Structures, waves and turbulences in the solar wind

- Solar wind and heliospheric magnetic field
- The heliosphere, structure and dynamics
- Fluctuations: scales and parameters
- Magnetoacoustic and Alfvénic fluctuations
- Turbulence spectra and radial evolution
- Ideal MHD invariants and dissipation
- Cross-helicity, anisotropy, compressibility
- Scaling and intermittency

The Sun’s open magnetic field lines

MHD model field during Ulysses crossing of ecliptic plane in early 1995

Mikic & Linker, 1999
Length scales in the solar wind

**Macrostructure - fluid scales**
- Heliocentric distance: $r = 150 \text{ Gm (1AU)}$
- Solar radius: $R_s = 696000 \text{ km (215 R_s)}$
- Alfvén waves: $\lambda = 30 - 100 \text{ Mm}$

**Microstructure - kinetic scales**
- Coulomb free path: $l \sim 0.1 - 10 \text{ AU}$
- Ion inertial length: $V_\parallel/\Omega_p (c/\omega_p) \sim 100 \text{ km}$
- Ion gyroradius: $r_L \sim 50 \text{ km}$
- Debye length: $\lambda_D \sim 10 \text{ m}$
- Helios spacecraft: $d \sim 3 \text{ m}$

Microscales vary with solar distance!

Solar wind stream structure and heliospheric current sheet

![Diagram of solar wind stream structure and heliospheric current sheet](Alfven, 1977)

Parker, 1963
**Stream interaction region**

Dynamic processes in interplanetary space

- Wave amplitude steepening ($n \sim r^2$)
- Compression and rarefaction
- Velocity shear
- Nonlinearity by advection ($V \cdot \nabla V$)
- Shock formation (co-rotating)

Schwenn, 1990

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**Spatial and temporal scales**

<table>
<thead>
<tr>
<th>Phenomenon</th>
<th>Frequency ($s^{-1}$)</th>
<th>Period (day)</th>
<th>Speed (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar rotation:</td>
<td>$4.6 \times 10^{-7}$</td>
<td>25</td>
<td>2</td>
</tr>
<tr>
<td>Solar wind expansion:</td>
<td>$5 - 2 \times 10^{-6}$</td>
<td>2 - 6</td>
<td>800 - 250</td>
</tr>
<tr>
<td>Alfvén waves:</td>
<td>$3 \times 10^{-4}$</td>
<td>1/24</td>
<td>50 (1AU)</td>
</tr>
<tr>
<td>Ion-cyclotron waves:</td>
<td>1 - 0.1</td>
<td>1 (s)</td>
<td>($V_A$) 50</td>
</tr>
</tbody>
</table>

Turbulent cascade: generation + transport

→ inertial range → kinetic range + dissipation
Phase velocities of MHD modes

\[ \omega^2 - \omega^2 (k c_{ms})^2 + (k c_s)^2 (k \cdot \mathbf{V}_A)^2 = 0 \]

\[ \omega = k \cdot \mathbf{V}_A \]

Weak turbulence, superposition of magnetohydrodynamic waves

- Magnetosonic waves compressible
  - parallel slow and fast
  - perpendicular fast

\[ C_{ms} = (c_s^2 + V_A^2)^{-1/2} \]

- Alfvén wave incompressible parallel and oblique

\[ V_A = B/(4 \pi \rho)^{1/2} \]

Broad band in \( k \) and random phases.
Typical day in April 1995 of Ulysses plasma and field observations in the polar (42° north) heliosphere at 1.4 AU

- Sharp changes in field direction
- Large Component variations
- Weak compressive fluctuations

Horbury & Tsurutani, 2001

Alfvénic fluctuations (Helios)

\[ \delta V = \pm \delta V_A \]

Neubauer et al., 1977
Alfvénic fluctuations (Ulysses)

Elsässer variables:
\[ \mathbf{Z}_\pm = \mathbf{V} \pm \mathbf{V}_A \]
Turbulence energy:
\[ e_\pm = \frac{1}{2} (\mathbf{Z}_\pm)^2 \]
Cross helicity:
\[ \sigma_c = (e_+ - e_-)/(e_+ + e_-) \]

Ulysses observed many such waves (4-5 per hour) in fast wind over the poles:
- Arc polarized waves
- Phase-steepened

Rotational discontinuity:
\[ \Delta \mathbf{V} = \pm \Delta \mathbf{V}_A \]
Finite jumps in velocities over gyrokinetic scales
Arc-polarized Alfvén waves

Slowly rotating Alfvén wave lasts about 15 minutes

Rotational discontinuity RD lasts only 3 minutes

Arc-polarized Alfvén waves

Elsässer variables: $Z^\pm = V^\pm V_A$
Turbulence energy: $e^\pm = 1/2 (Z^\pm)^2$
Elsässer ratio: $r_e = e^-/e^+$

Radial variation of $e^\pm(r)$; wave amplitude at 1-h period is not sufficient to drive fast wind!

