0 \quad \text{motion independent along axis of rotation, geostrophic motion} \]

(Proudman 1916, Taylor 1921)

**Ekman layer:**

At fixed boundary \( u = 0 \), violation of P.-T. theorem necessary for motion close to boundary allow viscous stresses \( \nu \nabla^2 u \) for gradients of \( u \) in \( z \)-direction

Ekman layer of thickness \( \delta_l \sim E^{1/2}L \sim 0.2 \, \text{m for Earth core} \)
inside tangent cylinder: $g \parallel \Omega$:
Coriolis force opposes convection
outside tangent cylinder:
P.-T. theorem leads to columnar convection cells
$\exp(\text{i} m \varphi - \omega t)$ dependence at onset of convection,
$2m$ columns which drift in $\varphi$-direction
inclined outer boundary violates Proudman-Taylor theorem
$\sim$ columns close to tangent cylinder around inner core
inclined boundaries, Ekman pumping and inhomogeneous thermal buoyancy
lead to secondary circulation along convection columns:
poleward in columns with $\omega_z < 0$, equatorward in columns with $\omega_z > 0$
$\sim$ negative helicity north of the equator and positive one south
Convection in rotating spherical shell continued

\[ \omega_z > 0 \quad \text{and} \quad \omega_z < 0 \]

cyclones / anticyclones

\[ H < 0 \]

\[ H > 0 \]
Taylor’s constraint

\[ 2\rho \Omega \times u = -\nabla \rho + \rho g + (\nabla \times B) \times B / 4\pi \quad \text{magnetostrophic regime} \]

\[ \nabla \cdot u = 0, \quad \rho = \text{const}; \quad \Omega = \omega_0 e_z \]

Consider \( \varphi \)-component and integrate over cylindrical surface \( C(s) \)

\[ \frac{\partial p}{\partial \varphi} = 0 \text{ after integration over } \varphi, \quad g \text{ in meridional plane} \]

\[ 2\rho \Omega \int_{C(s)} u \cdot ds = \frac{1}{4\pi} \int_{C(s)} \left( (\nabla \times B) \times B \right)_\varphi ds = 0 \]

\[ \int_{C(s)} \left( (\nabla \times B) \times B \right)_\varphi ds = 0 \quad \text{(Taylor 1963)} \]

net torque by Lorentz force on any cylinder \( \parallel \Omega \) vanishes

\( B \) not necessarily small, but positive and negative parts of the integrand cancelling each other out

violation by viscosity in Ekman boundary layers \( \sim \) torsional oscillations around Taylor state
Benchmark dynamo

\[ Ra = 10^5 = 1.8 \, Ra_{\text{crit}}, \quad E = 10^{-3}, \quad Pr = 1, \quad Pm = 5 \]

radial magnetic field at outer radius
radial velocity field at \( r = 0.83r_0 \)
axisymmetric magnetic field
axisymmetric flow

(Christensen et al. 2001)
Conversion of toroidal field into poloidal field

(Olson et al. 1999)
Generation of toroidal field from poloidal field

(Olson et al. 1999)
Field line bundle in the benchmark dynamo

(cf Aubert)
Introducing the strongly driven dynamo model

$$Ra = 1.2 \times 10^8 = 42 \, Ra_{\text{crit}}$$,
$$E = 3 \times 10^{-5}$$,
$$Pr = 1$$,
$$Pm = 2.5$$

- Radial magnetic field at outer radius
- Radial velocity field at $r = 0.93r_0$
- Axisymmetric magnetic field
- Axisymmetric flow

(Christensen et al. 2001)
Comparison of the radial magnetic field at the CMB

GU Denver model (Jackson et al. 2000)

Spectrally filtered simulation at
\[ E = 3 \cdot 10^{-5}, \ Ra = 42 \ Ra_{\text{crit}}, \ Pm = 1, \ Pr = 1 \]

Full numerical simulation

Reversing dynamo at
\[ E = 3 \cdot 10^{-4}, \ Ra = 26 \ Ra_{\text{crit}}, \ Pm = 3, \ Pr = 1 \]
Dynamical Magnetic Field Line Imaging / Movie 2