Alfvén waves in polar solar wind

Average values over 0.1 AU wide intervals of hourly variances of $Z^\pm$
Alfvén waves and solar wind streams in the ecliptic plane

- High Alfvén wave flux in fast streams
- Developed isotropic turbulence in slow streams

Tu et al., GRL, 17, 283, 1990

Compressive fluctuations in the solar wind

Marsch and Tu, JGR, 95, 8211, 1990

Kolmogorov-type turbulence
### Solar wind turbulence

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coronal Hole (open)</th>
<th>Current sheet (closed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alfvén waves:</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Density fluctuations:</td>
<td>weak (&lt;3%)</td>
<td>intense (&gt;10%)</td>
</tr>
<tr>
<td>Magnetic/kinetic turbulent energy:</td>
<td>≈ 1</td>
<td>&gt; 1</td>
</tr>
<tr>
<td>Spectral slope:</td>
<td>flat (-1)</td>
<td>steep (-5/3)</td>
</tr>
<tr>
<td>Wind speed:</td>
<td>high</td>
<td>low</td>
</tr>
<tr>
<td>$T_p$ ($T_e$):</td>
<td>high (low)</td>
<td>low (high)</td>
</tr>
<tr>
<td>Wave heating:</td>
<td>strong</td>
<td>weak</td>
</tr>
</tbody>
</table>

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**Magnetic field power spectrum**

- Power laws with index of about -1, -5/3 and -3
- Abrupt decline at $f_c$ indicates cyclotron absorption
- Steep spectrum at high frequencies above 2 Hz is mainly due to whistler waves

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Denskat et al., JGR **54**, 60, 1983
Integral invariants of ideal MHD

\[ E = \frac{1}{2} \int d^3x \left( V^2 + V_A^2 \right) \quad \text{Energy} \]

\[ H_c = \int d^3x \left( \mathbf{V} \cdot \mathbf{V}_A \right) \quad \text{Helicity} \]

\[ H_m = \int d^3x \left( \mathbf{A} \cdot \mathbf{B} \right) \quad \text{Magnetic helicity} \]

\[ \mathbf{B} = \nabla \times \mathbf{A} \]

Elsässer variables: \( \mathbf{Z}^\pm = \mathbf{V} \pm \mathbf{V}_A \)

\[ E^\pm = \frac{1}{2} \int d^3x \left( \mathbf{Z}^\pm \right)^2 = \int d^3x \ e^\pm(x) \]

Correlation length of turbulence

Correlation function:

\[ C_{AA}(\mathbf{x}, t, \mathbf{x}', t') = \langle A(\mathbf{x}, t) A(\mathbf{x}', t') \rangle \]

for any field \( A(\mathbf{x}, t) \).

If stationarity and homogeneity, then

\[ \tau = t - t', \quad \mathbf{r} = \mathbf{x} - \mathbf{x}' \]

\[ C_{AA}(\mathbf{x}, t, \mathbf{x}', t') = C_{AA}(\mathbf{r}, \tau) \]
Turbulence in the heliosphere

Questions and problems:

- Nature and origin of the fluctuations
- Distribution and spectral transfer of turbulent energy
- Spatial evolution with heliocentric distance
- Intermittency and microphysics of dissipation

Alfvénic correlations: Alfvénicity (cross helicity)

$$\sigma_c = \frac{(e^+ - e^-)}{(e^+ + e^-)} = \frac{2<\delta V \cdot \delta V_A>}{<\delta V^2 + (\delta V_A)^2>}$$

Magnetic versus kinetic energy: Alfvén ratio

$$r_A = \frac{e_V}{e_B} = \frac{<\delta V^2>}{<\delta V_A^2>}$$

Evolution of cross helicity

$$\sigma_c = \frac{2<\delta V \cdot \delta V_A>}{<\delta V^2 + (\delta V_A)^2>} = \frac{(e^+ - e^-)}{(e^+ + e^-)}$$

Roberts et al., J. Geophys. Res. 92, 12023, 1987

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Roberts et al., J. Geophys. Res. 92, 12023, 1987
**Alfvén ratio**

\[ r_A(k) = \frac{e_v(k)}{e_B(k)} \]

\[ e_A(k) = \frac{1}{2} \int d^3k \ e^{-i \mathbf{k} \cdot \mathbf{r}} \langle \mathbf{A}(0) \cdot \mathbf{A}(\mathbf{r}) \rangle \]

Spectrum

Marsch and Tu, J. Geophys. Res., 95, 8211, 1990

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**Spectral indices and spatial evolution of turbulence**

- Spectra steepen!
- \( e^+ >> e^- \), Alfvén waves dominate!