Figure 5. Snapshots from (a): DMFI movie 1 of model C and (b): movie 2 of model T. Left-hand panels: top view. Right-hand panels: side view. The inner (ICB) and outer (CMB) boundaries of the model are colour-coded with the radial magnetic field (a red patch denotes outwards oriented field). In addition, the outer boundary is made selectively transparent, with a transparency level that is inversely proportional to the local radial magnetic field. Field lines are also colour-coded in order to indicate \( e_z \)-parallel (red) and antiparallel (blue) direction. The radial magnetic field as seen from the Earth’s surface is represented in the upper-right inserts, in order to keep track of the current orientation and strength of the large-scale magnetic dipole. Colour maps for (a): ICB field from \(-0.12 \) to \( 0.12 \), in units of \( (\rho\mu)_{1/2}/\Omega_1 D \), CMB field from \(-0.03 \) to \( 0.03 \), Earth’s surface field from \(-210^{-4} \) to \( 210^{-4} \). For (b): ICB field from \(-0.72 \) to \( 0.72 \), CMB field from \(-0.36 \) to \( 0.36 \), Earth’s surface field from \(-1.8 \times 10^{-3} \) to \( 1.8 \times 10^{-3} \).

Vortices into columns elongated along the \( e_z \) axis of rotation, due to the Proudman–Taylor constraint. The sparse character of the magnetic energy distribution results from the tendency of field lines to cluster at the edges of flow vortices due to magnetic field expulsion (Weiss 1966; Galloway & Weiss 1981). Since magnetic field lines correlate well with the flow structures in our models, we will subsequently visualize the magnetic field structure alone. The supporting movies of this paper (see Fig. 1 for time window and Figs 5–9 for extracts) present DMFI field lines, together with radial magnetic flux patches at the inner boundary (which we will refer to as ICB) and the selectively transparent outer boundary (CMB). We will first introduce the concept of a magnetic vortex, which is defined as a field line structure resulting from the interaction with a flow vortex. By providing illustrations of magnetic cyclones and anticyclones, DMFI provides a dynamic, field-line based visual confirmation of previously published dynamo mechanisms (Kageyama & Sato 1997; Olson et al. 1999; Sakuraba & Kono 1999; Ishihara & Kida 2002), and allows the extension of such descriptions to time-dependent, spatially complex dynamo regimes.

3.1.1 Magnetic cyclones

A strong axial flow cyclone (red isosurface in Fig. 4) winds and stretches field lines to form a magnetic cyclone. Fig. 6 relates DMFI visualizations of magnetic cyclones, as displayed in Figs 4 and 5, with a schematic view inspired by Olson et al. (1999). A magnetic cyclone can be identified by the anticlockwise motion of field lines clustered close to the equator, moving jointly with fairly stable high-latitude CMB flux patches concentrated above and below the centre of the field line cluster. Model C (movie 1, Fig. 5a) exhibits very large-scale magnetic cyclones (times 4.3617, 4.3811), which suggest an axial vorticity distribution biased towards flow cyclones. Inside these vortices, the uneven distribution of buoyancy along \( e_z \) creates a thermal wind secondary circulation (Olson et al. 1999), which is represented in red on Fig. 6. This secondary circulation concentrates CMB flux at high latitudes, giving rise to relatively long-lived (several vortex turnovers) flux patches similar to those found in geomagnetic field models. Simultaneously, close to the equatorial plane, the secondary circulation concentrates field lines into bundles and also pushes them towards the outer boundary, where...
Reversals

500 years before midpoint  
midpoint  
500 years after midpoint

(Glatzmaier and Roberts 1995)
Reversals continued

Figure 12. The ... constrained and not observed with all field regularizations

(Aubert et al. 2008)
The geodynamo as a bistable oscillator

(Hoyng, Ossendrijver & Schmitt 2001)
Introduction

Basic electrodynamics

Kinematic turbulent dynamos

MHD dynamos

Geodynamo simulations

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