Marsch and Tu, JGR, 95, 8211, 1990
Spectral evolution of Alfvénic fluctuations

- Steepening by cascading
- Ion heating by wave sweeping
- Dissipation by wave absorption

Tu and Marsch, J. Geophys. Res., 100, 12323, 1995

Spectral evolution and turbulent cascade: slope steepening
Power spectrum evolution

Turbulence spectrum:
\[ e^\pm(f) = \frac{1}{2} (\delta Z^\pm)^2 \sim (f/f_0)^{-\alpha} \]

Horbury et al., JGR 101, 405, 1996

Radial variation of spectral features

• Turbulence intensity declines with solar distance
• Wave amplitudes are consistent between Helios and Ulysses in fast streams from coronal holes
• Variation of spectral breakpoint (decreases) as measured by various S/C
• Slower radial evolution of spectra over the poles

Horbury & Tsurutani, 2001
Kolmogorov phenomenology for isotropic homogeneous turbulence

Energy cascade:
Turbulent energy (per unit mass density), \( e \approx (\delta Z)^2 \), at scale \( i \) is transported by a hierarchy of turbulent eddies of ever decreasing sizes to the dissipation range at scale \( i_d \).

- energy transfer rate: \( \varepsilon \sim (\delta Z)^2 / \tau \)
- turnover time: \( \tau \sim / (\delta Z) \)
- wavenumber: \( k \sim 1 / \)
- energy spectrum: \( E_k \sim (\delta Z)^2 \)

\[ \varepsilon \sim \delta Z / (\delta Z)^2 \sim E_k^{3/2} k^{5/2} \]

Scale invariance: \( \varepsilon = \varepsilon \) (dissipation rate) \( \rightarrow \) \( E_k \sim k^{-5/3} \)

Spectral properties of 3-D magnetohydrodynamic turbulence

Direct numerical simulation with a spectral code with 512\(^3\) modes
Compensated normalized spectrum shows Kolmogorov scaling and sheet-like dissipative structures

\[ E_k \sim \varepsilon^{2/3} k^{-5/3} \quad \text{Kolmogorov, 1941} \]
\[ E_k \sim (\varepsilon \nu_A)^{1/2} k^{-3/2} \quad \text{Kraichnan, 1965} \]

MHD turbulence dissipation through absorption of dispersive kinetic waves

- Viscous and Ohmic dissipation in collisionless plasma (coronal holes and fast solar wind) is hardly important
- Waves become dispersive (at high frequencies beyond MHD) in the multi-fluid or kinetic regime
- Turbulence dissipation involves absorption (or emission by instability) of kinetic plasma waves!
- Cascading and spectral transfer of wave and turbulence energy is not well understood in the dispersive dissipation domain!

Anisotropy and dimension

- Particle pitch-angle scattering is weaker than for isotropic MHD consistent with observations of ESPs and CRs
- Compressible fluctuations are described by 2-D MHD

Correlations: Alfvén waves and 2-D turbulence

Matthaeus et al., J. Geophys. Res., 95, 20673, 1990
**Structure function and scaling**

Voyager 2 near 8.5 AU

\[ S_p(\tau) = \langle |V(\tau)-V(0)|^p \rangle = \tau^{s(p)} \]

Scaling exponent \( s(p) \) of speed increments

\[ s(p) = 1 - \ln\left[ \frac{1}{3} + \frac{2}{3}p \right] \]

P-model of fractal cascade; \( P=1/2 \) no intermittency

Burlaga, JGR, 96, 5847, 1991

**Probability distribution functions**

Helios: fast SW, \( V_x \) radial component of flow velocity

Non-Gaussian statistics at small scales!

Marsch and Tu, Annales Geophys., 12, 1127, 1994
Radial evolution of intermittency

Helios, fast solar wind: $B_x$ radial component of magnetic field, $B_y$, $B_z$.

Flatness (Gaussian, 3):

$$\mathcal{F}(\tau) = \frac{\langle S^4 \rangle}{\langle S^2 \rangle^2}$$

Structure function:

$$S^P_T = \langle |V(t+\tau) - V(t)|^P \rangle$$

Summary

- Solar wind is an almost isotropic turbulent magnetofluid
- Alfvénic fluctuations dominate, with an admixture of weak compressive (magnetosonic) fluctuations
- Turbulence develops towards Kolmogorov spectra, but intermittency prevails at small (below hourly) scales
- Alfvén ratio, cross-helicity, anisotropy evolve radially, as does the average energy spectrum
- Origin of the fluctuations: coronal sources for Alfvén waves, compressive waves from pressure imbalances and stream interactions, cascading by velocity shear
- Structure functions and probability distribution reveal non-gaussian statistics